

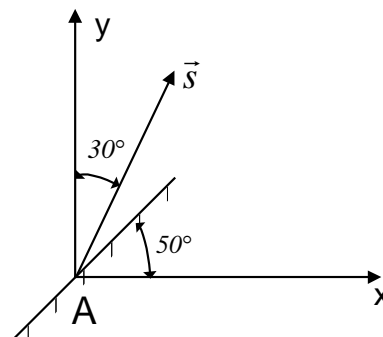
OTPORNOST MATERIJALA I

1. KOLOKVIJ, 13.11.2008.

GRUPA B

1. U točki „A“ napregnutog tijela poznata su naprezanja u tri smjera $\sigma_x = 65 \text{ MPa}$, $\sigma_y = 30 \text{ MPa}$ i $\sigma_s = 80 \text{ MPa}$.
- odrediti i skicirati vektor punog naprezanja na ravninu koja s osi x zatvara kut od 50° .
 - odrediti veličine i skicirati smjerove glavnih deformacija.

Ako je $E = 2 \cdot 10^5 \text{ MPa}$ i $\nu = 0,3$.

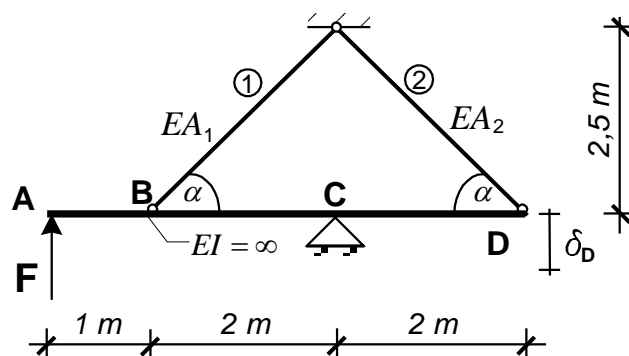


2. Treba odrediti silu F i naprezanja u štapovima „1“ i „2“ ako je pomak $\delta_D = 1,3 \text{ mm}$.

$$E = 2 \cdot 10^5 \text{ MPa}$$

$$A_1 = 80 \text{ mm}^2$$

$$A_2 = 60 \text{ mm}^2$$



3. Za zadani spoj prema slici treba odrediti najveće dopušteno opterećenje, ako je zadano:

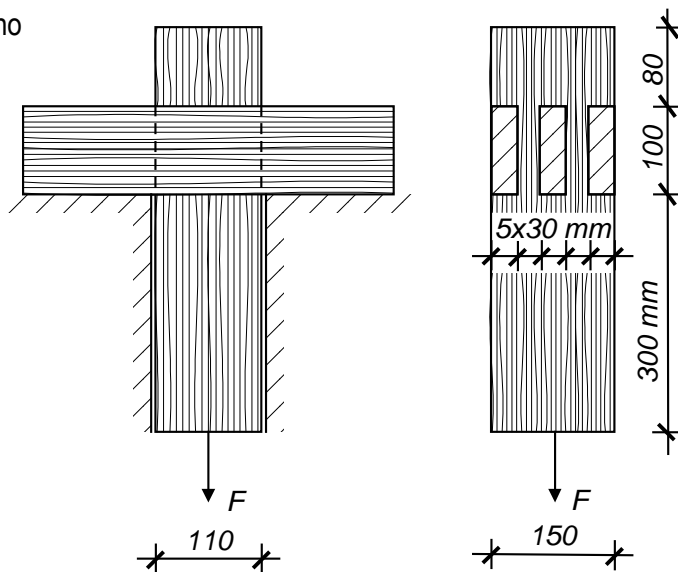
$$\sigma_{vl,dop}'' = 10 \text{ MPa}$$

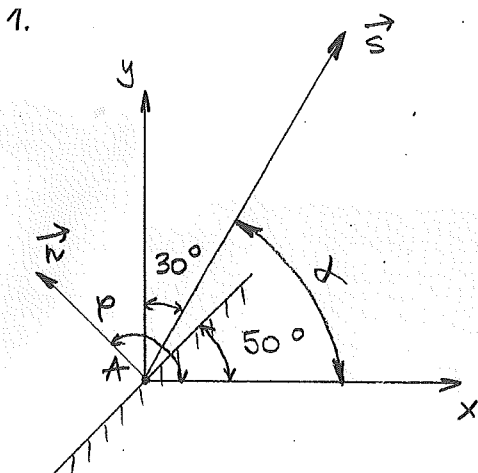
$$\tau_{dop}'' = 1,5 \text{ MPa}$$

$$\tau_{dop}^\perp = 3,5 \text{ MPa}$$

$$\sigma_{tl,dop}'' = 12 \text{ MPa}$$

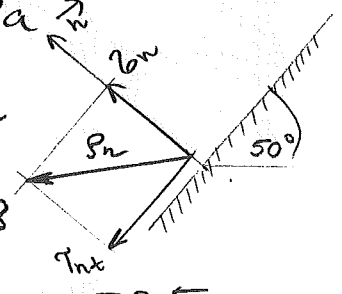
$$\sigma_{tl,dop}^\perp = 5 \text{ MPa}$$





$$\begin{aligned} \sigma_x &= 65 \text{ MPa} \\ \sigma_y &= 30 \text{ MPa} \\ \sigma_s &= 80 \text{ MPa} \\ E &= 2 \cdot 10^5 \text{ MPa} \\ \nu &= 0,3 \end{aligned}$$

a) $\sigma_s = \sigma_x \cos^2 \alpha + \sigma_y \sin^2 \alpha + \tau_{xy} \sin 2\alpha$ $\alpha = 90^\circ - 30^\circ = 60^\circ$
 $80 = 65 \cos^2 60^\circ + 30 \sin^2 60^\circ + \tau_{xy} \sin 120^\circ$ $\tau_{xy} = 47,63 \text{ MPa}$
 $\sigma_n = \sigma_x \cos^2 \varphi + \sigma_y \sin^2 \varphi + \tau_{xy} \sin 2\varphi$ $\varphi = 90^\circ + 50^\circ = 140^\circ$
 $\sigma_n = 65 \cos^2 140^\circ + 30 \sin^2 140^\circ + 47,63 \cdot \sin 280^\circ = 3,63 \text{ MPa}$
 $\tau_{nt} = \frac{\sigma_y - \sigma_x}{2} \sin 2\varphi + \tau_{xy} \cos 2\varphi = 25,50 \text{ MPa}$
 $\rho_n = \sqrt{\sigma_n^2 + \tau_{nt}^2} = \sqrt{3,63^2 + 25,5^2} = 25,76 \text{ MPa}$



b) $\epsilon_x = \frac{1}{E} (\sigma_x - \nu \sigma_y) = \frac{1}{2 \cdot 10^5} (65 - 0,3 \cdot 30) = 0,00028$
 $\epsilon_y = \frac{1}{E} (\sigma_y - \nu \sigma_x) = \frac{1}{2 \cdot 10^5} (30 - 0,3 \cdot 65) = 0,0000525$
 $\epsilon_{xy} = \frac{1+\nu}{E} \tau_{xy} = \frac{1+0,3}{2 \cdot 10^5} \cdot 47,63 = 0,00031$

$$\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \frac{1}{2} \sqrt{(\epsilon_y - \epsilon_x)^2 + 4\epsilon_{xy}^2} = (1,6625 \pm 3,3021) \cdot 10^{-4}$$

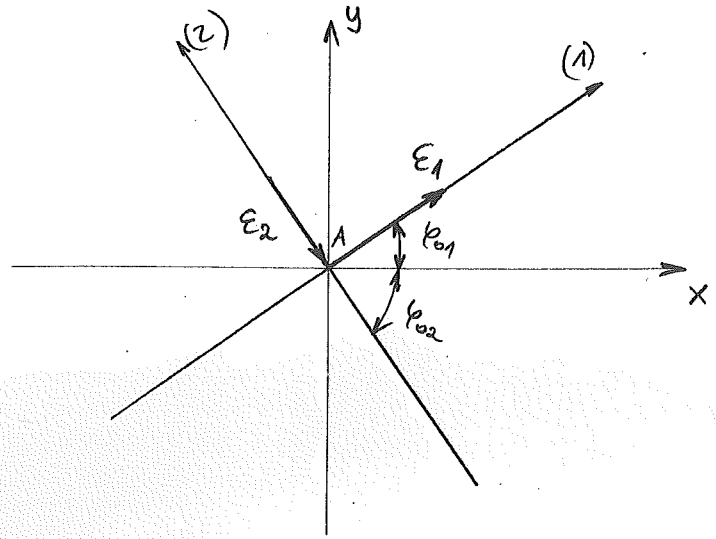
$$\begin{aligned} \epsilon_1 &= 4,9646 \cdot 10^{-4} \\ \epsilon_2 &= -1,6396 \cdot 10^{-4} \end{aligned}$$

$$\begin{aligned} \epsilon_x + \epsilon_y &= \epsilon_1 + \epsilon_2 \\ 3,325 \cdot 10^{-4} &= 3,325 \cdot 10^{-4} \quad \checkmark \end{aligned}$$

$$\operatorname{tg} \varphi_{0i} = \frac{\epsilon_{xy}}{\epsilon_i - \epsilon_y}$$

$$\begin{aligned} \varphi_{01} &= 34,93^\circ \\ \varphi_{02} &= -55,07^\circ \end{aligned}$$

$$\begin{aligned} |\varphi_{01}| + |\varphi_{02}| &= 90^\circ \\ 34,93^\circ + 55,07^\circ &= 90^\circ \end{aligned}$$



2. Treba odrediti silu F i naprezanja u štapovima „1“ i „2“ ako je pomak $\delta_D = 1,3$ mm.

$$E = 2 \cdot 10^5 \text{ MPa}$$

$$A_1 = 80 \text{ mm}^2$$

$$A_2 = 60 \text{ mm}^2$$

$$\Delta l_1 = \Delta l_2 = \sqrt{2^2 + 2,5^2} = 3,201 \text{ m}$$

$$\text{tg} \alpha = \frac{2,5}{2} \Rightarrow \alpha = 51,34^\circ$$

U.R. $\sum M_C = 0$

$$S_1 \cdot \sin \alpha \cdot 2 + S_2 \cdot \sin \alpha \cdot 2 - F \cdot 3 = 0$$

U.D.

$$\delta_B = \delta_D; \delta_B = \frac{\Delta l_1}{\sin \alpha}; \delta_D = \frac{\Delta l_2}{\sin \alpha}$$

$$\Delta l_1 = \Delta l_2 = 1,3 \cdot \sin \alpha = 1,015 \text{ mm}$$

$$\frac{S_1 \cdot l_1}{E \cdot A_1} = \frac{S_2 \cdot l_2}{E \cdot A_2} \Rightarrow S_1 = \frac{A_1}{A_2} \cdot S_2 = \frac{80}{60} \cdot S_2 = 1,333 S_2$$

$$\frac{S_2 \cdot l_2}{E \cdot A_2} = 1,015 \Rightarrow S_2 = \frac{1,015 \cdot E \cdot A_2}{l_2} = \frac{1,015 \cdot 2 \cdot 10^5 \cdot 60}{3,201 \cdot 10^3} = 3,805 \text{ kN}$$

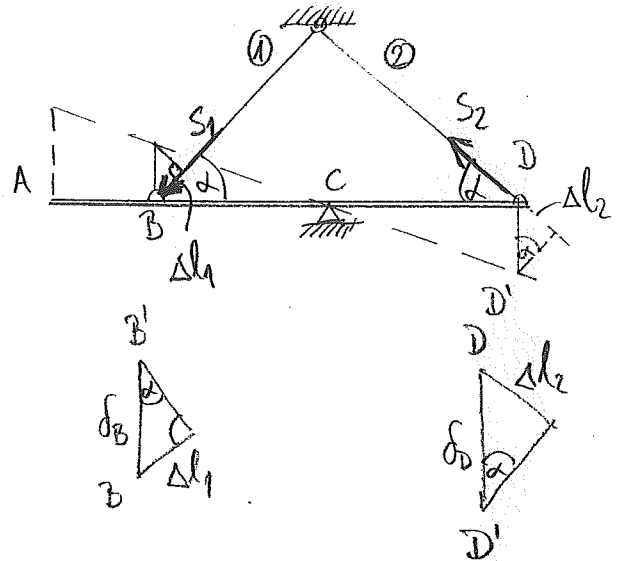
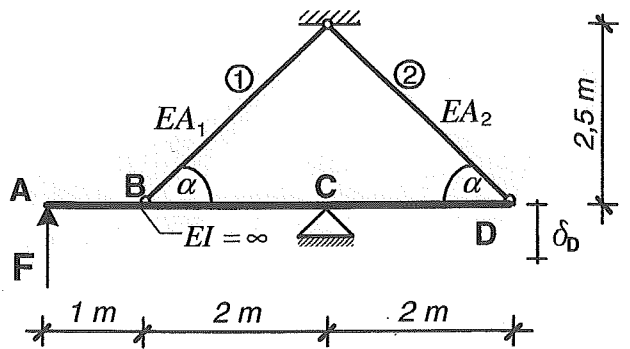
$$S_1 = 1,333 \cdot 3,805 = 5,073 \text{ kN}$$

12. U.R.

$$F = \frac{S_1 \cdot \sin \alpha \cdot 2 + S_2 \cdot \sin \alpha \cdot 2}{3} = \frac{2 \cdot \sin \alpha (S_1 + S_2)}{3} = 4,62 \text{ kN}$$

$$\sigma_1 = \frac{-S_1}{A_1} = \frac{-5,073 \cdot 10^3}{80} = -63,4 \text{ MPa}$$

$$\sigma_2 = \frac{S_2}{A_2} = \frac{3,805 \cdot 10^3}{60} = 63,4 \text{ MPa}$$



- 3 Za zadani spoj prema slici treba odrediti najveće dopušteno opterećenje, ako je zadano:

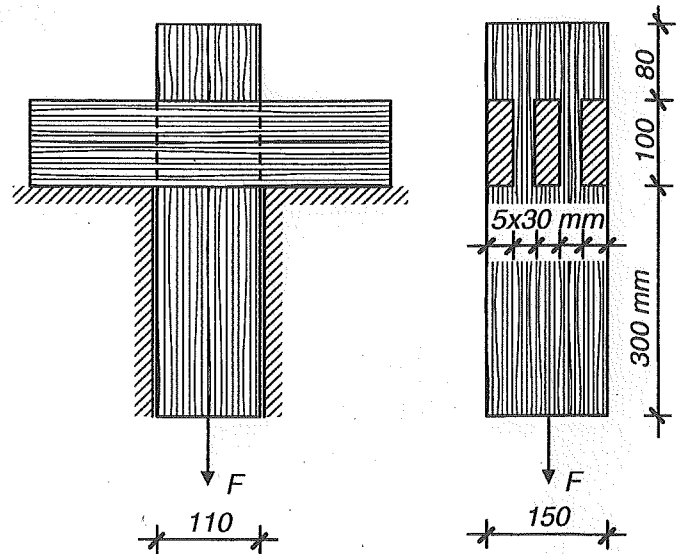
$$\sigma_{vl,dop}'' = 10 \text{ MPa}$$

$$\tau_{dop}'' = 1,5 \text{ MPa}$$

$$\tau_{dop}^\perp = 3,5 \text{ MPa}$$

$$\sigma_{tl,dop}'' = 12 \text{ MPa}$$

$$\sigma_{tl,dop}^\perp = 5 \text{ MPa}$$



$$\sigma_{vl}'' = \frac{F}{2 \cdot 30 \cdot 110} \leq 10 \text{ MPa} \Rightarrow F \leq 66 \text{ kN}$$

$$\tau'' = \frac{F}{4 \cdot 110 \cdot 80} \leq 1,5 \Rightarrow F \leq 52,8 \text{ kN}$$

$$\tau^\perp = \frac{F}{2 \cdot 3 \cdot 30 \cdot 100} \leq 3,5 \Rightarrow F \leq 63 \text{ kN}$$

$$\sigma_{tl}'' = \frac{F}{3 \cdot 30 \cdot 110} \leq 12 \Rightarrow F \leq 118,8 \text{ kN}$$

$$\sigma_{tl}^\perp = \frac{F}{3 \cdot 30 \cdot 110} \leq 5 \Rightarrow F \leq 49,5 \text{ kN}$$