M

OTPORNOST MATERIJALA I

GRUPA F

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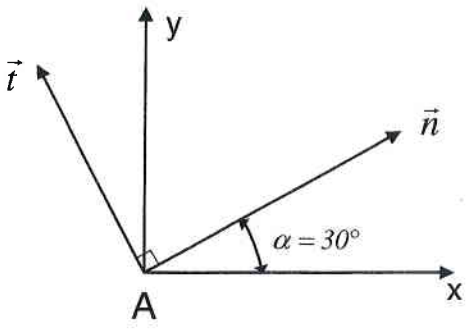
Prezime i ime:_____

1. Za napregnuti element u točki A određene su rel. deformacije ε_x=0,0004 i ε_y=-0,0003 koje su ujedno i glavne deformacije. Treba odrediti tenzor naprezanja i deformacija za koordinatni sustav n,t (te skicirati) koji je u odnosu na x,y KS zaokrenut za kut α=30°, te promjenu pravog kuta između n i t.

Ako je $E = 2 \cdot 10^5\ MPa$ i $\nu = 0{,}3$.

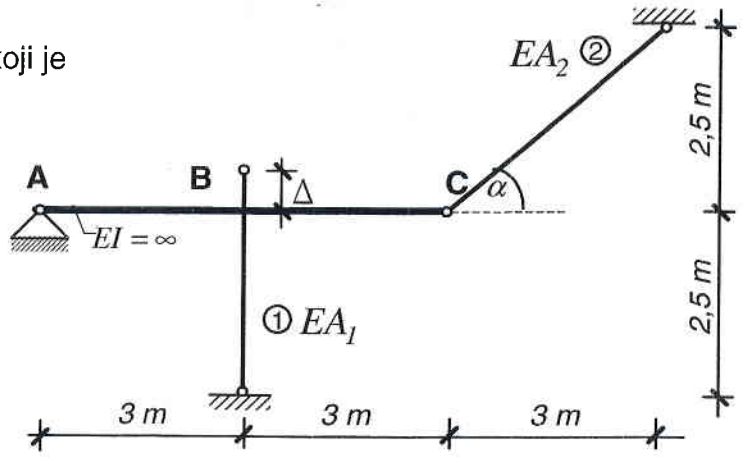


2. Treba odrediti naprezanja štapova „1" i „2", te pomak točke „C" nakon montiranja štapa „1" koji je izveden duži za Δ=1 mm, ako je zadano:

$E = 2 \cdot 10^5\ MPa$

$A_1 = 50\ mm^2$

$A_2 = 80\ mm^2$

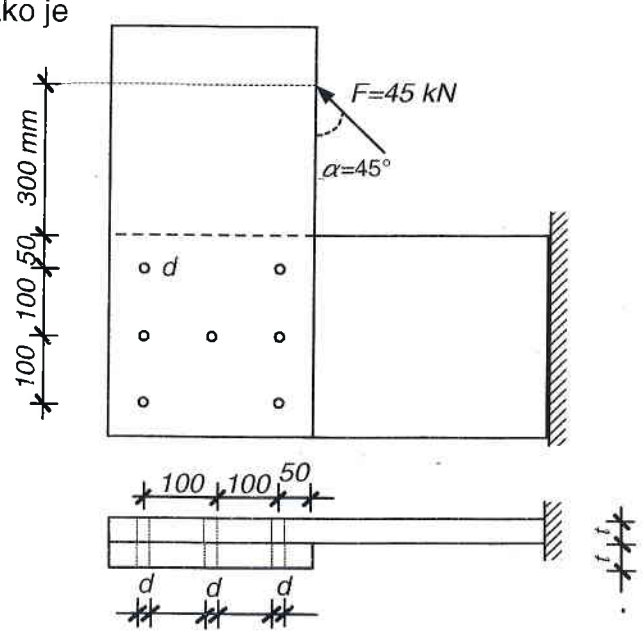


3. Dimenzionirati spoj prema slici (odrediti „d" i „t"), ako je zadano:

F = 45 kN

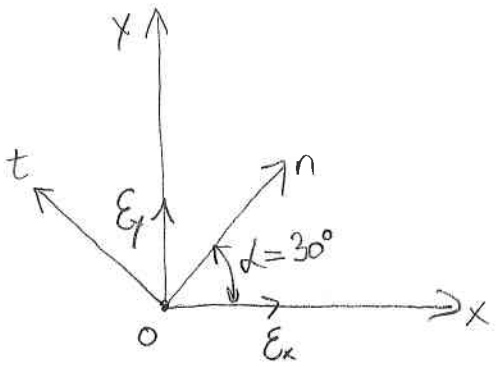
$\sigma_{ob,dop} = 200\ MPa$

$\tau_{dop} = 70\ MPa$



① F $\varepsilon_x = 0{,}0004$ $\quad E = 2\cdot10^5$ MPa

$\varepsilon_y = -0{,}00003$ $\quad \nu = 0{,}3$



$$\sigma_x = \frac{E}{1-\nu^2}\left(\varepsilon_x + \nu\varepsilon_y\right) = \frac{2\cdot10^5}{1-0{,}3^2}\left(4 - 0{,}3\cdot3\right)\cdot10^{-4}$$

$$\sigma_x = 68{,}13\ \text{MPa}$$

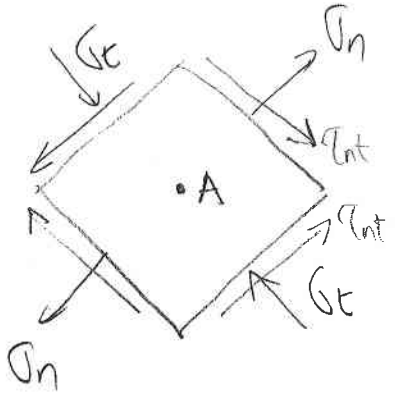
$$\sigma_y = \frac{E}{1-\nu^2}\cdot\left(\varepsilon_y + \nu\varepsilon_x\right) = \frac{2\cdot10^5}{1-0{,}3^2}\left(-3 + 0{,}3\cdot4\right)\cdot10^{-4}$$

$$\sigma_y = -39{,}56\ \text{MPa}$$

$$\sigma_n = \sigma_x\cdot\cos^2 30° + \sigma_y\cdot\sin^2 30° = 41{,}21\ \text{MPa}$$

$$\sigma_t = \sigma_x\cdot\cos^2 120° + \sigma_y\cdot\sin^2 120° = -12{,}64\ \text{MPa}$$

$$\tau_{nt} = \frac{\sigma_y - \sigma_x}{2}\cdot\sin 60° = -46{,}63\ \text{MPa}$$



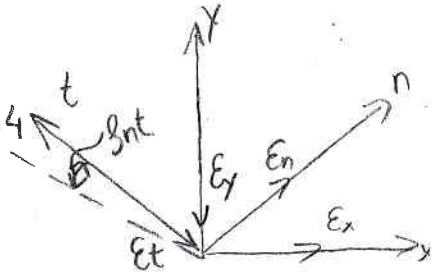
$$\varepsilon_n = \frac{1}{E}\left(\sigma_n - \nu\sigma_t\right) = \frac{1}{2\cdot10^5}\left(41{,}21 + 0{,}3\cdot12{,}64\right) = 2{,}25\cdot10^{-4}$$

$$\varepsilon_t = \frac{1}{E}\left(\sigma_t - \nu\sigma_n\right) = \frac{1}{2\cdot10^5}\left(-12{,}64 - 0{,}3\cdot41{,}21\right) = -1{,}25\cdot10^{-4}$$

$$\varepsilon_{nt} = \frac{\tau_{nt}}{2G} = \frac{1+\nu}{E}\tau_{nt} = \frac{1+0{,}3}{2\cdot10^5}\left(-46{,}63\right) = -3{,}03\cdot10^{-4}$$

$$\gamma_{nt} = 2\varepsilon_{nt} = -6{,}06\cdot10^{-4}\ \text{rad} = -0{,}035° = 0°\ 2'\ 6''$$ (Powiększyć ↺)

$$\sigma_{ij} = \begin{bmatrix} 41{,}21 & -46{,}63 \\ -46{,}63 & -12{,}64 \end{bmatrix}\ \text{MPa} \quad ; \quad \varepsilon_{ij} = \begin{bmatrix} 2{,}25 & -3{,}03 \\ -3{,}03 & -1{,}25 \end{bmatrix}\cdot10^{-4}$$



2) F

$\Delta = 1\,mm$

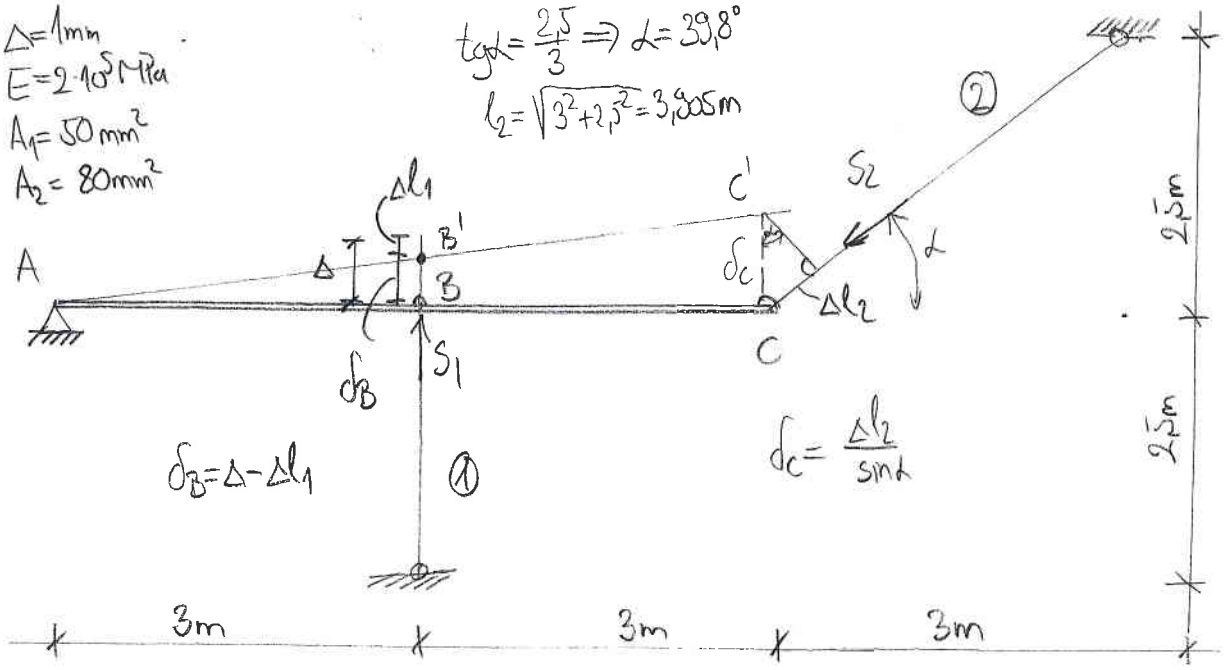
$E = 2 \cdot 10^5\,MPa$

$A_1 = 50\,mm^2$

$A_2 = 80\,mm^2$

$\operatorname{tg}\alpha = \frac{2,5}{3} \Rightarrow \alpha = 39,8°$

$l_2 = \sqrt{3^2 + 2,5^2} = 3,905\,m$



U.R. $\sum M_A = 0$

$S_1 \cdot 3 - S_2 \cdot \sin\alpha \cdot 6 = 0 \Rightarrow S_1 = 2 \cdot S_2 \cdot \sin\alpha$

U.D.

$\frac{\delta_B}{3} = \frac{\delta_C}{6} \Rightarrow \delta_B = \frac{1}{2}\delta_C \qquad \Delta - \Delta l_1 = \frac{1}{2}\frac{\Delta l_2}{\sin\alpha}$

$\Delta - \frac{S_1 l_1}{E A_1} = \frac{1}{2\sin\alpha} \cdot \frac{S_2 l_2}{E A_2}$

$\Delta - \frac{2 \cdot S_2 \cdot \sin\alpha \cdot l_1}{E \cdot A_1} = \frac{1}{2\sin\alpha} \frac{S_2 l_2}{E A_2} \Rightarrow S_2 = 1,958\,kN$

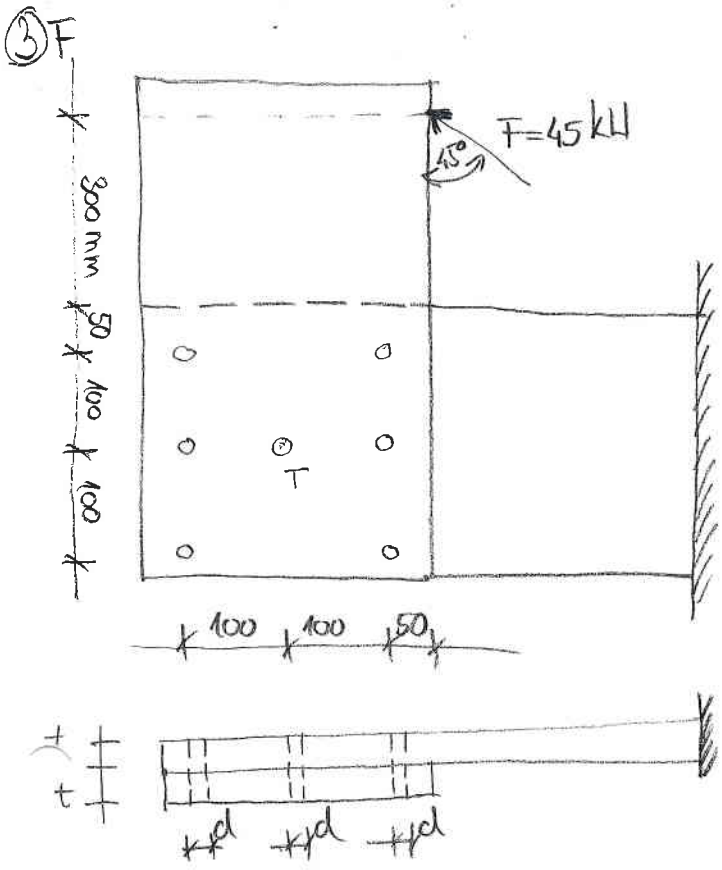
$S_1 = 2,507\,kN$

$\delta_C = \frac{1}{\sin\alpha} \cdot \frac{S_2 l_2}{E A_2} = \frac{1}{\sin 39,8} \cdot \frac{1,958 \cdot 10^3 \cdot 3905}{2 \cdot 10^5 \cdot 80} = 0,749\,mm$

$\sigma_1 = \frac{-2507}{50} = -50,14\,MPa$ (tlak)

$\sigma_2 = \frac{-1958}{80} = -24,48\,MPa$ (tlak)

③



$\tau_{dop} = 70\ MPa$; $\sigma_{d,dop} = 200\ MPa$

$M = F \cdot \sin\alpha \cdot 450 + F \cdot \cos\alpha \cdot 150 =$

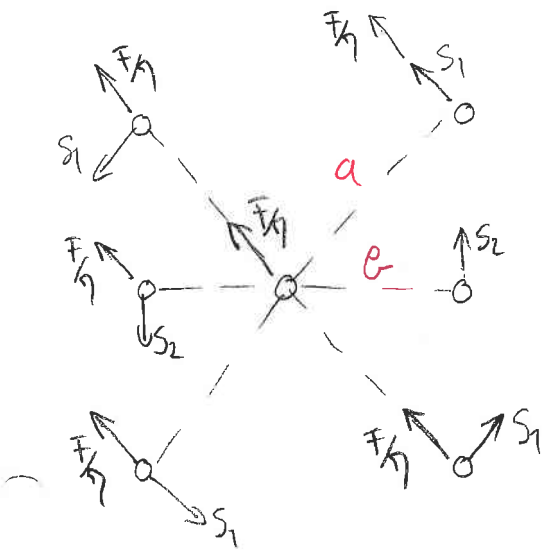
$= 4\, S_1 \cdot a + 2 \cdot S_2 \cdot b$

$a = 100\sqrt{2}\ mm$

$b = 100\ mm$

$\frac{S_1}{a} = \frac{S_2}{b} \Rightarrow S_1 = \frac{a}{b} S_2 = 1{,}414\, S_2$

$\frac{F}{\eta} = 6{,}43\ kN$



$45 \cdot 10^3 \left(\sin 45° \cdot 450 + \cos 45° \cdot 150 \right) =$

$= 4 \cdot 1{,}414 \cdot S_2 \cdot 100\sqrt{2} + 2 \cdot S_2 \cdot 100$

$1{,}9092 \cdot 10^5 = 1000\, S_2$

$S_2 = 19{,}09\ kN$

$S_1 = 27\ kN$

$S_{max} = 27 + 6{,}43 = 33{,}43\ kN$

$\tau_{max} = \frac{S_{max}}{\frac{d^2 \pi}{4}} \leq \tau_{dop}$

$d \geq \sqrt{\frac{4\, S_{max}}{\pi \cdot \tau_{dop}}} = \sqrt{\frac{4 \cdot 33{,}43 \cdot 10^3}{\pi \cdot 70}}$

$d \geq 24{,}66\ mm$

$d_{rd} = 25\ mm$

$\sigma_{d,dop} = \frac{S_{max}}{t\, d} \leq \sigma_{d,dop}$

$t \geq \frac{S_{max}}{d \cdot \sigma_{d,dop}} = \frac{33{,}43 \cdot 10^3}{25 \cdot 200}$

$t \geq 6{,}69\ mm$

$t \geq 8\ mm$