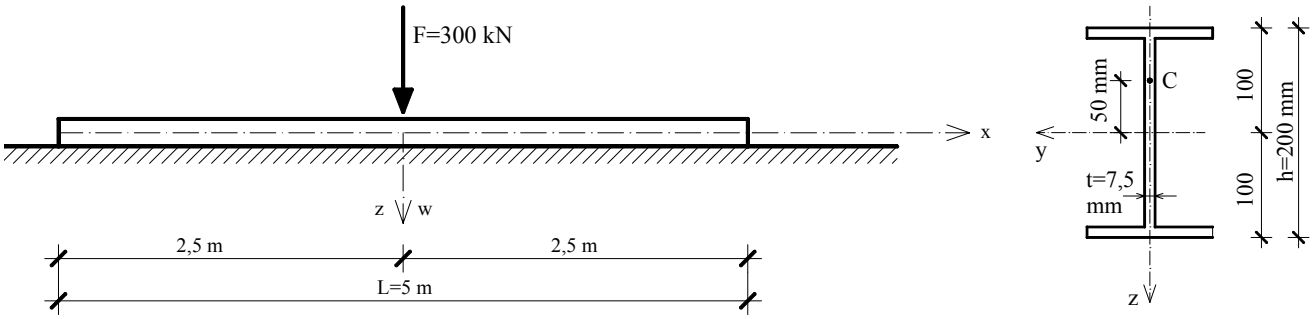
Za zadani nosač položen na elastičnu podlogu, prikazan na slici, treba odrediti progibnu liniju nosača te dijagrame unutrašnjih sila. Nadalje, treba odrediti najveća normalna i najveća posmična naprezanja u nosaču te glavna naprezanja u točki „C“ presjeka x=0.
Zadani su: koeficijent podloge $k = 900\,MPa$, modul elastičnosti $E = 2{,}1 \cdot 10^5\,MPa$, aksijalni moment tromosti $I_y = 2{,}14 \cdot 10^7\,mm^4$, statički moment površine polovice presjeka $S_y = 1{,}17 \cdot 10^5\,mm^3$



Parametar α koji ovisi o odnosu krutosti elastične podloge i nosača:

$$\alpha = \sqrt[4]{\frac{k}{4 \cdot E \cdot I_y}} = \sqrt[4]{\frac{900}{4 \cdot 2{,}1 \cdot 10^5 \cdot 2{,}14 \cdot 10^7 \cdot 10^{-12}}} = 2{,}66\,\frac{1}{m}$$

$\alpha \cdot L = 2{,}66 \cdot 5 = 13{,}3 > 5 \;\rightarrow$ nosač se može promatrati kao beskonačno dug

Jednadžba elastične linije nosača:

$$w = \frac{F}{8 \cdot E \cdot I_y \cdot \alpha^3} \cdot e^{-\alpha x} \cdot \left[\cos(\alpha x) + \sin(\alpha x)\right]$$

Jednadžba za kutove zaokreta:

$$\varphi = w' = -\frac{F}{4 \cdot E \cdot I_y \cdot \alpha^2} \cdot e^{-\alpha x} \cdot \sin(\alpha x)$$

Jednadžba za momente savijanja:

$$M = -E \cdot I_y \cdot w'' = \frac{F}{4 \cdot \alpha} \cdot e^{-\alpha x} \cdot \left[\cos(\alpha x) - \sin(\alpha x)\right]$$

Jednadžba za poprečne sile:

$$T(x) = -E \cdot I_y \cdot w''' = -\frac{F}{2} \cdot e^{-\alpha x} \cdot \cos(\alpha x)$$

Da bismo odredili tražene dijagrame, odredimo vrijednosti u karakterističnim presjecima:

<u>Progibna linija</u>

Za x=0 $\qquad w = \frac{F}{8 \cdot E \cdot I_y \cdot \alpha^3} = 0{,}433\;\; mm$

Nultočke progibne linije (točke u kojima je progib 0):

$$w = 0 \;\Rightarrow\; \cos(\alpha x) + \sin(\alpha x) = 0 \;\Rightarrow\; tg(\alpha x) = -1 \;\Rightarrow\; \alpha x_0 = \frac{3}{4}\pi, \frac{7}{4}\pi, \frac{11}{4}\pi \ldots$$

$$x_0 = \frac{3 \cdot \pi}{4 \cdot \alpha} = 0{,}885 \ \ m$$

$$x_0 = \frac{7 \cdot \pi}{4 \cdot \alpha} = 2{,}066 \ \ m$$

$$x_0 = \frac{11 \cdot \pi}{4 \cdot \alpha} = 3{,}246 \ \ m$$

Nagib tangente na progibnu liniju

Za x=0 $\varphi = w' = 0$

Nultočke: $\varphi = 0 \ \Rightarrow \ \sin(\alpha x) = 0 \ \ \Rightarrow \ \ \alpha x_0 = 0, \pi, 2\pi, 3\pi ...$

$$x_0 = 0$$

$$x_0 = \frac{\pi}{\alpha} = 1{,}180 \ \ m$$

$$x_0 = \frac{2\pi}{\alpha} = 2{,}360 \ \ m$$

$$x_0 = \frac{3\pi}{\alpha} = 3{,}541 \ \ m$$

Moment savijanja

Za x=0 $M = M_{\max} = \frac{F}{4 \cdot \alpha} = 28{,}195 \ \ kNm$

Nultočke: $M = 0 \ \ \Rightarrow \ \ \cos(\alpha x) - \sin(\alpha x) = 0 \ \ \Rightarrow \ \ tg(\alpha x) = 1 \ \ \Rightarrow \alpha x_0 = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4} ...$

$$x_0 = \frac{\pi}{4\alpha} = 0{,}295 \ \ m$$

$$x_0 = \frac{5\pi}{4\alpha} = 1{,}475 \ \ m$$

$$x_0 = \frac{9\pi}{4\alpha} = 2{,}656 \ \ m$$

Poprečna sila

Za x=0 $T = \frac{-F}{2} \ ; \ |T_{\max}| = \frac{F}{2} = 150 \ \ kN$

Nultočke: $T = 0 \ \ \Rightarrow \ \ \cos(\alpha x) = 0 \ \ \Rightarrow \ \ \alpha x_0 = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} ...$

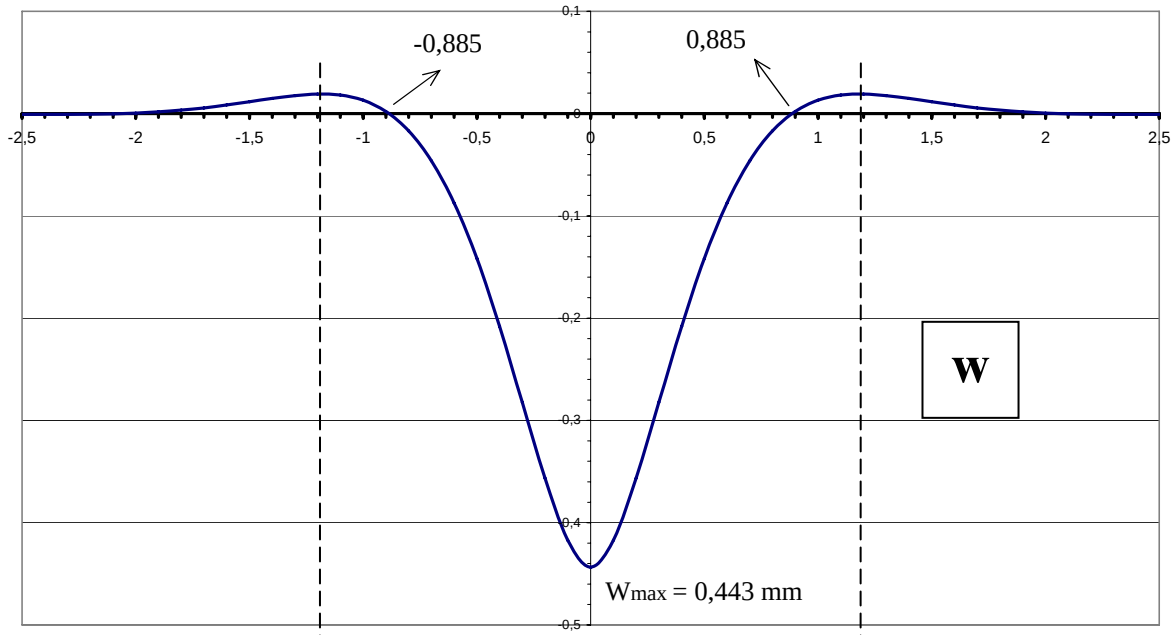
$$x_0 = \frac{\pi}{2\alpha} = 0{,}590 \ \ m$$

$$x_0 = \frac{3\pi}{2\alpha} = 1{,}771 \ \ m$$

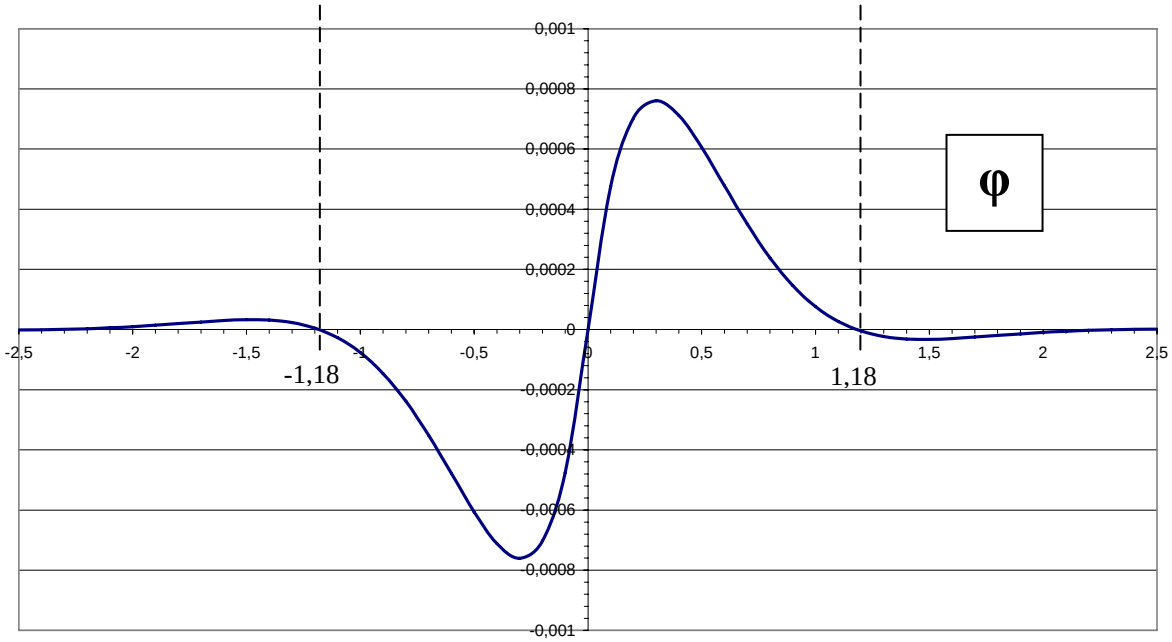
$$x_0 = \frac{5\pi}{2\alpha} = 2{,}951 \ \ m$$

Gore navedene jednadžbe vrijede za $x \geq 0$. Za dio za koji je $x \leq 0$ koristit ćemo činjenicu da su funkcije progiba i momenta simetrične a kuta zaokreta i poprečne sile antimetrične.

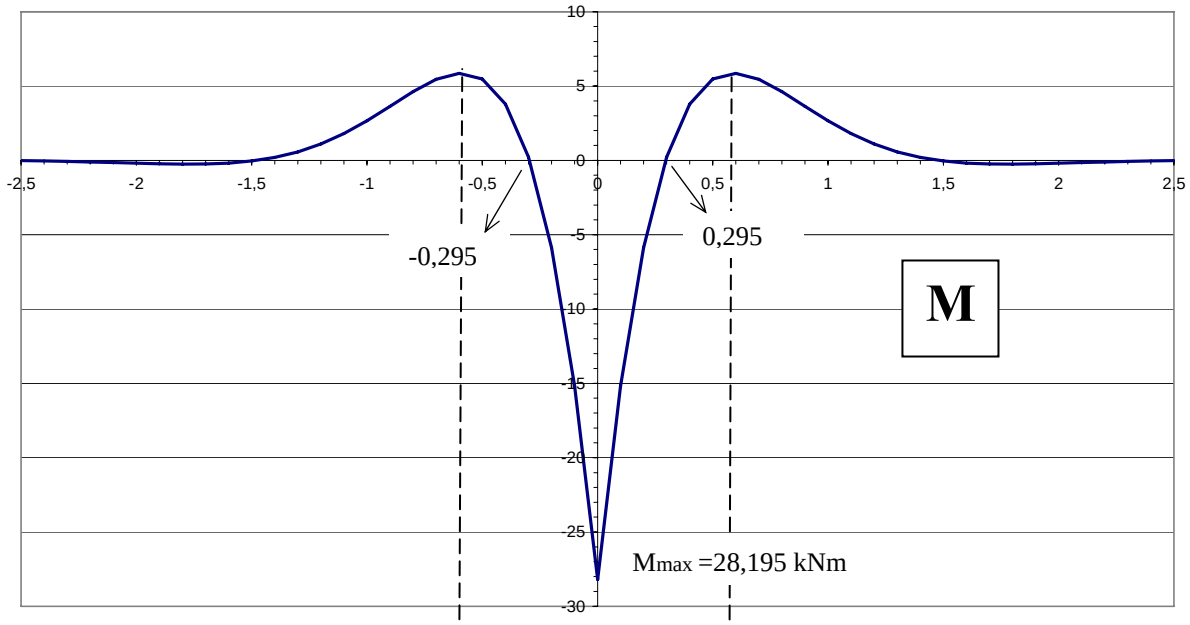
Progibi



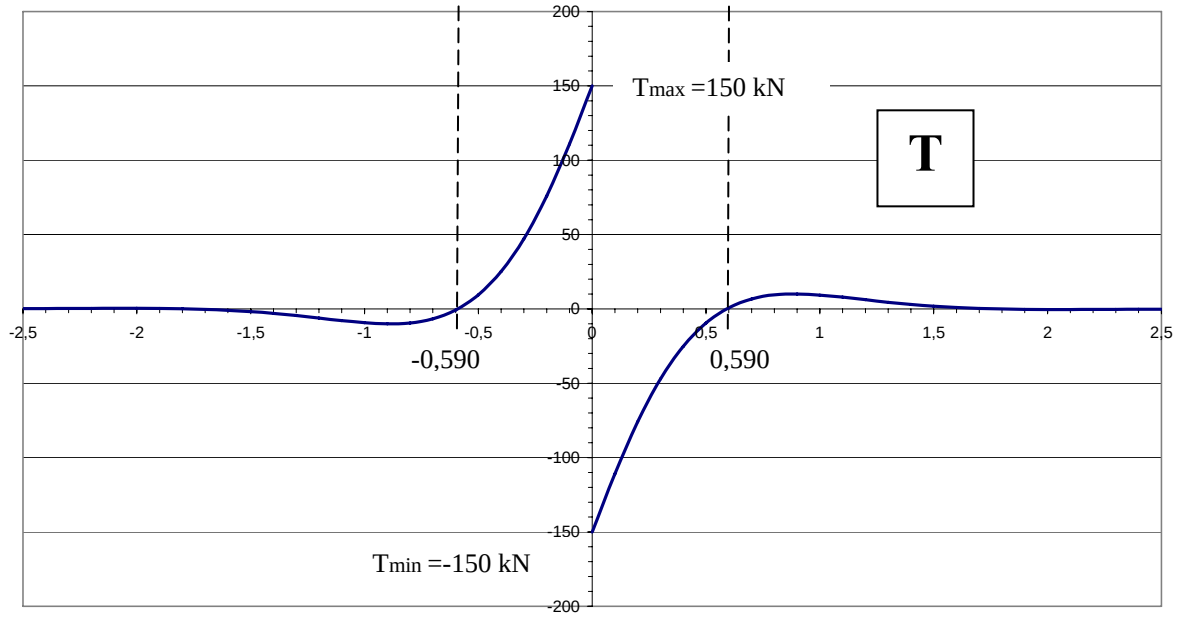
Kut zaokreta



Moment sile



Poprečna sila



Najveća normalna naprezanja:

$$\sigma_{\max} = \frac{M_{\max}}{I_y} \cdot \frac{h}{2} = 131{,}75 \ \ MPa$$

Najveća posmična naprezanja:

$$\tau_{\max} = \frac{T_{\max} \cdot S_y}{I_y \cdot t} = 109{,}35 \ \ MPa$$

Glavna naprezanja u točki „C“, desno od sile F:

$$\sigma_{x(C)} = \frac{M_{\max}}{I_y} \cdot z_C = -65{,}88 \ \ MPa$$

$$\tau_{xz(C)} = \frac{T_C \cdot S_{y(C)}}{I_y \cdot t}; \quad S_{y(C)} = S_y - 7{,}5 \cdot 50 \cdot \frac{50}{2} = 1{,}076 \cdot 10^5 \ \ mm^3$$

$$\tau_{xz(C)} = -100{,}56 \ \ MPa$$

$$\sigma_{1,2}^C = \frac{\sigma_x^C}{2} \pm \frac{1}{2} \cdot \sqrt{\left(\sigma_x^C\right)^2 + 4 \cdot \left(\tau_{xz}^C\right)^2} = -32{,}94 \pm 105{,}82 \ \ (MPa)$$

$$\sigma_1^C = \underline{+72{,}88 \ \ MPa}$$

$$\sigma_2^C = \underline{-138{,}76 \ \ MPa}$$

Kontrola: $\sigma_1^C + \sigma_2^C = \sigma_x^C + \sigma_y^C \qquad \left(\sigma_y^C = 0\right)$

Smjer glavnih naprezanja: $tg(2\varphi_o) = \frac{2 \cdot \tau_{xz}^C}{\sigma_x^C} = +3{,}0526 \quad \Rightarrow \quad \varphi_o = +35{,}93^o$

$$tg\varphi_{01} = \frac{\tau_{xz}^C}{\sigma_1^C - \sigma_y^C} = -1{,}380 \quad \Rightarrow \quad \varphi_{o1} = -54{,}07°$$

$$tg\varphi_{02} = \frac{\tau_{xz}^C}{\sigma_2^C - \sigma_y^C} = +0{,}724 \quad \Rightarrow \quad \varphi_{o2} = +35{,}93°$$

Kontrola: $|\varphi_{01}| + |\varphi_{02}| = 90^o$