OTPORNOST MATERIJALA 1 — grupa B

2. KOLOKVIJ 08. siječnja 2009. godine

Prezime i ime:_____

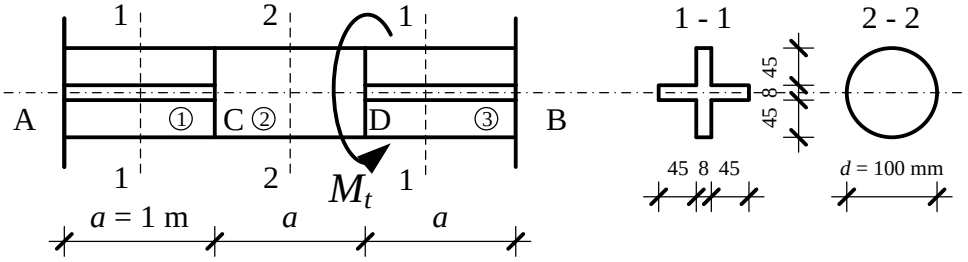
1. Na štap prikazan na slici djeluje moment torzije M_t.

Treba nacrtati dijagram momenata torzije i odrediti najveća posmična naprezanja po pojedinim dijelovima štapa, ako je zadano:

$\varphi_C = 10^{-2}$ rad

$E = 2 \cdot 10^5$ MPa

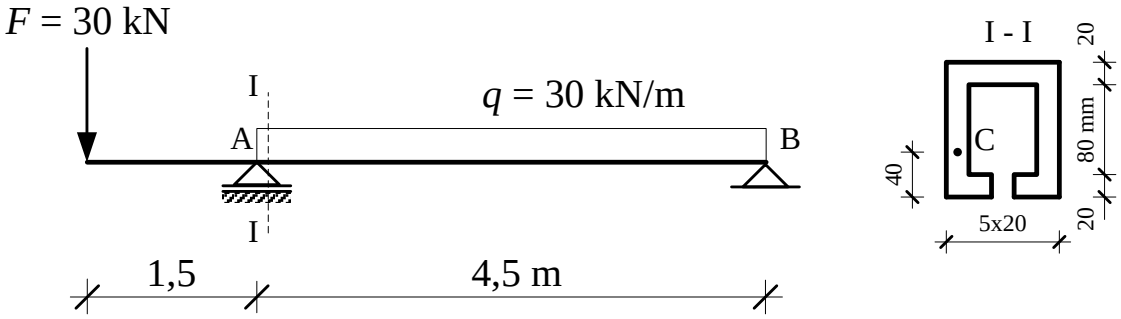
$\nu = 0,3.$



2. Za nosač prikazan na slici treba odrediti:

a) Maksimalno normalno i posmično naprezanje, te nacrtati odgovarajuće dijagrame naprezanja u kritičnim presjecima.

b) Veličinu i smjer glavnih naprezanja u točki **C** presjeka **I-I**, te skicirati trajektorije naprezanja u području točke **C**.

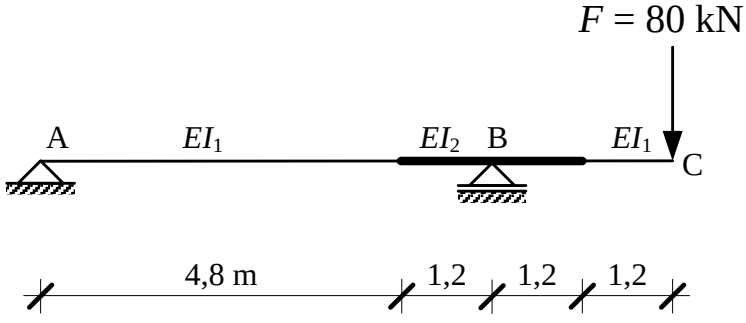


3. Grafoanalitičkim postupkom treba odrediti progib točke **C** i kut nagiba na elastičnu liniju presjeka **A**.

Zadano je: $E = 2,1 \cdot 10^5$ MPa

$I_1 = 3 \cdot 10^8$ mm^4

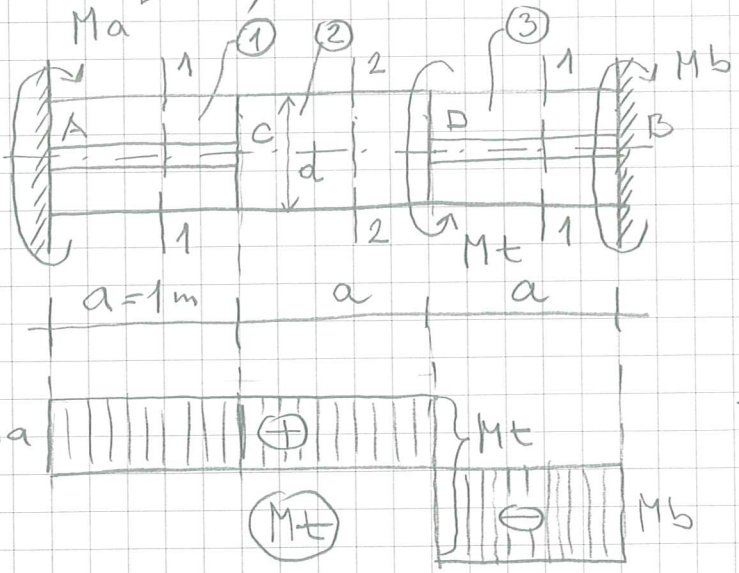
$I_2 = 1,2 \cdot I_1$.

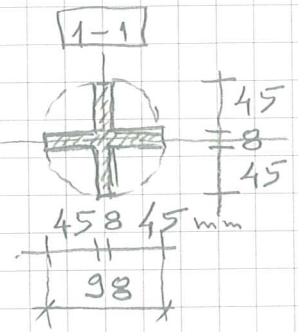


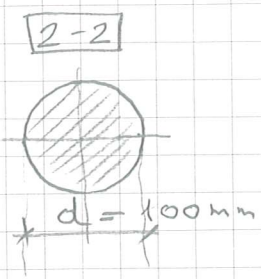
1. B) $\varphi_c' = 10^{-2}\ rad = 0{,}01\ rad = 0° 34' 22{,}6''$

$E = 2 \cdot 10^5\ MPa \qquad G = \frac{E}{2(1+\nu)} = 0{,}769 \cdot 10^5\ MPa$

$\nu = 0{,}30$







$J_{t_1} = \frac{1}{3}\sum t_i^3 \cdot s_i = \frac{1}{3}(8^3 \cdot 98 + 8^3 \cdot 90)$

$J_{t_1} = 32085{,}3\ mm^4$

$J_{p_2} = \frac{\pi \cdot d^4}{32} = 9{,}817 \cdot 10^6\ mm^4$

U.R. $\quad Ma + Mb = Mt \quad (1)$

$\varphi_c' = \frac{Ma \cdot a}{G J_{t_1}} \rightarrow Ma = \frac{\varphi_c' \cdot G \cdot J_{t_1}}{a} = \underline{24{,}674\ Nm}$

U.D. $\quad \varphi_A' = \frac{Ma \cdot a}{G J_{t_1}} + \frac{Ma \cdot a}{G J_{p_2}} + \frac{Ma \cdot a}{G J_{t_1}} - \frac{Mt \cdot a}{G J_{t_1}} = \phi \ \Big/ \frac{G \cdot J_{t_1}}{a} \quad (2)$

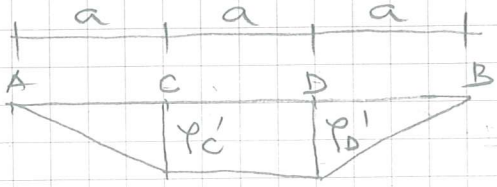
$Ma(1 + 0{,}002513 + 1) = Mt \rightarrow Mt = 2{,}002513 \cdot Ma$

$\underline{Mt = 49{,}41\ Nm} \qquad Mb = Mt - Ma = \underline{24{,}736\ Nm}$

$\tau_{max}^{①} = \frac{Ma}{J_{t_1}} \cdot t_{max} = \underline{6{,}152\ MPa}$

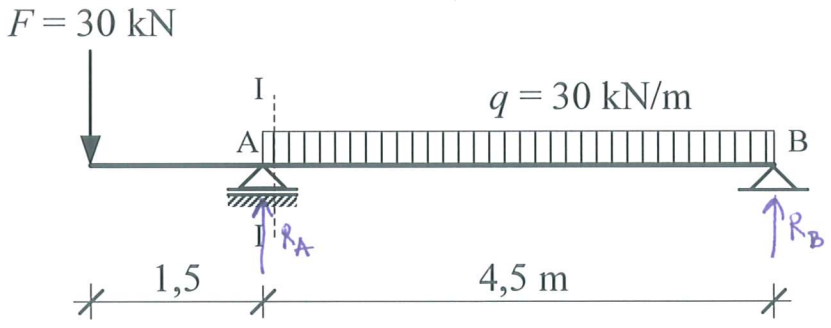
$\tau_{max}^{②} = \frac{Ma}{J_{p_2}} \cdot \frac{d}{2} = \underline{0{,}126\ MPa}$

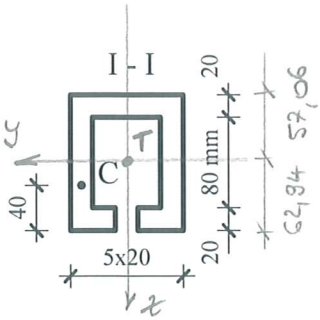
$\tau_{max}^{③} = \frac{Mb}{J_{t_1}} \cdot t_{max} = \underline{6{,}168\ MPa}$



$\varphi_D' = \varphi_{max}' = \frac{Mb \cdot a}{G J_{t_1}} = \underline{0{,}01003\ rad} = \underline{0° 34' 28''} = \varphi_{max}$

2.B)





$\sum M_A = 0$

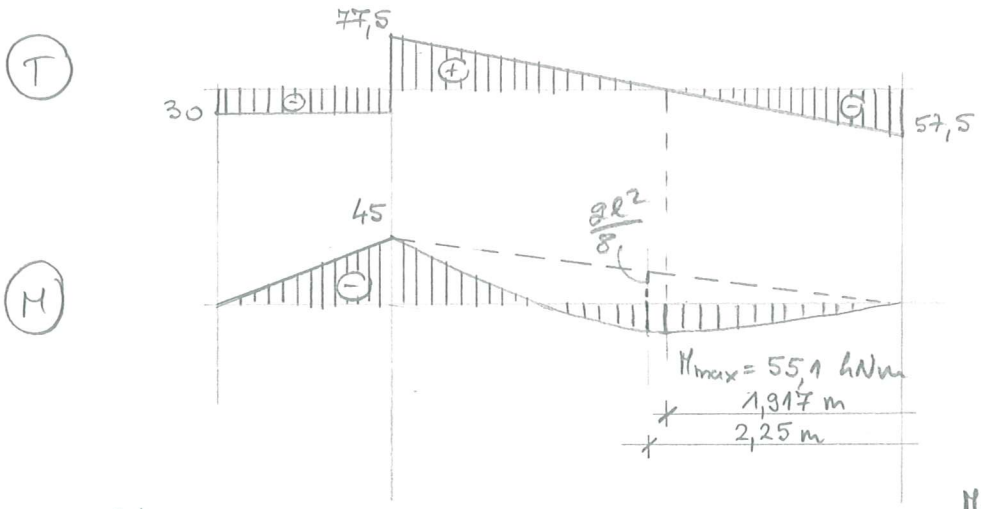
$30 \cdot 1,5 - 30 \cdot \frac{4,5^2}{2} + R_B \cdot 4,5 = 0$

$R_B = 57,5$ kN

$\sum M_B = 0$

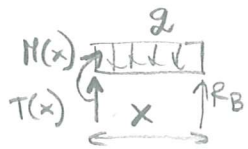
$R_A \cdot 4,5 - 30 \cdot 6 - \frac{30 \cdot 4,5^2}{2} = 0$

$R_A = 107,5$ kN



$\sum F_y^{desno} = 0 \quad R_B - 30x = -T(x) = 0$

$x = 1,917$ m



$M_{max} = 55,1$ kNm

$M_{max} + 30 \frac{1,917^2}{2} - R_B \cdot 1,917 = 0$

$z_T = \frac{\sum A_i z_i}{\sum A_i} = \frac{100 \cdot 120 \cdot 60 - 60 \cdot 80 \cdot 60 - 20 \cdot 20 \cdot 10}{100 \cdot 120 - 60 \cdot 80 - 20 \cdot 20}$

$= 62,94$ mm

$J_y = \frac{100 \cdot 120^3}{12} + 100 \cdot 120 \cdot 2,94^2 - \frac{60 \cdot 80^3}{12} - 60 \cdot 80 \cdot 2,94^2 - \frac{20 \cdot 20^3}{12} - 20 \cdot 20 \cdot 52,94^2 =$

$= 1,077 \cdot 10^7$ mm^4

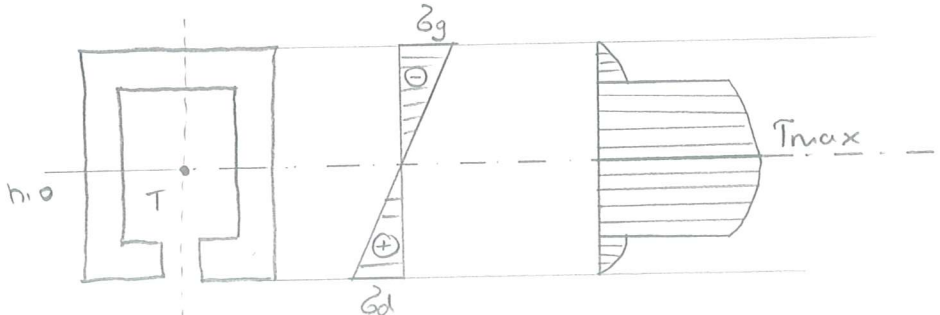
a) $\sigma_{max} = \frac{M_{max}}{J_y} \cdot z_{max}$

$\sigma_g = \frac{55,1 \cdot 10^6}{1,077 \cdot 10^7} \cdot (-57,06) = -291,92$ MPa

$\sigma_d = \frac{55,1 \cdot 10^6}{1,077 \cdot 10^7} \cdot 62,94 = 322,01$ MPa

$\tau_{max} = \frac{T_{max} \cdot S_{ymax}}{J_y \cdot b} = \frac{77,5 \cdot 10^3 \cdot 1,216 \cdot 10^5}{1,077 \cdot 10^7 \cdot 40} = 21,88$ MPa

$S_{ymax} = 100 \cdot 20 \cdot 47,06 + 2 \cdot 20 \cdot \frac{37,06^2}{2} = 1,216 \cdot 10^5$ mm^3



b) $\sigma_x^c = \frac{M_{1-1}}{J_y} z_c = \frac{-45 \cdot 10^6}{1{,}077 \cdot 10^7} \cdot 22{,}94 = -95{,}85$ MPa

$\tau_{xz}^c = \frac{T_{11} \cdot S_{yc}}{J_y \cdot b} = \frac{77{,}5 \cdot 10^3 \cdot 1{,}111 \cdot 10^5}{1{,}077 \cdot 10^7 \cdot 40} = 19{,}99$ MPa

$S_{yc} = 2(40 \cdot 20 \cdot 52{,}94 + 20 \cdot 20 \cdot 32{,}94) = 1{,}111 \cdot 10^5 \text{ mm}^3$

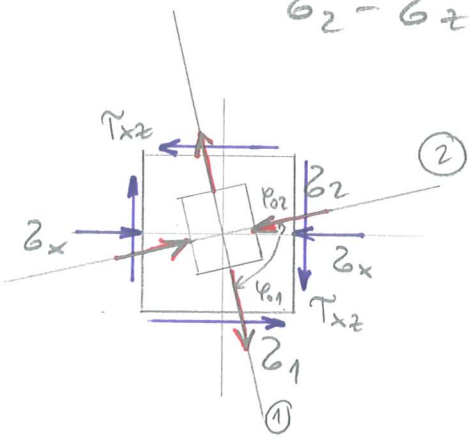
$\sigma_{1,2} = \frac{\sigma_x + \sigma_z}{2} \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_z)^2 + 4\tau_{xz}^2} =$

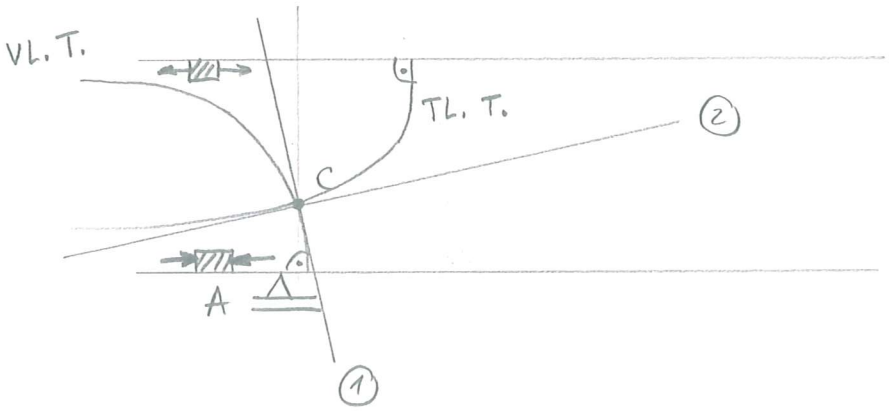
$= \frac{-95{,}85}{2} \pm \frac{1}{2}\sqrt{95{,}85^2 + 4 \cdot 19{,}99^2} = -47{,}93 \pm 51{,}93$

$\sigma_1 = 4{,}00$ MPa $\quad \sigma_2 = -99{,}86$ MPa

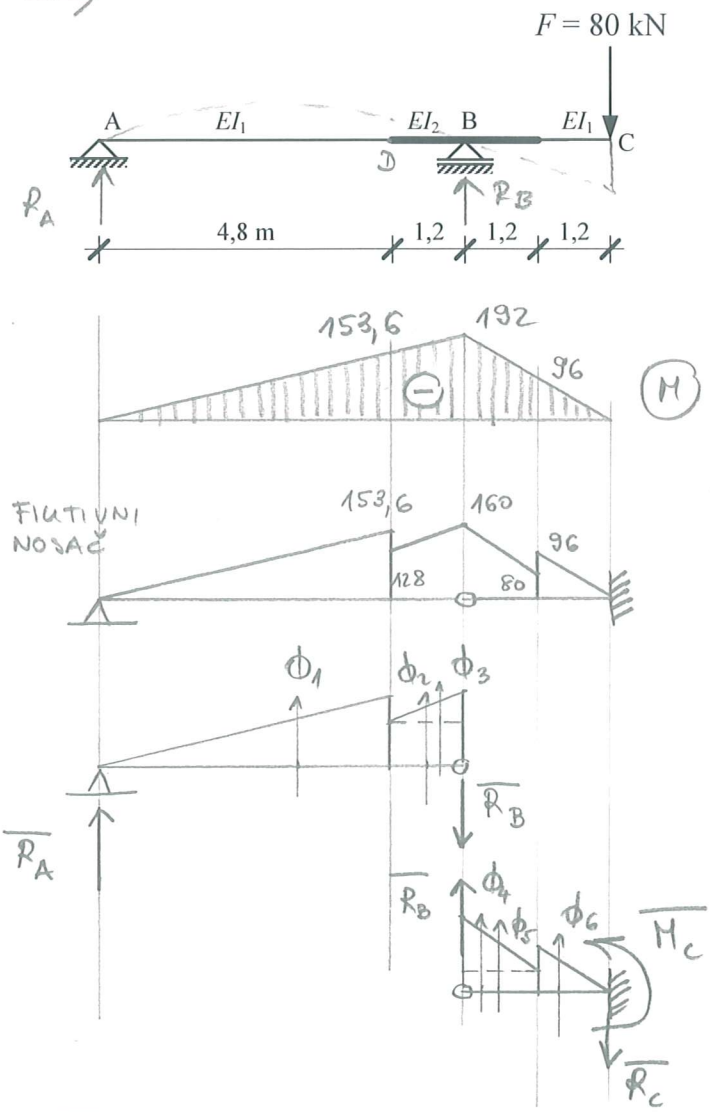
$tg\varphi_{01} = \frac{\tau_{xz}}{\sigma_1 - \sigma_z} = \frac{19{,}99}{4} = 4{,}9975 \Rightarrow \varphi_{01} = 78{,}67°$

$tg\varphi_{02} = \frac{\tau_{xz}}{\sigma_2 - \sigma_z} = \frac{19{,}99}{-99{,}86} = -0{,}2001 \Rightarrow \varphi_{02} = -11{,}31°$





3.B)



$E = 2{,}1 \cdot 10^5$ MPa

$J_1 = 3 \cdot 10^8$ mm^4

$J_2 = 1{,}2 \cdot J_1$

$\Sigma M_A = 0$

$F \cdot 8{,}4 - R_B \cdot 6 = 0 \quad R_B = 112$ kN

$R_A = -32$ kN

$\frac{192}{6} = \frac{M_D}{4{,}8} \quad M_D = -153{,}6$ kNm

$J_0 = J_1$

$M^* = M \frac{J_0}{J_1} = 153{,}6 \cdot \frac{1}{1{,}2} = 128$ kNm

$= \frac{192}{1{,}2} = 160$ kNm

$= \frac{96}{1{,}2} = 80$ kNm

$\Phi_1 = \frac{153{,}6 \cdot 4{,}8}{2} = 368{,}64$ kNm^2

$\Phi_2 = 128 \cdot 1{,}2 = 153{,}6$ kNm^2

$\Phi_3 = \frac{32 \cdot 1{,}2}{2} = 19{,}2$ kNm^2

$\Phi_4 = \frac{80 \cdot 1{,}2}{2} = 48$ kNm^2

$\Phi_5 = 80 \cdot 1{,}2 = 96$ kNm^2

$\Phi_6 = \frac{96 \cdot 1{,}2}{2} = 57{,}6$ kNm^2

$\Sigma \overline{M}_A = 0$

$\overline{R}_B \cdot 6 - \Phi_1 \cdot \frac{2}{3} 4{,}8 - \Phi_2 (4{,}8 + 0{,}6) - \Phi_3 \cdot (4{,}8 + 0{,}8) = 0$

$\overline{R}_B = 352{,}768$ kNm^2

$\Sigma \overline{F}_y = 0$

$\overline{R}_A = -188{,}672$ kNm^2

$\Sigma \overline{M}_C = 0$

$24 \cdot \overline{R}_B + \Phi_4 \cdot \left(1{,}2 + \frac{2}{3} \cdot 1{,}2\right) + \Phi_5 \cdot \left(1{,}2 + \frac{1.2}{2}\right) + \Phi_6 \cdot \frac{1.2 \cdot 2}{3} - \overline{M}_C = 0$

$\overline{M}_C = 1161{,}52$ kNm^3

$w_C = \frac{\overline{M}_C}{E I_0} = \frac{1161{,}52 \cdot 10^{12}}{2{,}1 \cdot 10^5 \cdot 3 \cdot 10^8} = 18{,}44$ mm

$\varphi_A = \frac{\overline{R}_A}{E I_0} = \frac{-188{,}672 \cdot 10^9}{2{,}1 \cdot 10^5 \cdot 3 \cdot 10^8} = -2{,}995 \cdot 10^{-3}$ rad