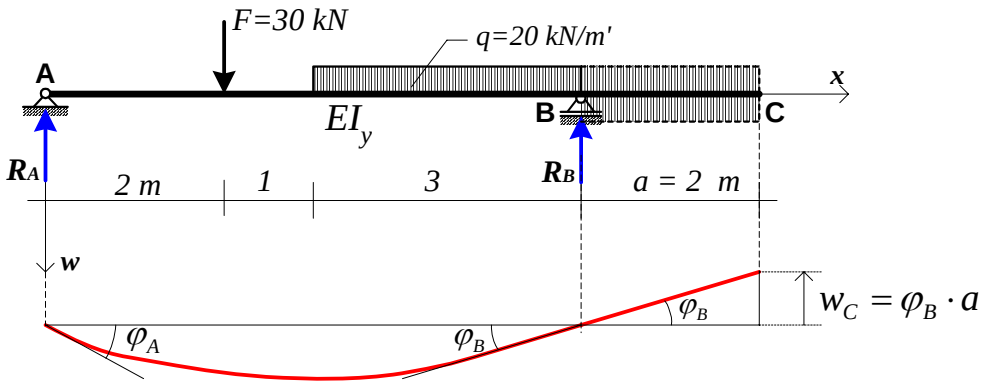
ZADATAK:

Za zadani nosač treba analitičkim postupkom odrediti progib nosača u točki C i kut zaokreta presjeka A i B.

Zadano je: $E \cdot I_y = const = 16{,}80 \cdot 10^{12}\ \ Nmm^2$



RJEŠENJE:

Određivanje reakcija iz uvjeta ravnoteže:

$\Sigma M_A = 0 \qquad F \cdot 2 + q \cdot 3 \cdot 4{,}5 - R_B \cdot 6 = 0 \quad \Rightarrow \quad R_B = 55{,}0\ \ kN$

$\Sigma M_B = 0 \qquad R_A \cdot 6 - F \cdot 4 - q \cdot 3 \cdot 1{,}5 = 0 \quad \Rightarrow \quad R_A = 35{,}0\ \ kN$

Kontrola: $\Sigma V = 0 \qquad F + q \cdot 3 = R_A + R_B$

Diferencijalna jednadžba elastične linije nosača:

$$EI_y \cdot \frac{d^2 w}{dx^2} = -M(x) \qquad (1)$$

Jednadžba momenta savijanja:

$$M(x) = R_A \cdot x - F \cdot (x-2) - q \cdot \frac{(x-3)^2}{2} + R_B \cdot (x-6) + q \cdot \frac{(x-6)}{2} \qquad (2)$$

Vrijedi za: $(x - a_i) > 0$

$$(2) \Rightarrow (1) \qquad EI_y \cdot \frac{d^2 w}{dx^2} = -R_A \cdot x + F \cdot (x-2) + q \cdot \frac{(x-3)^2}{2} - R_B \cdot (x-6) - q \cdot \frac{(x-6)^2}{2} \quad \Big| \int$$

$$EI_y \cdot \frac{dw}{dx} = EI_y \cdot \varphi(x) = -R_A \cdot \frac{x^2}{2} + F \cdot \frac{(x-2)^2}{2} + q \cdot \frac{(x-3)^3}{6} - R_B \cdot \frac{(x-6)^2}{2} - q \cdot \frac{(x-6)^3}{6} + C \quad \Big| \int$$

$$EI_y \cdot w(x) = -R_A \cdot \frac{x^3}{6} + F \cdot \frac{(x-2)^3}{6} + q \cdot \frac{(x-3)^4}{24} - R_B \cdot \frac{(x-6)^3}{6} - q \cdot \frac{(x-6)^4}{24} + C \cdot x + D$$

Rubni uvjeti za određivanje konstanti integracije C i D:

$x = 0 \qquad w = 0 \quad \Rightarrow \quad D = 0$

$$x = 6\ m \qquad w = 0 \quad \Rightarrow \quad 0 = -R_A \cdot \frac{6^3}{6} + F \cdot \frac{(6-2)^3}{6} + q \cdot \frac{(6-3)^4}{24} + C \cdot 6 \quad \Rightarrow$$

$$C = 145{,}42\ \ kNm^2$$

Jednadžba elastične linije nosača:

$$EI_y \cdot w(x) = -R_A \cdot \frac{x^3}{6} + F \cdot \frac{(x-2)^3}{6} + q \cdot \frac{(x-3)^4}{24} - R_B \cdot \frac{(x-6)^3}{6} - q \cdot \frac{(x-6)^4}{24} + C \cdot x \qquad (3)$$

Određivanje progiba nosača u točki C: $x = 8\ m$

$$EI_y \cdot w_C = -35 \cdot \frac{8^3}{6} + 30 \cdot \frac{(8-2)^3}{6} + 20 \cdot \frac{(8-3)^4}{24} - 55 \cdot \frac{(8-6)^3}{6} - 20 \cdot \frac{(8-6)^4}{24} + 145{,}42 \cdot 8 = -309{,}14\ \ kNm^3$$

$$w_C = \frac{-309{,}14}{EI_y} = \frac{-309{,}14 \cdot 10^3 \cdot 10^9}{16{,}8 \cdot 10^{12}} = -18{,}40\ \ mm$$

Jednadžba kuta zaokreta presjeka (nagiba tangente elastične linije nosača):

$$EI_y \cdot \varphi(x) = -R_A \cdot \frac{x^2}{2} + F \cdot \frac{(x-2)^2}{2} + q \cdot \frac{(x-3)^3}{6} - R_B \cdot \frac{(x-6)^2}{2} - q \cdot \frac{(x-6)^3}{6} + C \qquad (4)$$

Određivanje kuta zaokreta presjeka A: $x = 0\ m$

$$EI_y \cdot \varphi_A = C = 145{,}42\ \ kNm^2$$

$$\varphi_A = \frac{145{,}42}{EI_y} = \frac{145{,}42 \cdot 10^3 \cdot 10^6}{16{,}8 \cdot 10^{12}} = +0{,}008656\ \ rad = +0^o\ 29'\ 45{,}4''$$

Određivanje kuta zaokreta presjeka B: $x = 8\ m$

$$EI_y \cdot \varphi_B = -35 \cdot \frac{8^2}{2} + 30 \cdot \frac{(8-2)^2}{2} + 20 \cdot \frac{(8-3)^3}{6} - 55 \cdot \frac{(8-6)^2}{2} - 20 \cdot \frac{(8-6)^3}{6} + 145{,}42 = -154{,}58\ \ kNm^2$$

$$\varphi_B = \frac{-154{,}58}{EI_y} = \frac{-154{,}58 \cdot 10^3 \cdot 10^6}{16{,}8 \cdot 10^{12}} = -0{,}009201\ \ rad = -0^o\ 31'\ 37{,}8''$$

Određivanje progiba nosača u točki C preko kuta zaokreta φ_B:

$$w_C = -\varphi_B \cdot a = -0{,}009201 \cdot 2000 = -18{,}40\ \ mm$$

Dobijemo isto kao i preko jednadžbe elastične linije nosača (3) !