

2. KOLOKVIJ iz "OTPORNOSTI MATERIJALA 1" GRUPA A

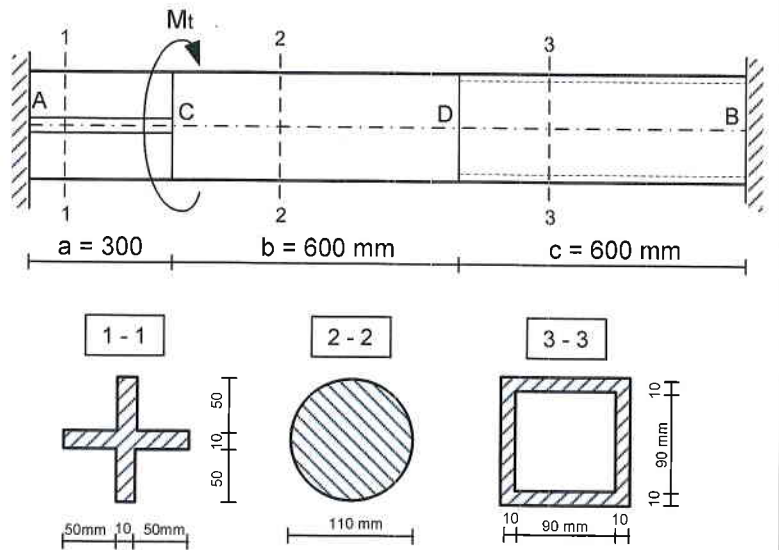
16. 01. 2017.

Ime i prezime: _____

1. Na štap prikazan na slici djeluje moment torzije M_t . Treba nacrtati dijagram momenata torzije i odrediti maksimalna naprezanja po pojedinim dijelovima štapa ako je kut zaokreta presjeka C $\varphi_C = 2^\circ$.

$$E = 2 \cdot 10^5 \text{ MPa}$$

$$\nu = 0.3$$

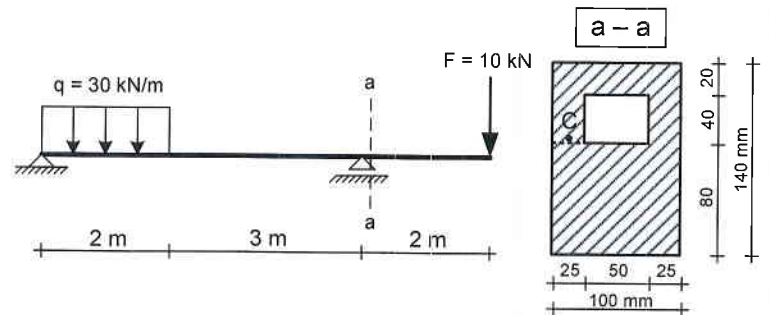


30

2. Za nosač prikazan na slici treba odrediti:

a) Maksimalno normalno i posmično naprezanje te nacrtati odgovarajuće dijagrame naprezanja u kritičnim presjecima

b) Veličinu i smjer glavnih naprezanja u točki C presjeka a - a



35

3. Grafoanalitičkim postupkom treba odrediti veličinu sile F uslijed koje je ukupni pomak točke C jednak 10 mm. Za izračunatu silu treba odrediti kut zaokreta točke A.

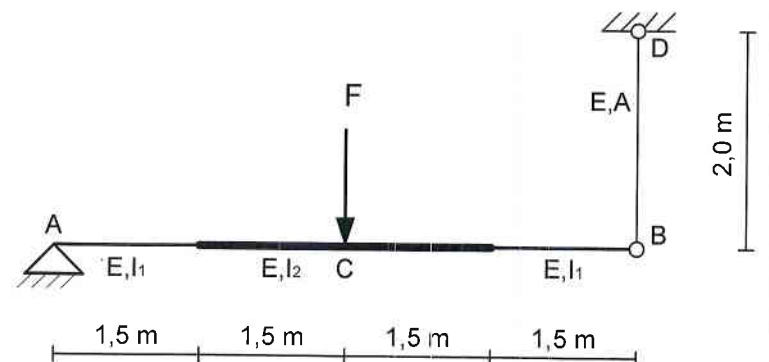
$$E = 2 \cdot 10^5 \text{ MPa}$$

$$I_1 = 1,2 \cdot 10^8 \text{ mm}^4$$

$$I_2 = 2I_1$$

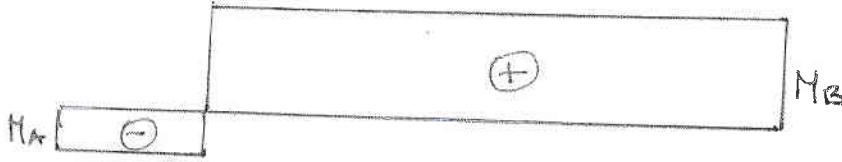
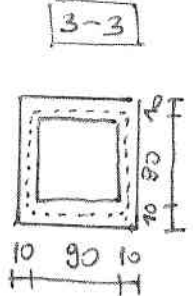
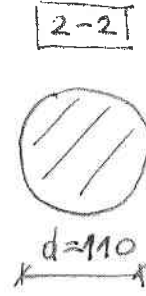
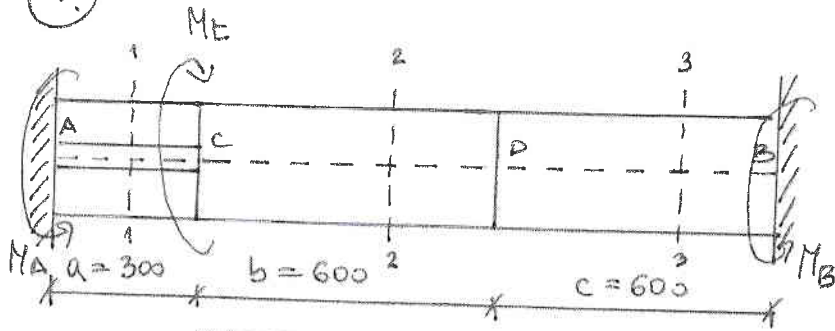
$$A = 200 \text{ mm}^2$$

$$w_C = 10 \text{ mm}$$



35

1.



$$E = 2 \cdot 10^5 \text{ MPa}$$

$$\nu = 0.3$$

$$G = \frac{E}{2(1+\nu)} = 76923 \text{ MPa}$$

U.R. $M_A + M_B = M_t$

$$\varphi_c' = \frac{M_A \cdot a}{G \cdot I_{t1}}$$

$$M_A = \frac{\varphi_c' \cdot G \cdot I_{t1}}{a}$$

$$I_{t1} = \frac{1}{3} (10^3 \cdot 110 + 2 \cdot 10^3 \cdot 50) = 70000 \text{ mm}^4$$

$$I_{p2} = \frac{\pi \cdot 110^4}{32} = 14,374 \cdot 10^6 \text{ mm}^4$$

$$I_{t3} = \frac{4 \cdot (100 \cdot 100)^2}{4 \cdot \frac{100}{10}} = 10 \cdot 10^6 \text{ mm}^4$$

$$\varphi_c = 2^\circ \quad \varphi_c' = \varphi_c \cdot \frac{\pi}{180} = 0.0349 \text{ rad}$$

$$M_A = \frac{0.0349 \cdot 76923 \cdot 70000}{300} = 0.627 \text{ kNm}$$

U.D. $\varphi_A = \frac{M_A \cdot a}{G \cdot I_{t1}} + \frac{M_A \cdot b}{G \cdot I_{p2}} + \frac{M_A \cdot c}{G \cdot I_{t3}} - \frac{M_t \cdot b}{G \cdot I_{p2}} - \frac{M_t \cdot c}{G \cdot I_{t3}} = \varphi \cdot \frac{G \cdot I_{t3}}{b}$

$$M_A \left(\frac{a}{b} \frac{I_{t3}}{I_{t1}} + \frac{I_{t3}}{I_{p2}} + \frac{c}{b} \right) = M_t \left(\frac{I_{t3}}{I_{p2}} + \frac{c}{b} \right) \quad c=b$$

$$73.124 M_A = 1,6957 M_t$$

$$M_t = 43,12 \text{ kNm}$$

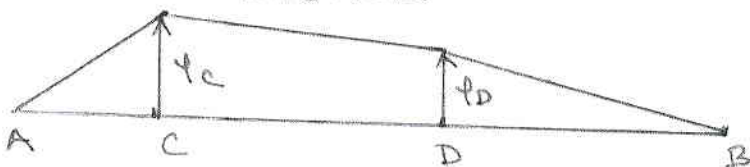
$$M_B = 42,50 \text{ kNm}$$

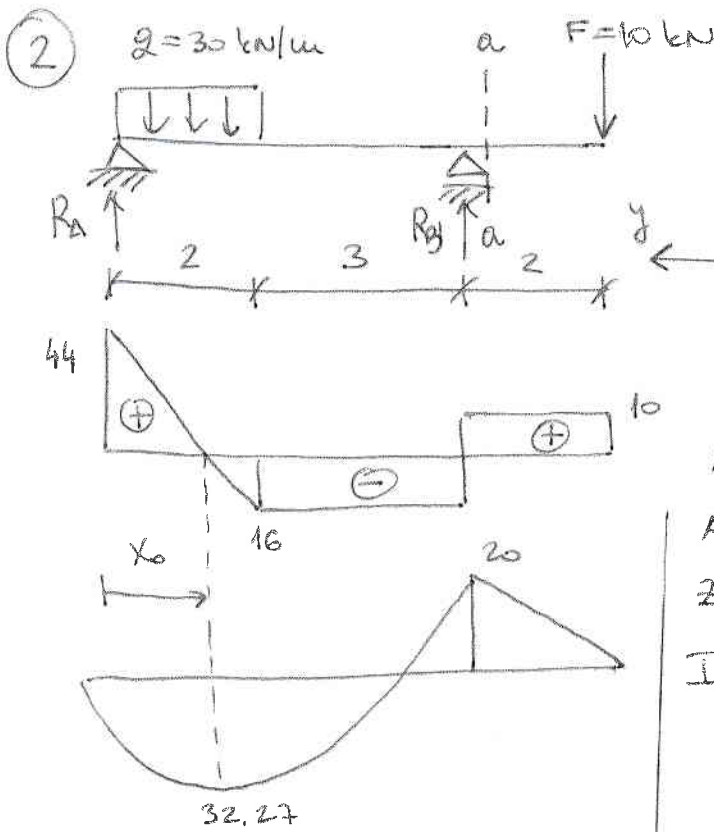
$$\tau_{\max}^{1-1} = \frac{M_A}{I_{t1}} \cdot t_{\max} = 89,6 \text{ MPa}$$

$$\varphi_D = \frac{M_B \cdot c}{G \cdot I_{p3}} = 0,03315 \text{ rad}$$

$$\tau_{\max}^{2-2} = \frac{M_B}{I_{p2}} \cdot \frac{d_1}{2} = 162,6 \text{ MPa}$$

$$\tau_{\max}^{3-3} = \frac{M_B}{2 \cdot A_0 \cdot t_{\min}} = 217,5 \text{ MPa}$$





$$\sum M_A = 0 \quad R_B = 26 \text{ kN}$$

$$\sum M_B = 0 \quad R_A = 44 \text{ kN}$$

$$q \cdot x_0 - R_A = 0 \quad x_0 = 1,467 \text{ m}$$

$$M_{\max} = R_A \cdot x_0 - \frac{q \cdot x_0^2}{2} = 32,27 \text{ kNm}$$

a) $M_{\max} = 32,27 \text{ kNm}$

$$T_{\max} = 44 \text{ kN}$$

$$\sigma_d = \frac{32,27 \cdot 10^6}{20,5 \cdot 10^6} \cdot 65 = 102,3 \text{ MPa}$$

$$\sigma_g = \frac{32,27 \cdot 10^6}{20,5 \cdot 10^6} \cdot (-75) = -118,1 \text{ MPa}$$

$$S_y = 100 \cdot 65 \cdot \frac{65}{2} = 211250 \text{ mm}^3$$

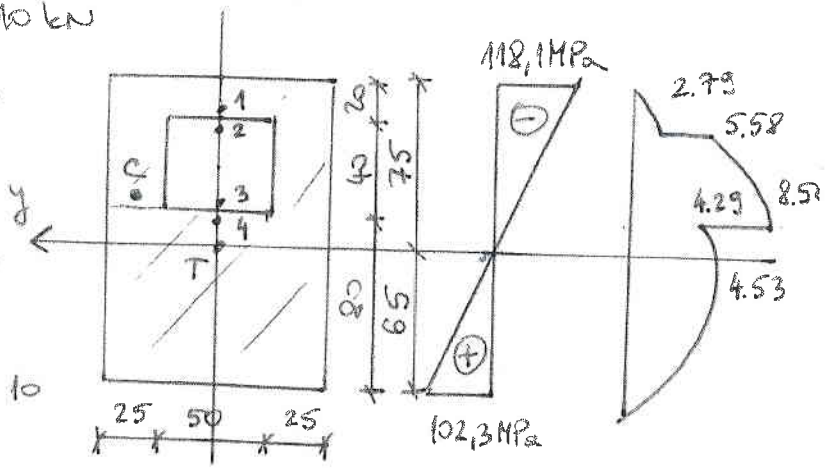
$$\tau_T = \frac{T_{\max} \cdot S_y}{I_y \cdot b} = 4,53 \text{ MPa}$$

$$S_{1,2} = 100 \cdot 20 \cdot 65 = 130000$$

$$S_{3,4} = 100 \cdot 60 \cdot 45 - 50 \cdot 40 \cdot 35 = 200000$$

$$\tau_1 = 2,79 \text{ MPa} \quad \tau_3 = 8,58 \text{ MPa}$$

$$\tau_2 = 5,58 \text{ MPa} \quad \tau_4 = 4,29 \text{ MPa}$$



$$A = 100 \cdot 140 - 50 \cdot 40 = 12000$$

$$z_T = 65 \text{ mm}$$

$$I_y = \frac{100 \cdot 140^3}{12} + 100 \cdot 140 \cdot 5^2 - \left[\frac{50 \cdot 40^3}{12} + 50 \cdot 40 \cdot 35^2 \right]$$

$$I_y = 20,5 \cdot 10^6 \text{ mm}^4$$

b) $M_{a-a} = -20 \text{ kNm}$

$$T_{a-a} = 10 \text{ kN}$$

$$\sigma_c = \frac{-20 \cdot 10^6}{20,5 \cdot 10^6} \cdot (-15) = 14,63 \text{ MPa}$$

$$\tau_c = \frac{T \cdot S_{3,4}}{I_y \cdot 50} = 1,95 \text{ MPa}$$

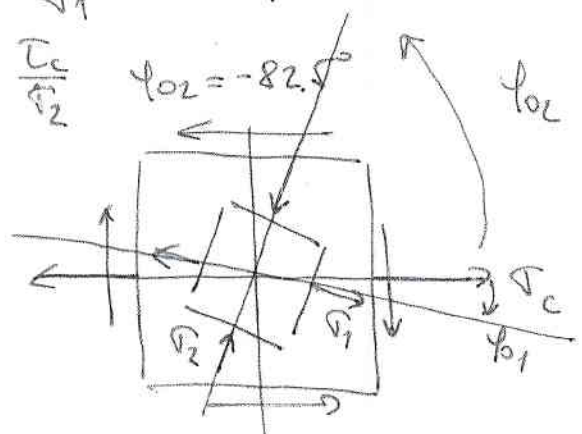
$$\sigma_{1,2} = \frac{\sigma_c}{2} \pm \frac{1}{2} \sqrt{\sigma_c^2 + 4\tau_c^2} = 7,32 \pm 7,57$$

$$\sigma_1 = 14,89 \text{ MPa}$$

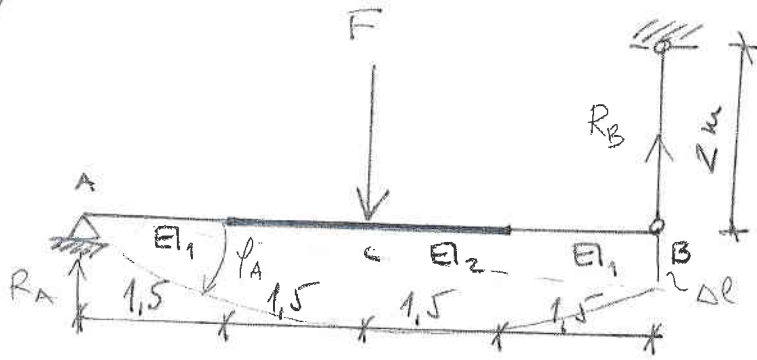
$$\sigma_2 = -0,25 \text{ MPa}$$

$$\phi_{01} = \frac{\tau_c}{\sigma_1} \quad \phi_{01} = 7,5^\circ$$

$$\phi_{02} = \frac{\tau_c}{\sigma_2} \quad \phi_{02} = -82,5^\circ$$



3.



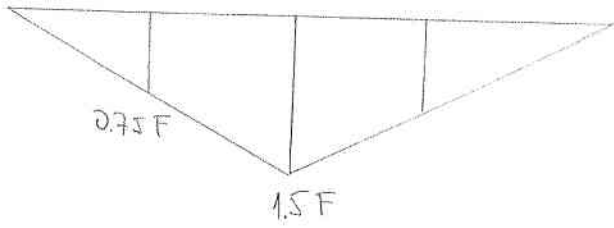
$$E = 2 \cdot 10^5 \text{ MPa}$$

$$I_1 = 1,2 \cdot 10^8 \text{ MPm}^4$$

$$I_2 = 2 I_1$$

$$A = 200 \text{ mm}^2$$

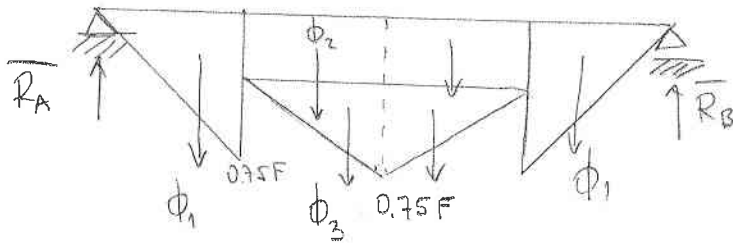
$$W_c = 10 \text{ mm}$$



$$R_A = R_B = \frac{F}{2} \text{ [N]}$$

$$\Delta l = \frac{R_B \cdot l}{E \cdot A} = \frac{F \cdot 2000}{2 \cdot 2 \cdot 10^5 \cdot 200}$$

$$\Delta l = 2,5 \cdot 10^{-5} F \text{ [mm]}$$



$$\frac{W_c'}{3} = \frac{\Delta l}{6}$$

$$W_c' = 0,5 \Delta l = 1,25 \cdot 10^{-5} F$$

$$I_0 = I_1 \Rightarrow M^* = M \frac{I_0}{I_2} = 1,5 F \cdot \frac{I_1}{2 I_1} = 0,75 F$$

$$\phi_1 = \frac{0,75 F \cdot 1,5}{2} = 0,5625 F$$

$$\phi_2 = 0,375 F \cdot 1,5 = 0,5625 F$$

$$\phi_3 = \frac{0,375 F \cdot 1,5}{2} = 0,28125 F$$

$$\bar{R}_A = \phi_1 + \phi_2 + \phi_3 = 1,40625 F \text{ [Nm}^2\text{]}$$

$$\bar{M}_c = \bar{R}_A \cdot 3 - \phi_1 \cdot (1,5 + 0,5) - \phi_2 \cdot \frac{1,5}{2} - \phi_3 \cdot 0,5 = 2,53125 F \text{ [Nm}^3\text{]}$$

$$W_c'' = \frac{\bar{M}_c}{E I_0} = \frac{2,53125 \cdot 10^9 F}{2 \cdot 10^5 \cdot 1,2 \cdot 10^8} = 1,055 \cdot 10^{-4} F$$

$$W_c = W_c' + W_c'' = 1,25 \cdot 10^{-5} F + 1,055 \cdot 10^{-4} F = 10$$

$$F = 84,76 \text{ kN}$$

$$\varphi_A' = \frac{\Delta l}{6000} = \frac{2,5 \cdot 10^{-5} \cdot 84760}{6000} = 3,532 \cdot 10^{-4} \text{ rad}$$

$$\varphi = \varphi_A' + \varphi_A''$$

$$= 5,32 \cdot 10^{-3} \text{ rad}$$

$$\varphi_A'' = \frac{\bar{R}_A}{E I_0} = \frac{1,40625 \cdot 84760 \cdot 10^6}{2 \cdot 10^5 \cdot 1,2 \cdot 10^8} = 4,966 \cdot 10^{-3} \text{ rad}$$