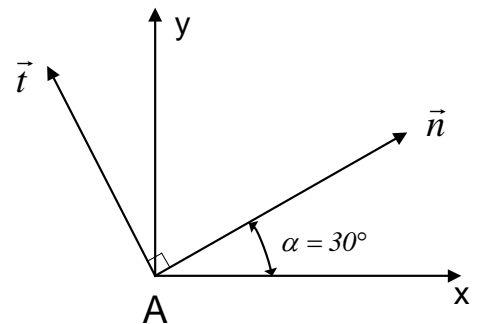


# OTPORNOST MATERIJALA I

1. KOLOKVIJ, 13.11.2008.

## GRUPA A

1. U točki „A“ napregnutog tijela izmjerene su relativne deformacije  $\varepsilon_n = 0,0008$   $\varepsilon_t = 0,0003$  i  $\varepsilon_y = 0,0006$ . Treba odrediti i skicirati smjer i veličinu glavnih napreznja, te vektor punog napreznja na ravnini određenu normalom  $y$ , ako je  $E = 2 \cdot 10^5 \text{ MPa}$  i  $\nu = 0,3$ .



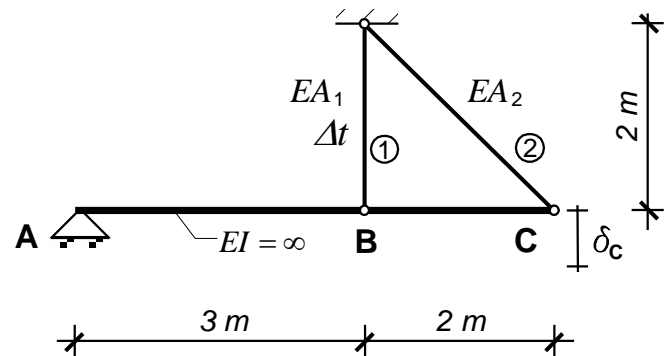
2. Za koliko stupnjeva mora narasti temperatura štapa „1“ da bi pomak  $\delta_c$  bio 2 mm. Treba odrediti i napreznja koja se pri tome pojavljuju u štapovima „1“ i „2“.

$$E = 2 \cdot 10^5 \text{ MPa}$$

$$A_1 = 140 \text{ mm}^2$$

$$A_2 = 100 \text{ mm}^2$$

$$\alpha_t = 1,2 \cdot 10^{-5} \text{ 1/K}$$



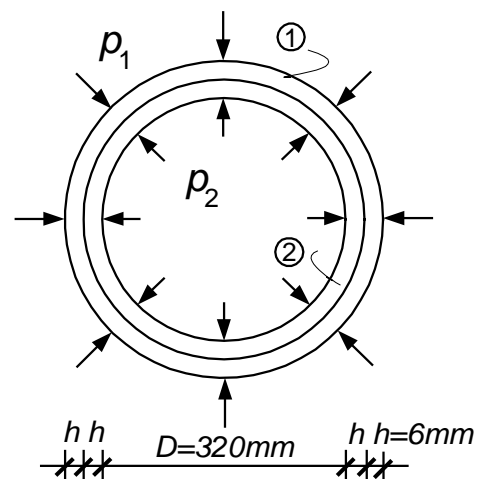
3. Cijev „1“ navučena je preko cijevi „2“ i međusobno se dodiruju. Treba odrediti napreznja u cijevima i promjenu promjera ako je zadano:

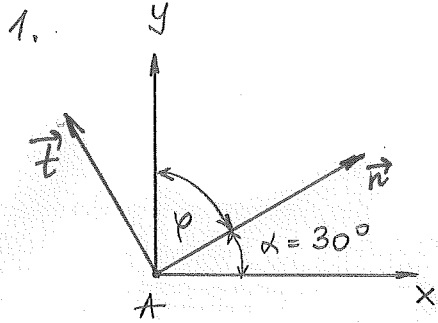
$$E_1 = 2,0 \cdot 10^5 \text{ MPa}$$

$$E_2 = 1,2 \cdot 10^5 \text{ MPa}$$

$$p_1 = 6 \text{ MPa}$$

$$p_2 = 2 \text{ MPa}$$





$$\varepsilon_n = 0,0008$$

$$\varepsilon_t = 0,0003$$

$$\varepsilon_y = 0,0006$$

$$E = 2 \cdot 10^5 \text{ MPa}$$

$$\nu = 0,3$$

$$\varepsilon_y = \varepsilon_n \cos^2 \varphi + \varepsilon_t \sin^2 \varphi + \varepsilon_{nt} \sin 2\varphi \quad \varphi = 90^\circ - 30^\circ = 60^\circ$$

$$0,0006 = 0,0008 \cos^2 60^\circ + 0,0003 \sin^2 60^\circ + \varepsilon_{nt} \sin 120^\circ$$

$$\varepsilon_{nt} = 0,0002$$

$$\varepsilon_x = \varepsilon_n \cos^2 \psi + \varepsilon_t \sin^2 \psi + \varepsilon_{nt} \sin 2\psi \quad \psi = -\alpha = -30^\circ$$

$$= 0,0008 \cos^2(-30^\circ) + 0,0003 \sin^2(-30^\circ) + 0,0002 \sin(-60^\circ)$$

$$\varepsilon_x = 0,0005$$

$$\varepsilon_n = \varepsilon_x \cos^2 \alpha + \varepsilon_y \sin^2 \alpha + \varepsilon_{xy} \sin 2\alpha$$

$$0,0008 = 0,0005 \cos^2 30^\circ + 0,0006 \sin^2 30^\circ + \varepsilon_{xy} \sin 60^\circ$$

$$\varepsilon_{xy} = 0,0003$$

$$\sigma_x = \frac{E}{1-\nu^2} (\varepsilon_x + \nu \varepsilon_y) = \frac{2 \cdot 10^5}{1-0,3^2} (0,0005 + 0,3 \cdot 0,0006) = 149,45 \text{ MPa}$$

$$\sigma_y = \frac{E}{1-\nu^2} (\varepsilon_y + \nu \varepsilon_x) = \frac{2 \cdot 10^5}{1-0,3^2} (0,0006 + 0,3 \cdot 0,0005) = 164,83 \text{ MPa}$$

$$\tau_{xy} = \frac{E}{1+\nu} \varepsilon_{xy} = \frac{2 \cdot 10^5}{1+0,3} 0,0003 = 46,15 \text{ MPa}$$

$$\rho_y = \sqrt{\sigma_y^2 + \tau_{xy}^2} = \sqrt{164,83^2 + 46,15^2} = 171,17 \text{ MPa}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \frac{1}{2} \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} = 157,14 \pm 46,79 \text{ MPa}$$

$$\sigma_1 = 203,93 \quad \sigma_2 = 110,35 \quad \sigma_x + \sigma_y = \sigma_1 + \sigma_2$$

$$314,28 = 314,28 \text{ kN}$$

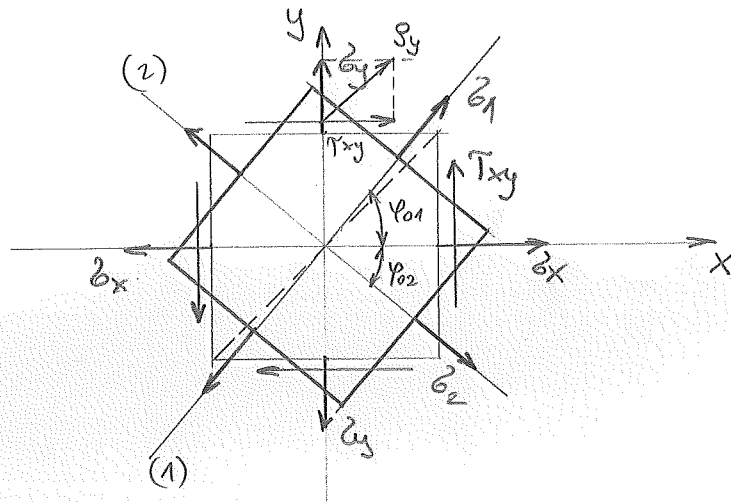
$$\operatorname{tg} \varphi_{oi} = \frac{\tau_{xy}}{\sigma_i - \sigma_y}$$

$$\varphi_{o1} = 49,73^\circ$$

$$\varphi_{o2} = -40,27^\circ$$

$$|\varphi_{o1}| + |\varphi_{o2}| = 90^\circ$$

$$90^\circ = 90^\circ$$



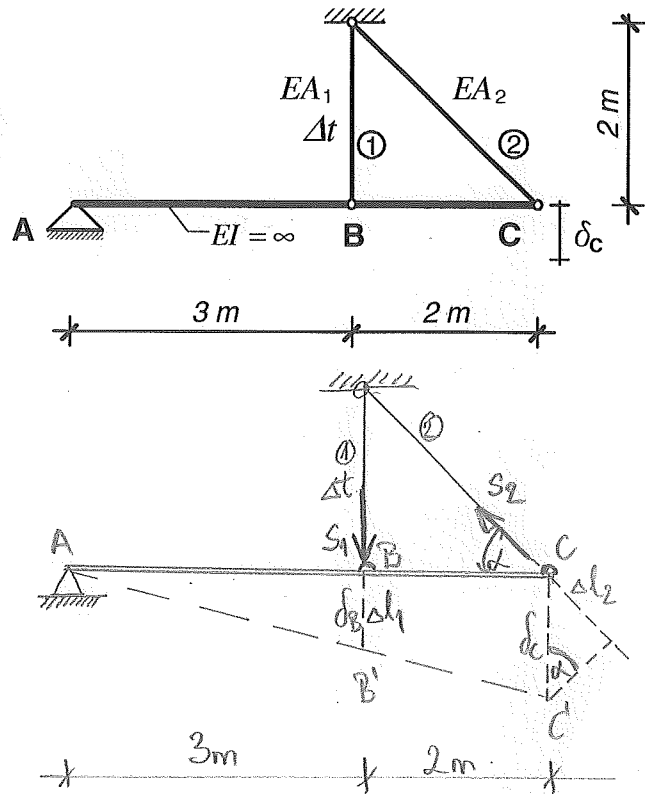
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$$\alpha = 45^\circ; l_2 = 2\sqrt{2} \text{ m}$$

$$\text{u.r. } \sum M_A = 0$$

$$S_2 \cdot \sin \alpha \cdot 5 - S_1 \cdot 3 = 0$$

$$S_1 = \frac{5}{3} \sin \alpha \cdot S_2$$

u.d.

$$\frac{\delta_B}{3} = \frac{\delta_c}{5} \Rightarrow \delta_B = \frac{3}{5} \delta_c = 1,2 \text{ mm} \quad \delta_B = \Delta l_1$$

$$\delta_c = \frac{\Delta l_2}{\sin \alpha} = \frac{1}{\sin \alpha} \cdot \frac{S_2 \cdot l_2}{E \cdot A_2} \Rightarrow S_2 = \frac{\delta_c \cdot E \cdot A_2 \cdot \sin \alpha}{l_2} = \frac{2 \cdot 2 \cdot 10^5 \cdot 100 \cdot \sin 45^\circ}{2\sqrt{2} \cdot 10^3} = 10 \text{ kN (vlak)}$$

$$S_1 = \frac{5}{3} \cdot \sin 45^\circ \cdot 10 = 11,78 \text{ kN (vlak)}$$

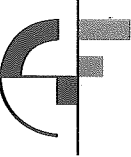
$$\delta_B = \Delta l_1 = \frac{-S_1 \cdot l_1}{E \cdot A_1} + \alpha_t \cdot \Delta t \cdot l_1$$

$$1,2 = \frac{-11,78 \cdot 10^3 \cdot 2 \cdot 10^3}{2 \cdot 10^5 \cdot 140} + 1,2 \cdot 10^{-5} \cdot \Delta t \cdot 2 \cdot 10^3$$

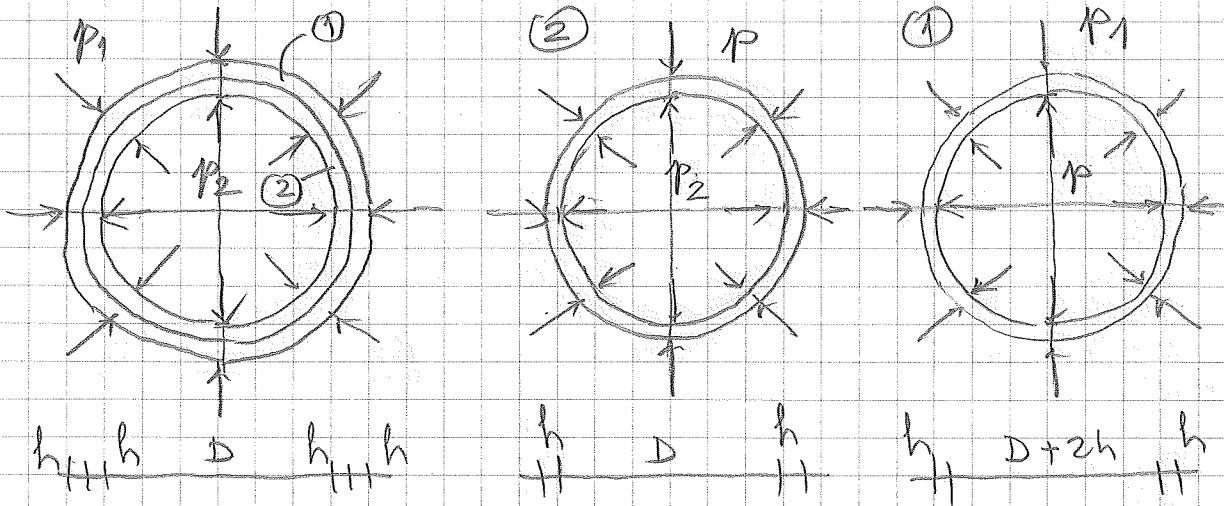
$$1,2 = -0,84 + 24 \cdot 10^{-2} \Delta t$$

$$\Delta t = 85 \text{ K}$$

$$\sigma_1 = \frac{S_1}{A_1} = \frac{-11,78 \cdot 10^3}{140} = -84,14 \text{ MPa}; \quad \sigma_2 = \frac{S_2}{A_2} = \frac{10 \cdot 10^3}{100} = 100 \text{ MPa}$$



3.  $E_1 = 2 \cdot 10^5 \text{ MPa}$        $D = 320 \text{ mm}$   
 $E_2 = 1,2 \cdot 10^5 \text{ MPa}$        $h = 6 \text{ mm}$   
 $p_1 = 6 \text{ MPa}$   
 $p_2 = 2 \text{ MPa}$



$$\epsilon_x^{(2)} = \epsilon_x^{(1)}$$

$$\epsilon_x^{(2)} = \frac{p_2 \cdot D}{2hE_2} - \frac{p(D+2h)}{2hE_2} \quad \epsilon_x^{(1)} = \frac{p(D+2h)}{2hE_1} - \frac{p_1(D+4h)}{2hE_1}$$

$$\frac{p_2 D}{2hE_2} - \frac{p(D+2h)}{2hE_2} = \frac{p(D+2h)}{2hE_1} - \frac{p_1(D+4h)}{2hE_1} \quad / \cdot \frac{2hE_1}{D+2h}$$

$$p \left( 1 + \frac{E_1}{E_2} \right) = p_2 \frac{D}{D+2h} \cdot \frac{E_1}{E_2} + p_1 \cdot \frac{D+4h}{D+2h}$$

$$p \cdot 2,6667 = p_2 \cdot 1,6065 + p_1 \cdot 1,0361 = 9,4297$$

$$p = \underline{\underline{3,536 \text{ MPa}}}$$

$$\sigma_x^{(2)} = \frac{p_2 \cdot D}{2h} - \frac{p(D+2h)}{2h} = 53,33 - 97,83 = \underline{\underline{-44,50 \text{ MPa}}}$$

$$\sigma_x^{(1)} = \frac{p(D+2h)}{2h} - \frac{p_1(D+4h)}{2h} = 97,83 - 172,0 = \underline{\underline{-74,17 \text{ MPa}}}$$

$$\epsilon_x^{(1)} = (44,444 - 81,524) \cdot 10^{-5} = -3708 \cdot 10^{-5}$$

$$\Delta D = \epsilon_x^{(1)} \cdot D = \underline{\underline{-0,1187 \text{ mm}}}$$

SMANJENJE PROMIJERA!