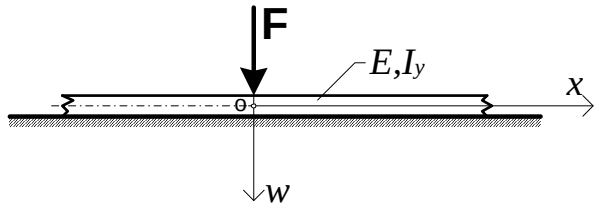
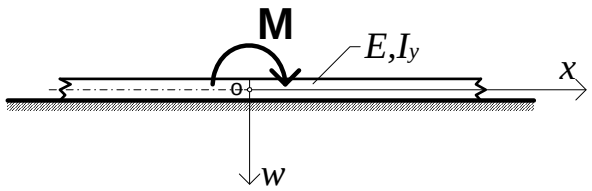
NOSAČ NA ELASTIČNOJ PODLOZI



$$w = \frac{F}{8 \cdot E \cdot I_y \cdot \alpha^3} \cdot e^{-\alpha x} \cdot \left[\cos(\alpha x) + \sin(\alpha x)\right] \quad x \geq 0$$



$$w = \frac{M}{4 \cdot E \cdot I_y \cdot \alpha^2} \cdot e^{-\alpha(x-a)} \cdot \sin\left[\alpha(x-a)\right] \quad x \geq 0$$

Modul podloge: $k_1 \ \left[MN/m^3\right]$

Koeficijent podloge: $k = k_1 \cdot b \ \left[Pa\right]$

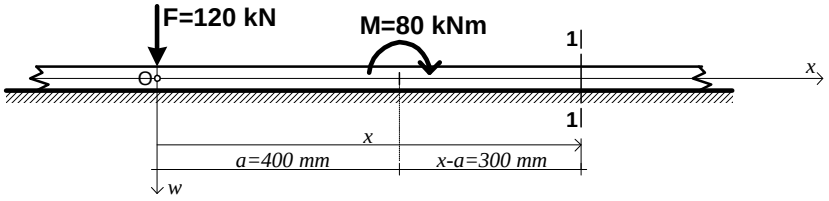
Širina nosača: $b \ \left[m\right]$

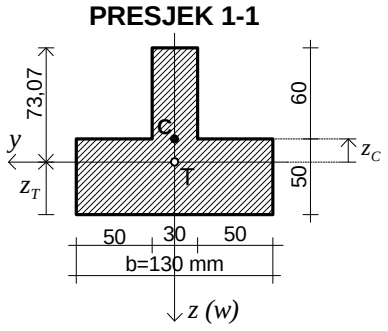
Parametar α koji ovisi o odnosu krutosti nosača i elastične podloge: $\alpha = \sqrt[4]{\dfrac{k}{4 \cdot E \cdot I_y}}$

ZADATAK 3.

Treba odrediti smjer i veličinu glavnih naprezanja u točki "C" presjeka 1-1 beskonačno dugog nosača na elastičnoj podlozi opterećenog prema slici.

Zadan je modul elastičnosti nosača $E = 2{,}0 \cdot 10^5 \ MPa$ i
modul podloge $k_1 = 8000 \ MN/m^3 = 8 \ N/mm^3$.





$A = 8300 \ mm^2$

$z_T = 36{,}93 \ mm$

$I_y = 6{,}16 \cdot 10^6 \ mm^4$

$z_C = -13{,}07 \ mm$

$S_y^C = 30 \cdot 60 \cdot (73{,}07 - 30) = 77562 \ mm^3$

Koeficijent podloge: $k = k_1 \cdot b = 8000 \cdot 0{,}13 = 1040 \ MN/m^2 = 1040 \ MPa \ (N/mm^2)$

Parametar α koji ovisi o odnosu krutosti nosača i elastične podloge:

$$\alpha = \sqrt[4]{\frac{k}{4 \cdot E \cdot I_y}} = \sqrt[4]{\frac{1040}{4 \cdot 2{,}0 \cdot 10^5 \cdot 6{,}16 \cdot 10^6}} = 3{,}811 \cdot 10^{-3} \ \frac{1}{mm}$$

Jednadžba elastične linije nosača:

$$w = \frac{F}{8 \cdot E \cdot I_y \cdot \alpha^3} \cdot e^{-\alpha x} \cdot \left[\cos(\alpha x) + \sin(\alpha x)\right] + \frac{M}{4 \cdot E \cdot I_y \cdot \alpha^2} \cdot e^{-\alpha(x-a)} \cdot \sin\left[\alpha(x-a)\right]$$

Jednadžba za kutove zaokreta:

$$w' = \varphi = -\frac{F}{4 \cdot E \cdot I_y \cdot \alpha^2} \cdot e^{-\alpha x} \cdot \sin(\alpha x) + \frac{M}{4 \cdot E \cdot I_y \cdot \alpha} \cdot e^{-\alpha(x-a)} \cdot \left\{\cos\left[\alpha(x-a)\right] - \sin\left[\alpha(x-a)\right]\right\}$$

Jednadžba za momente savijanja:

$$w'' = -\frac{F}{4 \cdot E \cdot I_y \cdot \alpha} \cdot e^{-\alpha x} \cdot \left[\cos(\alpha x) - \sin(\alpha x)\right] - \frac{M}{2 \cdot E \cdot I_y} \cdot e^{-\alpha(x-a)} \cdot \cos\left[\alpha(x-a)\right] = -\frac{M(x)}{E \cdot I_y}$$

Jednadžba za poprečne sile:

$$w''' = \frac{F}{2 \cdot E \cdot I_y} \cdot e^{-\alpha x} \cdot \cos(\alpha x) + \frac{M \cdot \alpha}{2 \cdot E \cdot I_y} \cdot e^{-\alpha(x-a)} \cdot \left\{\cos\left[\alpha(x-a)\right] + \sin\left[\alpha(x-a)\right]\right\} = -\frac{T(x)}{E \cdot I_y}$$

Izraz za momente savijanja:

$$M(x) = +\frac{F}{4 \cdot \alpha} \cdot e^{-\alpha x} \cdot \left[\cos(\alpha x) - \sin(\alpha x)\right] + \frac{M}{2} \cdot e^{-\alpha(x-a)} \cdot \cos\left[\alpha(x-a)\right]$$

Izraz za poprečne sile:

$$T(x) = -\frac{F}{2} \cdot e^{-\alpha x} \cdot \cos(\alpha x) - \frac{M \cdot \alpha}{2} \cdot e^{-\alpha(x-a)} \cdot \left\{\cos\left[\alpha(x-a)\right] + \sin\left[\alpha(x-a)\right]\right\}$$

Za $x = 700\ \ mm$: $\alpha \cdot x = 2{,}677$ $\quad \sin(\alpha x) = 0{,}45635$ $\quad \cos(\alpha x) = -0{,}8898$

$\alpha \cdot (x-a) = 1{,}1433$ $\quad \sin\left[\alpha(x-a)\right] = 0{,}9100$ $\quad \cos\left[\alpha(x-a)\right] = 0{,}4146$

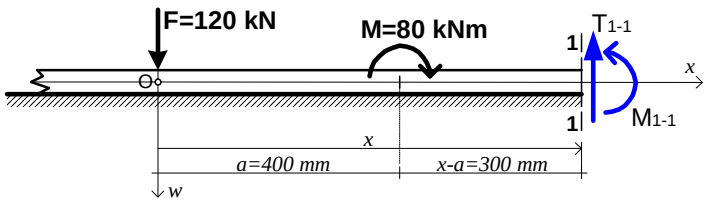
$e^{-\alpha x} = 0{,}06941$

$e^{-\alpha(x-a)} = 0{,}3188$

Moment savijanja i poprečna sila u presjeku 1-1 ($x = 700\ \ mm$):

$$M_{1-1} = +\frac{F}{4 \cdot \alpha} \cdot e^{-\alpha x} \cdot \left[\cos(\alpha x) - \sin(\alpha x)\right] + \frac{M}{2} \cdot e^{-\alpha(x-a)} \cdot \cos\left[\alpha(x-a)\right] = \underline{+4{,}55\ \ kNm}$$

$$T_{1-1} = -\frac{F}{2} \cdot e^{-\alpha x} \cdot \cos(\alpha x) - \frac{M \cdot \alpha}{2} \cdot e^{-\alpha(x-a)} \cdot \left\{\cos\left[\alpha(x-a)\right] + \sin\left[\alpha(x-a)\right]\right\} = \underline{-60{,}67\ \ kN}$$



Normalno naprezanje u točki C: $\sigma_x^C = \dfrac{M_{1-1}}{I_y} \cdot z_C = \underline{-9{,}65\ \ MPa}$ $\quad (tlak)$

Posmično naprezanje u točki C: $\tau_{xz}^C = \dfrac{T_{1-1} \cdot S_y^C}{I_y \cdot b_C} = \underline{-25{,}45\ \ MPa}$

Veličina glavnih naprezanja: $\sigma_{1,2}^C = \frac{\sigma_x^C}{2} \pm \frac{1}{2} \cdot \sqrt{\left(\sigma_x^C\right)^2 + 4 \cdot \left(\tau_{xz}^C\right)^2} = -4{,}825 \pm 25{,}903 \ \ (MPa)$

$\sigma_1^C = \underline{+21{,}08 \ \ MPa}$

$\sigma_2^C = \underline{-30{,}73 \ \ MPa}$

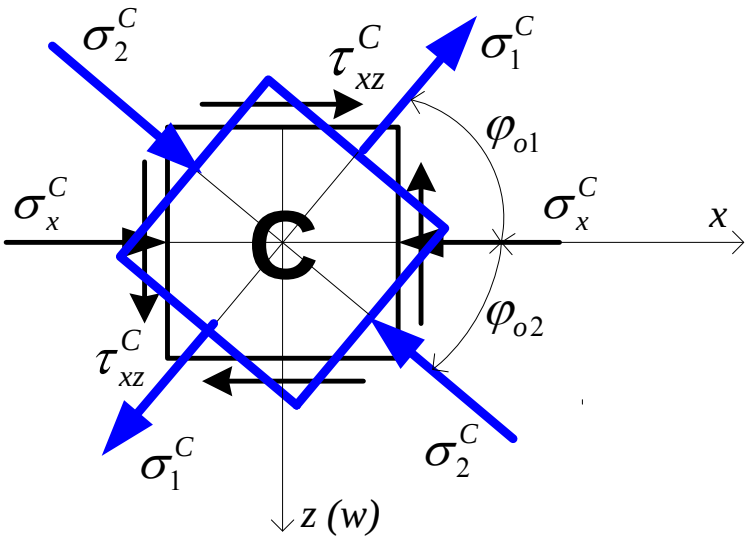
Kontrola: $\sigma_1^C + \sigma_2^C = \sigma_x^C + \sigma_y^C \qquad \left(\sigma_y^C = 0\right)$

Smjer glavnih naprezanja: $tg\left(2\varphi_o\right) = \frac{2 \cdot \tau_{xz}^C}{\sigma_x^C} = +5{,}2746 \quad \Rightarrow \quad \varphi_o = +39{,}63^o$

$tg\varphi_{01} = \frac{\tau_{xz}^C}{\sigma_2^C - \sigma_y^C} = -1{,}2073 \quad \Rightarrow \quad \varphi_{o1} = \underline{-50{,}36^o}$

$tg\varphi_{02} = \frac{\tau_{xz}^C}{\sigma_2^C - \sigma_y^C} = +0{,}8282 \quad \Rightarrow \quad \varphi_{o2} = \underline{+39{,}63^o}$

Kontrola: $\left|\varphi_{01}\right| + \left|\varphi_{02}\right| = 90^o$



U nastavku su još **4 primjera** izraza za progib u presjeku I-I, ovisno o položaju presjeka u odnosu na opterećenje nosača na elastičnoj podlozi (1. primjer je isti kao u prethodnom zadatku).

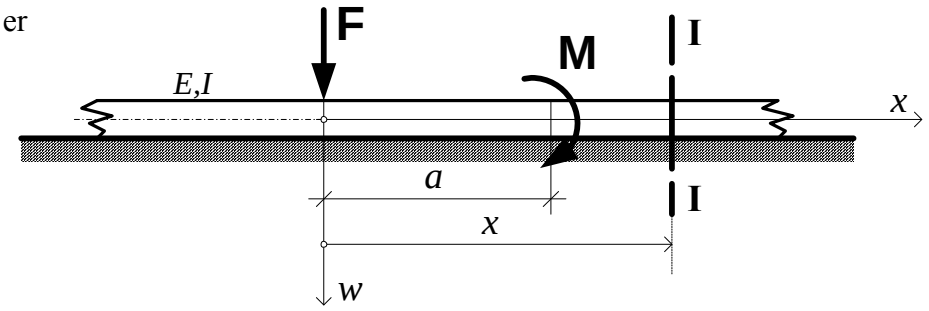
Kut zaokreta, moment savijanja i poprečnu silu u presjeku dobijemo nakon prve, druge i treće derivacija izraza za progib.

Kut zaokreta u presjeku I-I: $\varphi_{I-I} = w'\left(x\right)$

Moment savijanja u presjeku I-I: $M_{I-I} = -E \cdot I_y \cdot w''\left(x\right)$

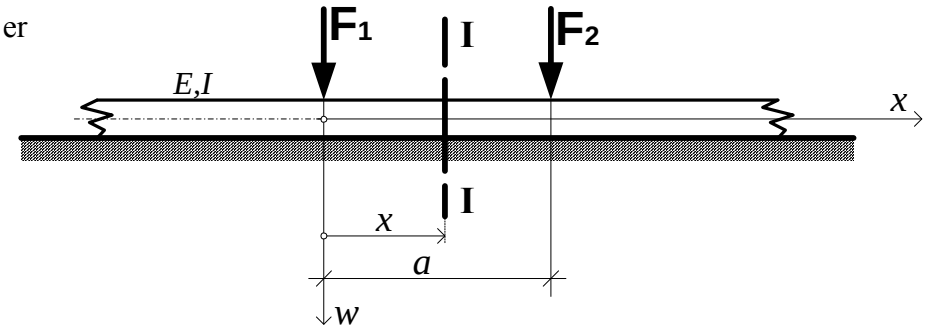
Poprečna sila u presjeku I-I: $T_{I-I} = -E \cdot I_y \cdot w'''\left(x\right)$

1. primjer



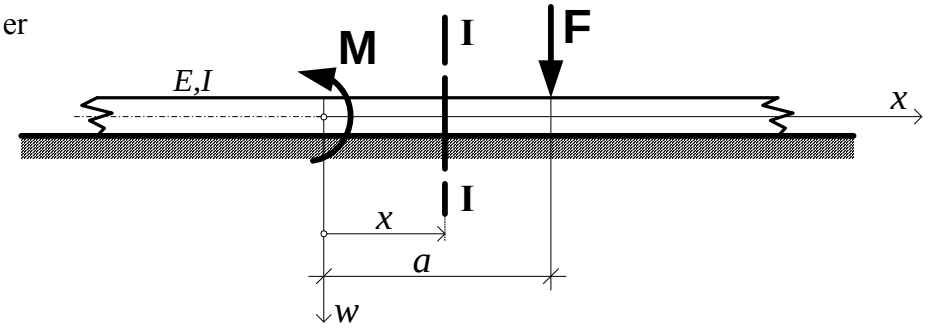
$$w = \frac{F}{8EI\alpha^3} \cdot e^{-\alpha x}(\sin\alpha x + \cos\alpha x) + \frac{M}{4EI\alpha^2} \cdot e^{-\alpha(x-a)} \sin\alpha(x-a)$$

2. primjer



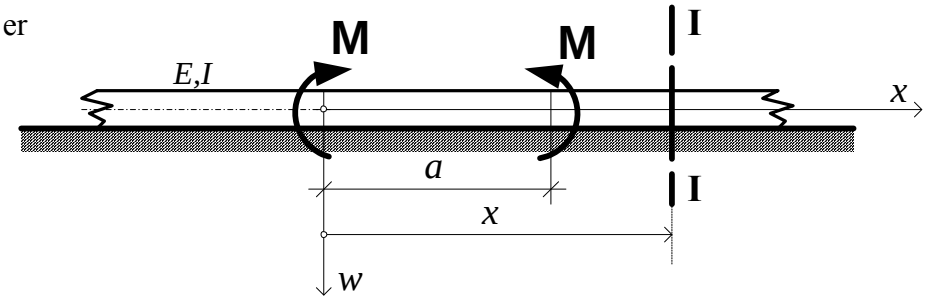
$$w = \frac{F_1}{8EI\alpha^3} \cdot e^{-\alpha x}(\sin\alpha x + \cos\alpha x) + \frac{F_2}{8EI\alpha^3} \cdot e^{-\alpha(a-x)}\left[\sin\alpha(a-x) + \cos\alpha(a-x)\right]$$

3. primjer



$$w = -\frac{M}{4EI\alpha^2} \cdot e^{-\alpha x} \sin\alpha x + \frac{F}{8EI\alpha^3} \cdot e^{-\alpha(a-x)}\left[\cos\alpha(a-x) + \sin\alpha(a-x)\right]$$

4. primjer



$$w = \frac{M}{4EI\alpha^2} \cdot e^{-\alpha x} \sin\alpha x - \frac{M}{4EI\alpha^2} \cdot e^{-\alpha(x-a)} \sin\alpha(x-a)$$