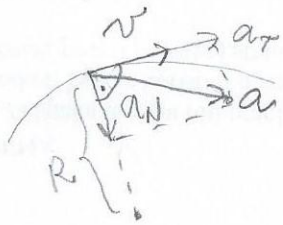
① 16.6.2014.



Rastavimo vektor ubrzanja: $\vec{a} = \vec{a}_N + \vec{a}_T$ $\quad \vec{a}_N \perp \vec{a}_T$ (okomiti)

$\vec{v} \parallel \vec{a}_T$ (kolinearni)

$\vec{v} \times \vec{a} = \vec{v} \times \vec{a}_N + \vec{v} \times \vec{a}_T$ ($\vec{v} \times \vec{a}_T = 0$ jer je $\alpha = 0$)

$|\vec{v} \times \vec{a}| = |\vec{v} \times \vec{a}_N| = |v| \cdot |a_N| \cdot \sin\alpha$ ($\sin\alpha = 1$)

$|\vec{v} \times \vec{a}| = v \cdot \frac{v^2}{R}$

$a_T = \frac{dv}{dt}$

$a_N = \frac{v^2}{R}$

$\vec{a} = \frac{d\vec{v}}{dt}$

$R = \frac{v^3}{|\vec{v} \times \vec{a}|}$

6 BODOVA

$\vec{r}(t) = (2 + 4\cos t)\vec{i} + (2 + 2\sin t)\vec{j}$

$\vec{v}(t) = \frac{d\vec{r}}{dt} = -4\sin t\,\vec{i} + 2\cos t\,\vec{j} \Rightarrow v(t) = \sqrt{16\sin^2 t + 4\cos^2 t}$

$\vec{a}(t) = \frac{d\vec{v}}{dt} = -4\cos t\,\vec{i} - 2\sin t\,\vec{j} \Rightarrow a(t) = \sqrt{16\cos^2 t + 4\sin^2 t}$

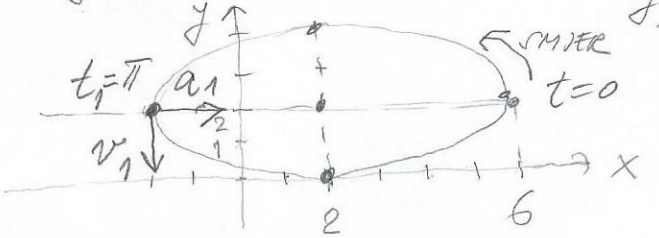
$x(t) = 2 + 4\cos t$

$y(t) = 2 + 2\sin t$

$\frac{x-2}{4} = \cos t \;/^2$

$\frac{y-2}{2} = \sin t \;/^2$ $\Big\}+ \Rightarrow \left(\frac{x-2}{4}\right)^2 + \left(\frac{y-2}{2}\right)^2 = 1$

TRAJEKTORIJA JE ELIPSA



$t_1 = \pi \Rightarrow x_1 = -2 \quad y_1 = +2$

$\vec{v}_1 = -2\vec{j} \Rightarrow v_1 = 2\ m/s$

$\vec{a}_1 = 4\vec{i} \quad a_1 = 4\ m/s^2$

$a_{1T} = \vec{a}_1 \cdot \frac{\vec{v}_1}{|\vec{v}_1|} = 0$ (vektori su okomiti!)

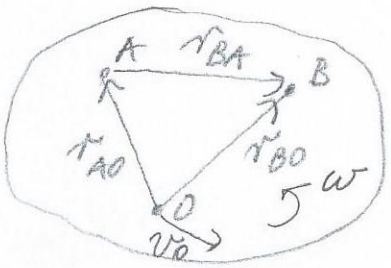
$\vec{a}_{1N} = \vec{a}_1 - \vec{a}_{1T} = \vec{a}_1 = 4\vec{i}$

$a_{1N} = \frac{v_1^2}{R_1} \Rightarrow R_1 = \frac{v_1^2}{a_{1N}} = \frac{4}{4} = 1$ $\quad \underline{R_1 = 1\ m}$

14 BODOVA

(2) TEOREM O IZBORU ISHODIŠTA

16.06.2014. – Promatra se gibanje krutog tijela i tri točke na tom tijelu. Ako je ishodište (POZNATA BRZINA)



$$\vec{v}_A = \vec{v}_O + \vec{\omega} \times \vec{r}_{AO} \quad \dots (1)$$

$$\vec{v}_B = \vec{v}_O + \vec{\omega} \times \vec{r}_{BO} \quad \dots (2)$$

POLOŽAJI TOČAKA POVEZANI SU VEKTORIMA

$$\vec{r}_{AO} + \vec{r}_{BA} = \vec{r}_{BO} \Rightarrow \vec{r}_{BA} = \vec{r}_{BO} - \vec{r}_{AO}$$

AKO OD (2) ODUZMEMO (1) =>

$$\vec{v}_B - \vec{v}_A = \vec{\omega} \times \vec{r}_{BO} - \vec{\omega} \times \vec{r}_{AO}$$

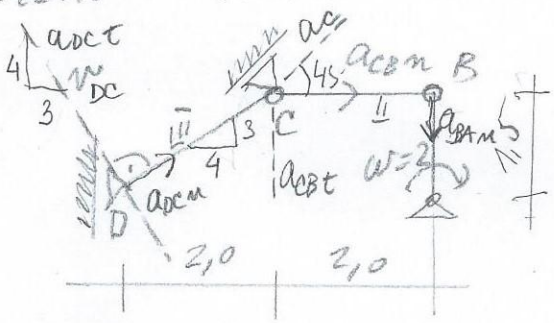
$$\vec{v}_B - \vec{v}_A = \vec{\omega} \times (\vec{r}_{BO} - \vec{r}_{AO})$$

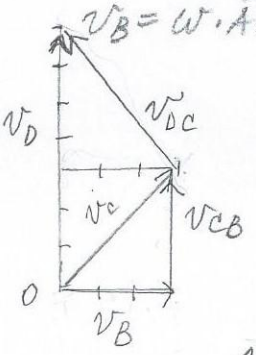
$$\vec{v}_B - \vec{v}_A = \vec{\omega} \times \vec{r}_{BA}$$

$$\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{BA}$$

DOBILI SMO BRZINU TOČKE B ODREĐENU POMOĆU BRZINE TOČKE A, ODNOSNO ISHODIŠTE JE SADA U TOČKI A, ŠTO ZNAČI DA TIJEKOM RJEŠAVANJA MOŽEMO BIRATI POKRETNO ISHODIŠTE.

5 BODOVA





$v_B = \omega \cdot \overline{AB} = 3 \text{ m/s}$ $\quad \vec{v}_B = 3\vec{i}$

$$\vec{v}_C = \vec{v}_B + \vec{v}_{CB}$$

$$\vec{v}_D = \vec{v}_C + \vec{v}_{DC}$$

OČITANO: $v_C = 4{,}2 \text{ m/s}$

$v_{CB} = 3 \text{ m/s} \Rightarrow \omega_{II} = \frac{3}{2} = 1{,}5 \text{ r/s}$

$\vec{\omega}_{II} = -1{,}5\vec{k}$

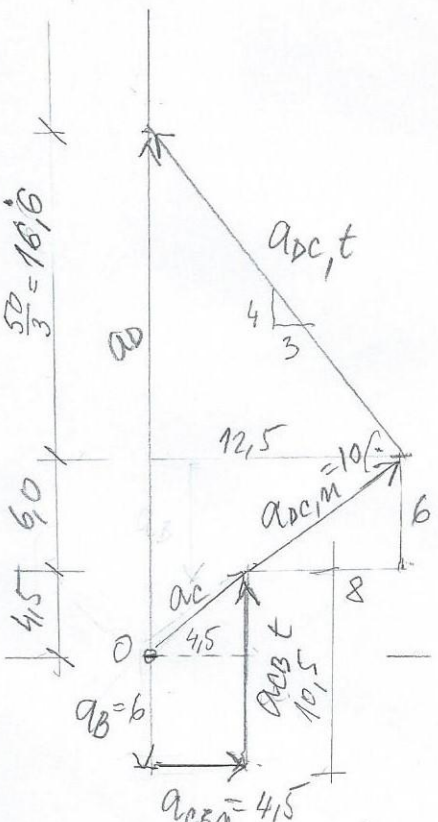
$v_D = 7 \text{ m/s}$

$v_{DC} = 5 \text{ m/s} \Rightarrow \omega_{III} = \frac{5}{2{,}5} = 2 \text{ r/s}$

$\vec{\omega}_{III} = -2\vec{k}$

BRZINE I $\vec{\omega}$

6 BODOVA



UBRZANJA: $\varepsilon_I = 0$

$$\vec{a}_B = \vec{a}_{BA,n} \qquad a_{BA\,n} = \omega_I^2 \cdot 1{,}5 = 6 \text{ m/s}^2$$

$$\vec{a}_C = \vec{a}_B + \vec{a}_{CB,n} + \vec{a}_{CB,t} \qquad a_{CBN} = \omega_{II}^2 \cdot 2 = 4{,}5 \text{ m/s}^2$$

OČITANO:

$a_{CB\,t} = 10{,}5 \Rightarrow \varepsilon_{II} = \frac{10{,}5}{2} = 5{,}25 \text{ r/s}^2$

$a_C = 6{,}4 \text{ m/s}$ $\qquad \vec{\varepsilon}_{II} = -5{,}25\vec{k}$

$$\vec{a}_D = \vec{a}_C + \vec{a}_{DC\,n} + \vec{a}_{DC\,t} \qquad a_{DC,n} = \omega_{III}^2 \cdot 2{,}5$$

OČITANO:

$a_{DC,n} = 10{,}0 \text{ m/s}^2$

$a_D = 27{,}17 \text{ m/s}^2$

$a_{DC,t} = 20{,}8 \text{ m/s}^2 \Rightarrow \varepsilon_{III} = \frac{20{,}8}{2{,}5} = 8{,}3$

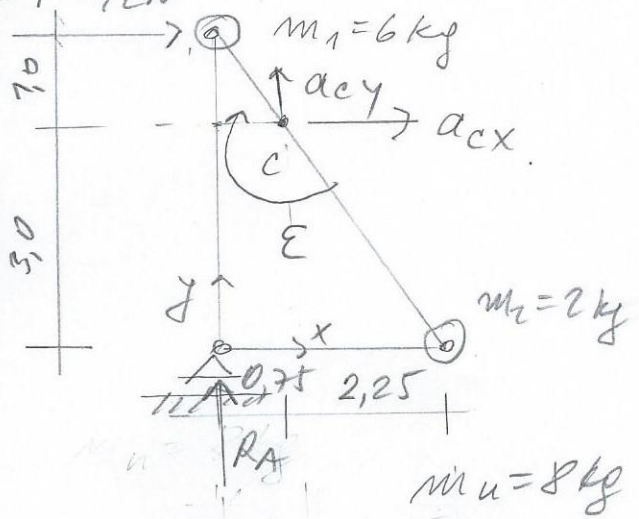
$\vec{\varepsilon}_{III} = -8{,}3 \text{ r/s}^2$

UBRZANJA i ε

9 BODOVA

③ 16.06.2014

F = 42 N



$m_u = 8\,kg$

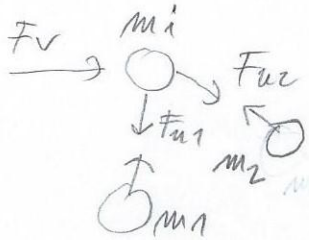
$8 \cdot x_c = 2 \cdot 3 \Rightarrow \underline{x_c = 0{,}75\,m}$

$8 \cdot y_c = 6 \cdot 4 \Rightarrow \underline{y_c = 3{,}0\,m}$

$J_c = 6(1 + 0{,}75^2) + 2(3^2 + 2{,}25^2) = 37{,}5$

$\underline{J_c = 37{,}5\ kgm^2}$

GIBANJE SUSTAVA ČESTICA pod djelovanjem sile u ravnini - primjena II Newtonovog aksioma



za svaku čestica

$m_i \vec{a}_i = \sum \vec{F}_v + \sum \vec{F}_u$

za cijeli sustav:

$m_u = \sum_{i=1}^{n} m_i$

Translacija:

$m_u \cdot \vec{a}_{CM} = \sum \vec{F}_v$

Rotacija oko CM

$J_{CM} \cdot \vec{\varepsilon} = \sum \vec{M}^{(F_v)}_{CM}$

$J_{CM} = \sum m_i \cdot d_{ic}^2$

J_{CM} ... moment tromosti mase

d_{ic} ... udaljenost i-te mase od centra mase | 4 BODA

TRANSLACIJA:

$m_u \vec{a}_c = \sum \vec{F}$ → $\vec{i}$ .... $8 \cdot a_{cx} = 42$

$\underline{a_{cx} = 5{,}25\ m/s^2}$

↓

$\vec{j}$ ... $8 \cdot a_{cy} = R_A$ ... (1)

ROTACIJA: $\underline{J_c \cdot \vec{\varepsilon} = \sum \vec{M}_c}$

$37{,}5 \cdot \varepsilon = F \cdot 1 + R_A \cdot 0{,}75$ ... (2)

(3) u (1) → $R_A = 8(-0{,}75 \cdot \varepsilon) = -6 \cdot \varepsilon$

u (2) → $37{,}5 \cdot \varepsilon = 42 - 6 \cdot \varepsilon \cdot 0{,}75$

$(37{,}5 + 4{,}5)\varepsilon = 42$

$\varepsilon = 1\,r/s^2 \Rightarrow \underline{\vec{\varepsilon} = -1\vec{k}\ r/s} \Rightarrow \underline{a_{cy} = -0{,}75\ m/s^2}$

KOMPATIBILNOST GIBANJA:

$\vec{a}_A = \vec{a}_c + \vec{a}_{AC,t}$

UVJET SPOJA:

$a_{Ay} = 0$

$0 = a_{cy} + 0{,}75 \cdot \varepsilon$

$a_{cy} = -0{,}75 \cdot \varepsilon$ ... (3)

$R_A = -6 \cdot 1 = -6\,N$ $\quad \underline{\vec{R}_A = -6\vec{j}\ (N)}$ $\quad \underline{\vec{a}_c = 5{,}25\vec{i} - 0{,}75\vec{j}}$

$\vec{a}_1 = \vec{a}_c + \vec{a}_{1c,t} = 5{,}25\vec{i} - 0{,}75\vec{j} + 1\vec{i} + 0{,}75 \cdot 1\vec{j}$

$\underline{\vec{a}_1 = 6{,}25\vec{i}\ (m/s^2)}$

16 BODOVA

(4) Djelovanje impulsa na tijelo – gibanje u ravnini. Gibanje se rastavlja na translaciju u centru mase tijela i rotaciju oko centra mase:

– Translacija (iz 2. N. aksioma !) $m\vec{v} = \vec{Q}$ količina gibanja

$$\frac{d}{dt}(m_u \vec{v}_c) = \sum \vec{F} = \vec{R} \Rightarrow d(m_u \cdot \vec{v}_c) = \vec{R}\,dt \Big/ \int_{t_1}^{t_2}$$

$$m\vec{v}_c\Big|_{t_1}^{t_2} = \int_{t_1}^{t_2} \vec{R}\,dt \Rightarrow m\vec{v}_c(t_2) - m\vec{v}_c(t_1) = \underbrace{\int_{t_1}^{t_2} \vec{R}\,dt}_{\text{impuls } \vec{S}}$$

– Rotacija oko CM.

$$\frac{d}{dt}(\vec{K}_c) = \sum \vec{M}_c^{(F_v)} \Big/ \cdot dt \int_{t_1}^{t_2}$$

K_c ... kinetički moment ili mom. količ. g.v.

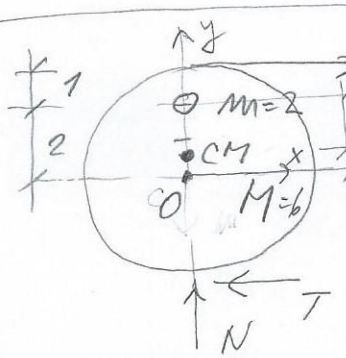
$$\vec{K}_{c2} - \vec{K}_{c1} = \int_{t_1}^{t_2} \vec{M}_c^{(F)}\,dt = \vec{M}_c^{(S)}$$

moment od impulsa na CM.

za gibanje u ravnini

$$\vec{K}_c = J_c \cdot \vec{\omega} \Rightarrow J_c\vec{\omega}_2 - J_c\vec{\omega}_1 = \sum \vec{M}^{(S)}$$

7 BODOVA



$S = 18\,Ns$ $\quad m_u = 8\,kg$ $\quad (M=6,\ m=2)$

$8 \cdot J_{CM} = 2 \cdot 2 \Rightarrow y_{CM} = 0{,}5\,m$

$$J_{CM} = 6 \cdot 0{,}5^2 + 2 \cdot 1{,}5^2 + \frac{1}{2} 6 \cdot 3^2 = 33\,kg\,m^2$$

ZA POČETNI TRENUTAK ! (nema integrala sila)

– TRANSLACIJA :

$$m_u \vec{v}_{c2} - m_u \vec{v}_{c1} = \vec{S} \quad \rightarrow \vec{i} \quad 8 v_{cx} = 18$$

$$v_{cx} = 2{,}25\,m/s$$

$\vec{j}$ ... $8 \cdot v_{cy} = 0 \Rightarrow v_{cy} = 0$

$$\vec{v}_c = 2{,}25\,\vec{i}$$

ROTACIJA

$$J_c \cdot \vec{\omega}_2 - J_c \vec{\omega}_1 = \sum \vec{M}_c^{S} \Rightarrow$$

$$33 \cdot \omega = 18 \cdot 2{,}5$$

$$\omega = 1{,}36\,s^{-1}$$

$$\vec{\omega} = -1{,}36\,\vec{k}$$

BRZINA CENTRA DISKA

$$\vec{v}_0 = \vec{v}_{CM} + \vec{v}_{OCM}$$

$$\vec{v}_0 = 2{,}25\,\vec{i} - 1{,}36 \cdot 0{,}5\,\vec{i} = +1{,}57\,\vec{i}$$

$$\vec{v}_B = \vec{v}_0 + \vec{v}_{BO} = 1{,}57\vec{i} - 1{,}36 \cdot 3\,\vec{j} = 1{,}57\,\vec{i} - 4{,}08\,\vec{j}$$

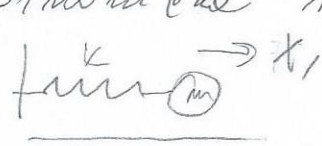
$$v_B = \sqrt{v_{Bx}^2 + v_{By}^2} \qquad v_B = \sqrt{1{,}5^2 + 4{,}08^2} = 4{,}37\,m/s$$

13 BODOVA

16.06.2014.

(5) Postupci određivanja dif. jednadžbe slobodnih oscilacija:

1) Dinamička ravnoteža – D'Alembertov princip



$\vec{F}_i = -m\vec{\ddot{x}}$ $\quad F_{el} = k \cdot x$ $\quad \underline{m\ddot{x} + kx = 0}$

2) Zakon očuvanja mehaničke energije

$E_{kin} + E_{pot} = const \;/\frac{d}{dt}$

3) Metoda virtualnog rada

$F_i \cdot \delta x + F_{el} \cdot \delta x = 0$ 3 boda

---

a) $E_{k\,red} = E_{k\,stv.}$

$\frac{1}{2} m_r \dot{x}^2 = \frac{1}{2} J_0 \cdot \left(\frac{\dot{x}}{3}\right)^2$

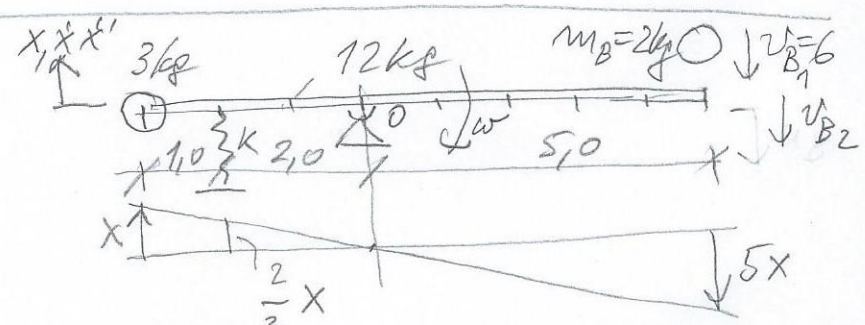
$m_r = \frac{103}{9} = \underline{11,4\ kg}$

$E_{pot_r} = E_{pot\,stv.}$

$\frac{1}{2} k_r x^2 = \frac{1}{2} k \left(\frac{2}{3}x\right)^2$

$k_r = 12000 \cdot \frac{4}{9} = \underline{5333,3\ N/m}$

dif. jed. $11,44\ddot{x} + 5333,3x = 0 \Rightarrow \underline{\ddot{x} + 466,02x = 0}$



$J_0 = \frac{12 \cdot 8^2}{12} + 12 \cdot 1^2 + 3 \cdot 3^2 = 103\ kgm^2$

b) $\Omega = \sqrt{\frac{k_r}{m_r}} = 21,5875\ r/s$

$T = \frac{2\pi}{\Omega} = 0,291\,s$

c) $x(t) = x_0 \cdot \cos \Omega t + \frac{v_0}{\Omega} \cdot \sin \Omega t$

UTROK OSCILACIJA – SRAZ (otvoreke su na crtezu)

$e = 1 = \frac{5\omega - v_{B2}}{v_{B_1}} \Rightarrow v_{B2} = 5\omega - 6 \quad ..(1)$

ŠTAP ROTIRA OKO O:

$K_{omji} = K_{poslije}$

$2 \cdot 6 \cdot 5 = J_0 \omega + m_B v_{B2} \cdot 5$

$60 = 103\omega + 10(5\omega - 6) \Rightarrow$

$120 = 153 \cdot \omega \Rightarrow \underline{\omega = 0,784\ r/s}$

$v_0 = 3\omega = 2,35\ m/s \quad (x_0 = 0)$

$x(t) = \frac{2,35}{21,59} \sin 21,59t$

$\underline{x(t) = 0,10886 \sin 21,59t}$

---

d) $F_0 = k \cdot \delta_0 = F_{ost} + F_0(t)$

statičke deformacija:

$\Sigma M_O = 0$

$3g \cdot 3 - 12g \cdot 1 - F_{ost} \cdot 2 = 0$

$F_{ost} = \frac{3}{2} g = 14,715\ N$

$F_0(t)_{MAX} = k \frac{2}{3} x(t)_{MAX}$

$F_0(t)_{MAX} = 12000 \cdot \frac{2}{3} \cdot 0,10886$

$= 870,88$

$\underline{F_{0\,MAX} = 885,6\ N}$

17 BODOVA

RJEŠENJA ISPITA 19.02.2014.

① $a_{1T} = 4{,}8\ m/s^2$, $a_{1N} = 6{,}4\ m/s^2$, $R_1 = 3{,}9\ m$ (14 BOD)

– ZA TEORIJSKO PITANJE VIDI STR. 11 SKRIPTE NA STRANICI PREDMETA www.grad.unizg.hr (6 BODOVA)

② $v_B = 1{,}5\ m/s$, $a_B = 2{,}1\ m/s^2$, $\omega_1 = 0$, $\vec{\omega}_2 = -0{,}5\vec{k}$, $\vec{\omega}_3 = \vec{k}$

$v_C = 2{,}1\ m/s$, $a_C = 1{,}06\ m/s^2$, $\vec{\varepsilon}_1 = 0{,}5\vec{k}$, $\vec{\varepsilon}_2 = -0{,}75\vec{k}$, $\vec{\varepsilon}_3 = \vec{k}$

16 BODOVA

4 BODA: TEORIJSKI DIO VIDI 4. PREDAVANJE, slide 9.

③ a) $v_{1x} = 5{,}196\ m/s$, $v_{1y} = 1{,}0\ m/s$

b) $d = 1{,}2\ m$

c) $v_2 = 6{,}9\ m/s$

} 15 BODOVA

TEORIJSKI DIO 5 BODOVA, 11. PREDAVANJE 1. slide

④ $A = 10{,}124\ N$, $F_1 = -12{,}656\ N$, $F_2 = 32{,}91\ N$, $F_3 = 7{,}594\ N$

16 BODOVA

TEORIJSKI DIO 4 BODA:

SAŽETO PRIKAZATI OSNOVNE RELACIJE IZ 10. PREDAVANJA, slide 5, 6 i 11 } 4. BODA

⑤ 1. $E_{pot}(t) = C_3 + \frac{1}{2}k \cdot \frac{24}{25}x^2(t)$ ------ 4 BODA

2. $\ddot{x} + 131{,}74x = 0$

3. $x(t) = 0{,}0624 \cos 11{,}48t + 0{,}0104 \sin 11{,}48t$

4. $x(t) = 0{,}0633 \cdot \sin(11{,}48t + \alpha)$

$\delta_{DMAX} = \frac{4}{5}(0{,}0624 + 0{,}0633) = 0{,}1005\ m$

$\delta_{DMAX} = 10{,}05\ cm$

} 16 BODOVA