

2. KOLOKVIJ iz "OTPORNOSTI MATERIJALA 1" GRUPA C

18. 01. 2017.

Ime i prezime: _____

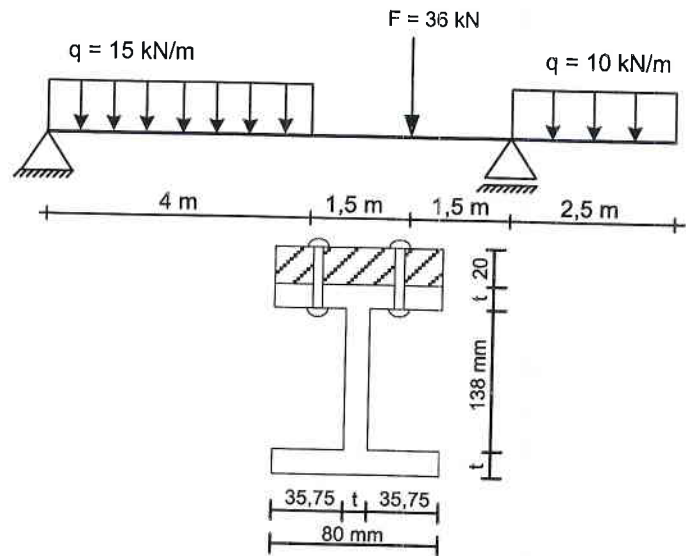
1. Za sastavljeni nosač na slici treba odrediti razmak zakovica „e“ ako je zadano:

$$\sigma_{dop} = 240 \text{ MPa}$$

$$\tau_{dop} = 120 \text{ MPa}$$

$$d = 12 \text{ mm}$$

$$t = 8,5 \text{ mm}$$

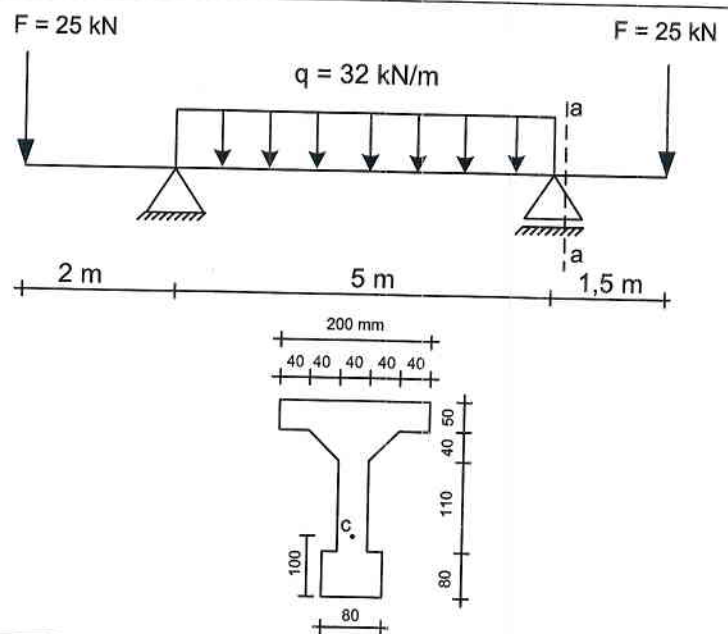


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2. Za nosač prikazan na slici treba odrediti:

a) Maksimalno normalno i posmično naprezanje te nacrtati odgovarajuće dijagrame naprezanja u kritičnim presjecima

b) Veličinu i smjer glavnih naprezanja u točki C presjeka a – a



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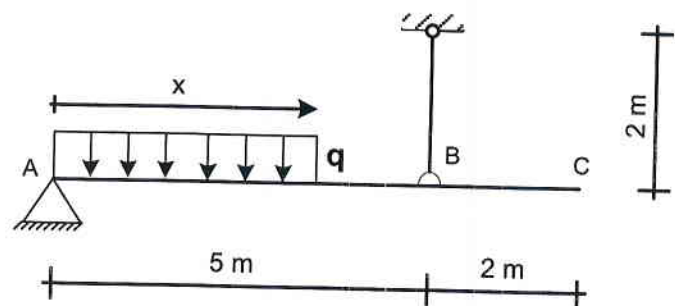
3. Odredite duljinu kontinuiranog opterećenja x tako da ukupni pomak točke B iznosi $\Delta_B = 3,125 \text{ mm}$. Za određenu vrijednost x **analitičkim postupkom** izračunajte ukupni pomak točke C i kut zaokreta točke A.

$$E = 2 \cdot 10^5 \text{ MPa}$$

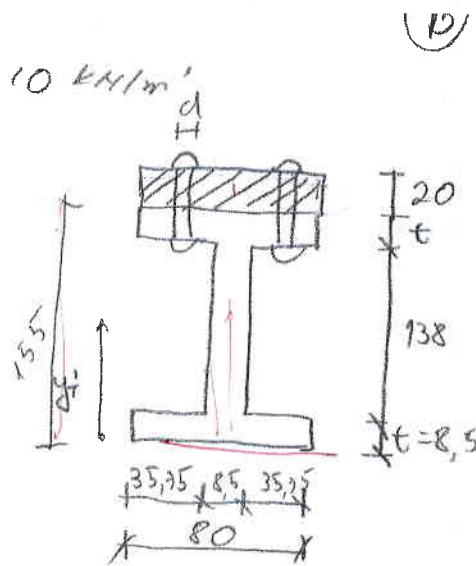
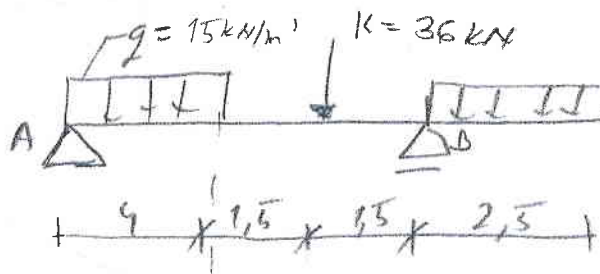
$$I = 5 \cdot 10^7 \text{ mm}^4$$

$$A = 40 \text{ mm}^2$$

$$q = 5 \text{ kN/m}$$

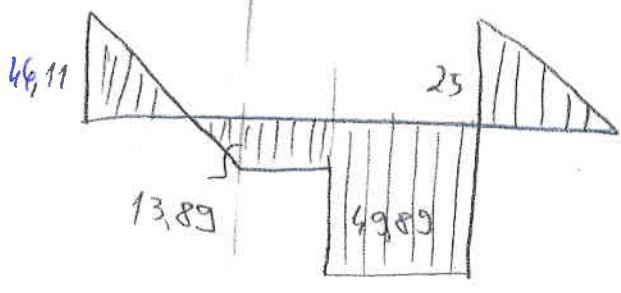


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$d = 12 \text{ mm}$
 $A = 2533 \text{ mm}^2$
 $y_T = 77,5 \text{ mm}$
 $I_y = 9,17 \cdot 10^6 \text{ mm}^4$
 $\sigma_{dop} = 120 \text{ MPa}$
 $\sigma_{dop} = 240 \text{ MPa}$

$\sum \Pi_A = d \quad R_B = 174,89 \text{ kN}$
 $\sum \Pi_B = d \quad R_A = 46,11 \text{ kN}$



$A_{uk} = 2533 + 20 \cdot 80 = 4133 \text{ mm}^2$
 $A_{uk} = 4133 \text{ mm}^2$

$y_T = \frac{2533 \cdot 77,5 + 20 \cdot 80 \cdot 165}{4133} = 111,37 \text{ mm}$

$I_y = 9,17 \cdot 10^6 + 2533 \cdot (111,37 - 77,5)^2 + \frac{80 \cdot 20^3}{12} + 80 \cdot 20 \cdot (165 - 111,37)^2$

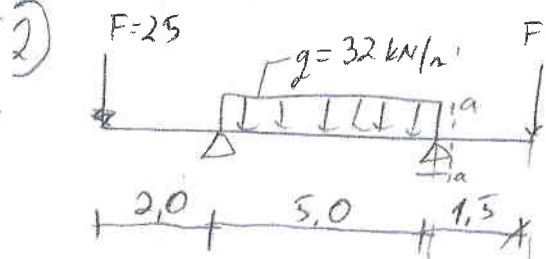
$I_y = 16,73 \cdot 10^6 \text{ mm}^4$

$S_y = 20 \cdot 80 \cdot (165 - 111,37) = 85808 \text{ mm}^3$

$1) \quad \tau_{max} = \frac{R_x}{2 \frac{d^2 \pi}{42}} \leq \sigma_{dop} \Rightarrow R_x \leq \sigma_{dop} \cdot \frac{d^2 \pi}{2} = 120 \cdot \frac{12^2 \pi}{2}$
 $R_x \leq 27,14 \text{ kN} \quad \boxed{17}$

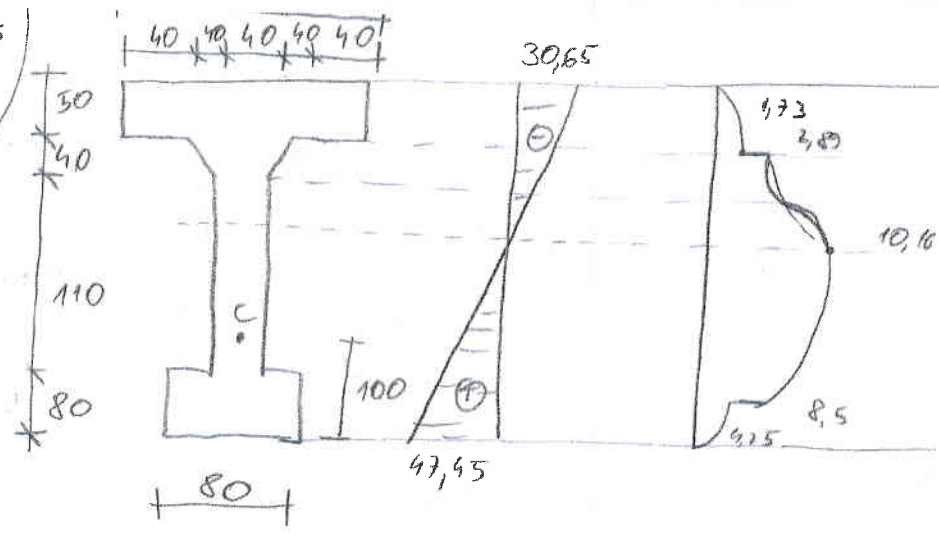
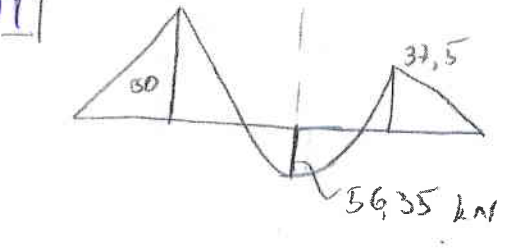
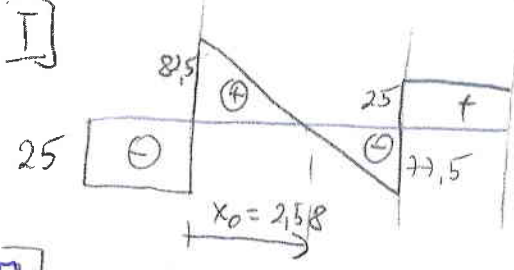
$2) \quad \sigma_{max} = \frac{R_x}{2 \cdot d \cdot t} \leq \sigma_{dop} \Rightarrow R_x \leq \sigma_{dop} \cdot 2 \cdot d \cdot t = 240 \cdot 2 \cdot 12 \cdot 8,5$
 $R_x \leq 48,96 \text{ kN}$

$R_x = \frac{T_{max} \cdot S_y}{I_y} \cdot e \Rightarrow e \leq \frac{R_x \cdot I_y}{T_{max} \cdot S_y} = \frac{27,14 \cdot 10^3 \cdot 16,73 \cdot 10^6}{48,96 \cdot 10^3 \cdot 85808}$
 $e \leq 106,1 \text{ mm} \quad e = 105 \text{ mm}$



$$\sum \Pi_A = \phi \rightarrow R_B = 102,5 \text{ kN}$$

$$\sum \Pi_B = \phi \rightarrow R_A = 107,5 \text{ kN}$$



$$A = 24000 \text{ mm}^2$$

$$z_T = \frac{200 \cdot 50 \cdot 255 + 150 \cdot 40 \cdot 155 + 2 \cdot \frac{1}{2} \cdot 40 \cdot 40 \cdot 216,67 + 80 \cdot 80 \cdot 90}{24000}$$

$$z_T = 170,11 \text{ mm}$$

$$I_y = \frac{80 \cdot 80^3}{12} + 80 \cdot 80 \cdot (170,11 - 40)^2 + \frac{40 \cdot 150^3}{12} + 40 \cdot 150 \cdot (170,11 - 155)^2$$

$$+ \frac{200 \cdot 50^3}{12} + 200 \cdot 50 \cdot (255 - 170,11)^2 + 2 \left[\frac{40 \cdot 40^3}{36} + \frac{1}{2} \cdot 40 \cdot 40 \cdot (216,67 - 170,11)^2 \right]$$

$$I_y = 2,02 \cdot 10^8 \text{ mm}^4$$

MAKSIMALNO NORMALNO NAPREZANJE

$$\sigma_{y_{grc}} = \frac{36,35 \cdot 10^6}{2,02 \cdot 10^8} \cdot (-109,89) = -39,65 \text{ MPa}$$

$$\sigma_{dofg} = \frac{56,35 \cdot 10^6}{2,02 \cdot 10^8} \cdot (170,11) = 47,45 \text{ MPa}$$

$$S_{y_{grc}} = 80 \cdot 80 \cdot (170,11 - 40) + 90,11 \cdot 40 \cdot (170,11 - 125,06) = 995082,22 \text{ mm}^3$$

$$\tau_{max} = \frac{82,5 \cdot 10^3 \cdot 995082,22}{2,02 \cdot 10^8 \cdot 40} = 10,16 \text{ MPa}$$

$$M_{a-a} = -37,5 \text{ kNm} \quad T_{a-a} = 25 \text{ kNm}$$

$$S_c = 80 \cdot 80 \cdot 130,11 + 40 \cdot 20 \cdot 80,11$$

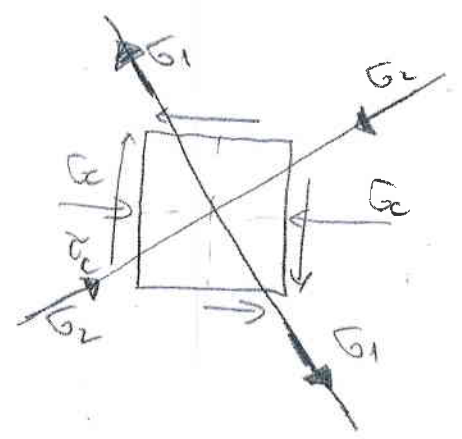
$$\sigma_c = \frac{-37,5 \cdot 10^6}{2,02 \cdot 10^8} \cdot 70,11 = -13,02 \text{ MPa}$$

$$\tau_c = \frac{T_{a-a} \cdot S_c}{I_y \cdot t_c} = \frac{25 \cdot 10^3 \cdot 896792}{2,02 \cdot 10^8 \cdot 40} = 2,77 \text{ MPa}$$

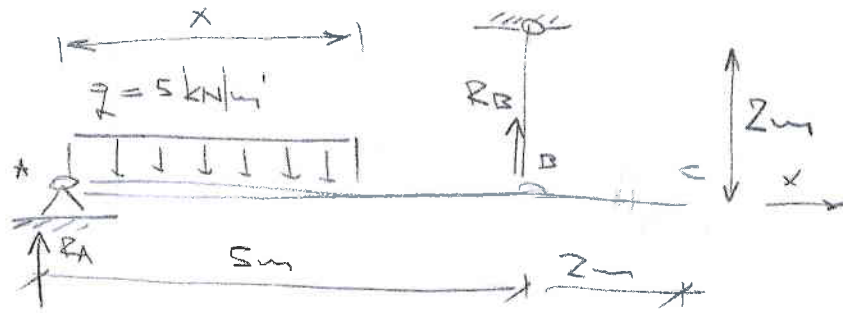
$$\sigma_{1,2} = \frac{\sigma_c}{2} \pm \frac{1}{2} \sqrt{\sigma_c^2 + 4 \cdot \tau_c^2} = \frac{-13,02}{2} \pm \frac{1}{2} \sqrt{(-13,02)^2 + 4 \cdot 2,77^2}$$

$$\sigma_1 = 0,56 \text{ MPa} \quad \sigma_2 = -13,58 \text{ MPa}$$

$$\tan \varphi_{01} = \frac{\tau_c}{\sigma_1} \Rightarrow \varphi_{01} = 78,57^\circ \quad \tan \varphi_{02} = \frac{\tau_c}{\sigma_2} \Rightarrow \varphi_{02} = -12,01^\circ$$



3. ZADATAK:



$$q = 5 \text{ kN/m}$$

$$E = 2 \cdot 10^5 \text{ MPa}$$

$$I = 5 \cdot 10^7 \text{ mm}^4$$

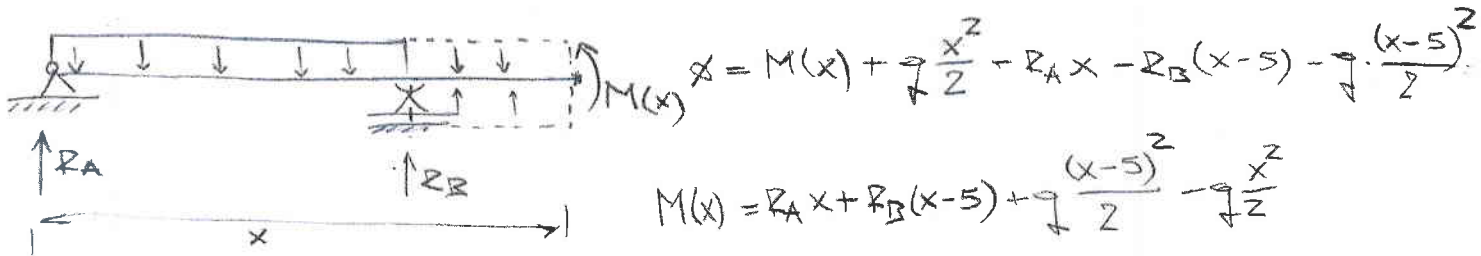
$$A = 40 \text{ mm}^2$$

$$J_B = 3,125 \text{ mm}$$

$$\Delta_B = \frac{R_B \cdot h}{E \cdot A} \rightarrow R_B = \frac{\Delta_B \cdot E \cdot A}{h} = \frac{3,125 \cdot 2 \cdot 10^5 \cdot 40}{2000}$$

$$R_B = 12,5 \text{ kN}$$

$$\sum M_A = 0 \rightarrow R_B \cdot 5 = q \cdot \frac{x^2}{2} \rightarrow x = 5 \text{ m} \quad (R_A = 12,5 \text{ kN})$$



$$0 = M(x) + q \frac{x^2}{2} - R_A x - R_B (x-5) - q \frac{(x-5)^2}{2}$$

$$M(x) = R_A x + R_B (x-5) + q \frac{(x-5)^2}{2} - q \frac{x^2}{2}$$

$$EI \frac{d^2 W}{dx^2} = q \frac{x^2}{2} - R_A x - R_B (x-5) - q \frac{(x-5)^2}{2} \quad || \int$$

$$EI \frac{dW}{dx} = q \frac{x^3}{6} - R_A \frac{x^2}{2} - R_B \frac{(x-5)^2}{2} - q \frac{(x-5)^3}{6} + C \quad || \int$$

$$EI \cdot W = q \frac{x^4}{24} - R_A \frac{x^3}{6} - R_B \frac{(x-5)^3}{6} - q \frac{(x-5)^4}{24} + Cx + D$$

$$W(0) = 0 \rightarrow D = 0$$

$$W(5) = 0 \rightarrow 0 = 5 \cdot \frac{5^4}{24} - 12,5 \cdot \frac{5^3}{6} + 5C \rightarrow C = 26,04 \text{ kNm}^2$$

$$W'_c = \frac{1}{EI} \left(5 \cdot \frac{7^4}{24} - 12,5 \cdot \frac{7^3}{6} - 12,5 \cdot \frac{2^3}{6} - 5 \cdot \frac{2^4}{24} + 26,04 \cdot 7 \right) = \frac{-52,095 \cdot 10^{12}}{2 \cdot 10^5 \cdot 5 \cdot 10^7}$$

$$W'_c = -5,21 \text{ mm}$$

$$W''_c = \frac{7}{5} \cdot 3,125 = 4,375 \text{ mm} \quad \left. \begin{array}{l} W'_c = -5,21 \text{ mm} \\ W''_c = 4,375 \text{ mm} \end{array} \right\} W_c = -0,835 \text{ mm}$$

$$\varphi'_A = \varphi'(0) = \frac{C}{EI} = \frac{26,04 \cdot 10^9}{2 \cdot 10^5 \cdot 5 \cdot 10^7} = 2,604 \cdot 10^{-3} \text{ rad} \quad \left. \begin{array}{l} \varphi'_A = 2,604 \cdot 10^{-3} \text{ rad} \\ \varphi''_A = 0,625 \cdot 10^{-3} \text{ rad} \end{array} \right\} \varphi_A = 3,229 \cdot 10^{-3} \text{ rad}$$

$$\varphi''_A = \frac{J_B}{5000} = 0,625 \cdot 10^{-3} \text{ rad}$$

