

# JEDNOSTAVNIJI STATIČKI NEODREĐENI SUSTAVI

## ZADATAK

Sistem prikazan na slici opterećen je momentom savijanja  $M_A = 60 \text{ kNm}$ .

Temperatura štapa  $\overline{BD}$  se poveća za  $\Delta T = 30 \text{ K}$ .

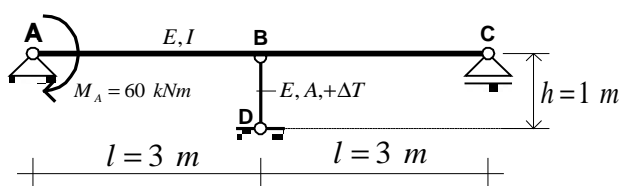
Treba odrediti dijagrame unutarnjih sila  $M, T, N$ .

Zadano je:  $E = 2,1 \cdot 10^5 \text{ MPa}$      $h = 1000 \text{ mm}$

$$I = 1,5 \cdot 10^9 \text{ mm}^4$$

$$A = 10^4 \text{ mm}^2$$

$$\alpha_T = 1,2 \cdot 10^{-5} / \text{K}$$

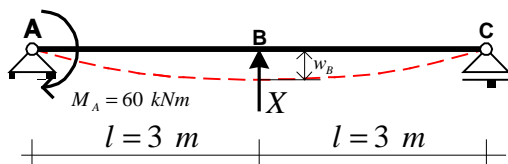


Sustav je jedanput statički neodređen.

Dopunska jednačba deformacija glasi:

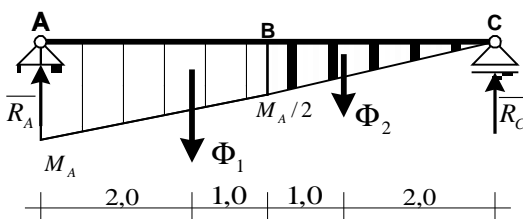
$$w_B = \Delta h \quad (1)$$

$$w_B = w_B^{M_A} + w_B^X = \Delta h \quad (1')$$



$$w_B^X = -\frac{X \cdot (2l)^3}{48 \cdot E \cdot I} = \frac{X \cdot l^3}{6 \cdot E \cdot I}$$

$$\Delta h = \frac{X \cdot h}{E \cdot A} - \alpha_T \cdot \Delta T \cdot h$$



$$w_B^{M_A} = \frac{\overline{M_B^{M_A}}}{E \cdot I}$$

$$\Phi_1 = \frac{1}{2} \cdot M_A \cdot 2l = 180 \text{ kNm}^2$$

$$\Phi_2 = \frac{1}{2} \cdot \frac{M_A}{2} \cdot l = 45 \text{ kNm}^2$$

$$\Sigma M_A = 0 \quad \overline{R_C} \cdot 2l - \Phi_1 \cdot 2,0 = 0 \quad \overline{R_C} = \frac{1}{3} \Phi_1 = 60 \text{ kNm}^2$$

$$\Sigma M_C = 0 \quad \overline{R_A} \cdot 2l - \Phi_1 \cdot 4,0 = 0 \quad \overline{R_A} = \frac{2}{3} \Phi_1 = 120 \text{ kNm}^2$$

$$\overline{M_B^{M_A}} = \overline{R_C} \cdot l - \Phi_2 \cdot 1,0 = 135 \text{ kNm}^3$$

$$w_B = w_B^{M_A} + w_B^X = \Delta h$$

$$\frac{\overline{M_B^{M_A}}}{E \cdot I} - \frac{X \cdot l^3}{6 \cdot E \cdot I} = \frac{X \cdot h}{E \cdot A} - \alpha_T \cdot \Delta T \cdot h \quad | \cdot EI$$

$$\overline{M_B^{M_A}} - \frac{X \cdot l^3}{6} = \frac{X \cdot h \cdot I}{A} - \alpha_T \cdot \Delta T \cdot h \cdot E \cdot I$$

$$\overline{M_B^{M_A}} + \alpha_T \cdot \Delta T \cdot h \cdot E \cdot I = X \cdot \left( \frac{l^3}{6} + \frac{I}{A} \cdot h \right)$$

$$X = \frac{\overline{M_B^{M_A}} + \alpha_T \cdot \Delta T \cdot h \cdot I \cdot E}{\frac{l^3}{6} + \frac{I}{A} \cdot h}$$

$$X = \frac{135 + 1,2 \cdot 10^{-5} \cdot 30 \cdot 1,0 \cdot 2,1 \cdot 10^5 \cdot 1,5 \cdot 10^9 \cdot 10^{-9}}{\frac{3^3}{6} + \frac{1,5 \cdot 10^9 \cdot 10^{-12}}{10^4 \cdot 10^{-6}}} = \frac{248,40}{4,5 + 0,15} = 53,42 \text{ kN}$$

### ODREĐIVANJE REAKCIJA

$$\Sigma M_A = 0 \quad R_C \cdot 2l - X \cdot l + M_A = 0 \quad R_C = -16,71 \text{ kN}$$

$$\Sigma M_C = 0 \quad R_A \cdot 2l - X \cdot l - M_A = 0 \quad R_A = -36,71 \text{ kN}$$

### DIJAGRAMI UNUTARNJIH SILA

$$M_B = R_C \cdot 3,0 = M_A + R_A \cdot 3,0 = -50,13 \text{ kNm}$$

