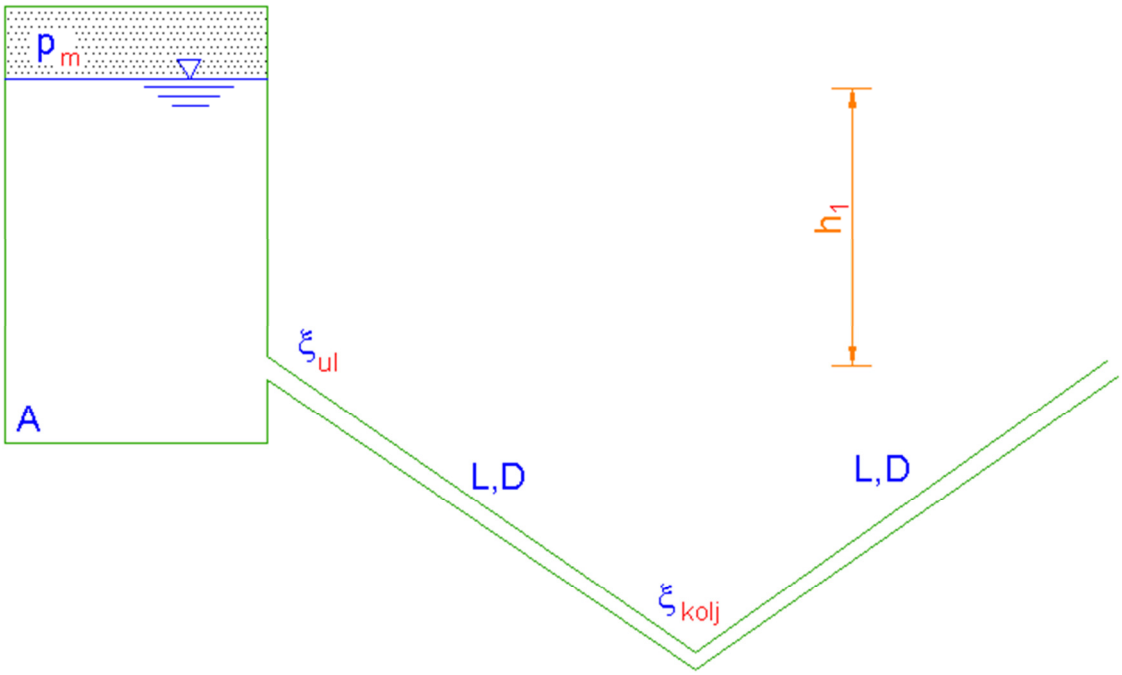
BJ REALNA TEKUĆINA

1. Potrebno je odrediti protok za sistem kao na slici te nacrtati piezometarsku i energetsku liniju.

Zadano je: $L = 200$ m; $h_1 = 5$ m; $D = 200$ mm; $\varepsilon = 0{,}5$ mm;
$\lambda = f(\varepsilon/D)$; $p_m = -20$ kPa; $\xi_{KOLJ} = 0{,}2$; $\xi_{UL} = 0{,}5$;
$\rho = 1000$ kg/m^3



Napomena: izraz $\lambda = f(\varepsilon/D)$ znači da Darcyev koeficijent trenja λ ovisi samo o relativnoj hrapavosti cijevi ε/D, odnosno zadani režim tečenja u cijevima je potpuno turbulentni.

$$\frac{\varepsilon}{D} = \frac{0{,}5}{200} = 0{,}0025 \xrightarrow{\text{Moodyjev dijagram}} \lambda = 0{,}025$$

Bernoullijeva jednadžba između presjeka $1-1(uzvodnog)$ *i* $2-2(nizvodnog)$

$$E_1 = E_2 + \Delta E_{1-2}$$

$$z_1 + \frac{p_1}{\rho g} + \frac{v_1^2}{2g} = z_1 + \frac{p_1}{\rho g} + \frac{v_1^2}{2g} + \Delta h$$

$$5 + \frac{p_m}{\rho g} = \frac{v^2}{2g} + \frac{v^2}{2g}\left(\xi_{UL} + \lambda \cdot \frac{2L}{D} + \xi_{KOLJ}\right)$$

$$5 + \frac{p_m}{\rho g} = \frac{v^2}{2g}\left(\xi_{UL} + \lambda \cdot \frac{2L}{D} + \xi_{KOLJ} + 1\right)$$

$$5-\frac{20}{1\cdot g}=\frac{v^2}{2g}\left(0,5+0,025\cdot\frac{2\cdot 200}{0,2}+0,2+1\right)$$

$$5-2,04=\frac{v^2}{19,62}\cdot 51,7$$

$$v=1,06\ m/s$$

$$\frac{v^2}{2g}=\frac{1,06^2}{2\cdot 9,81}=0,054\ m$$

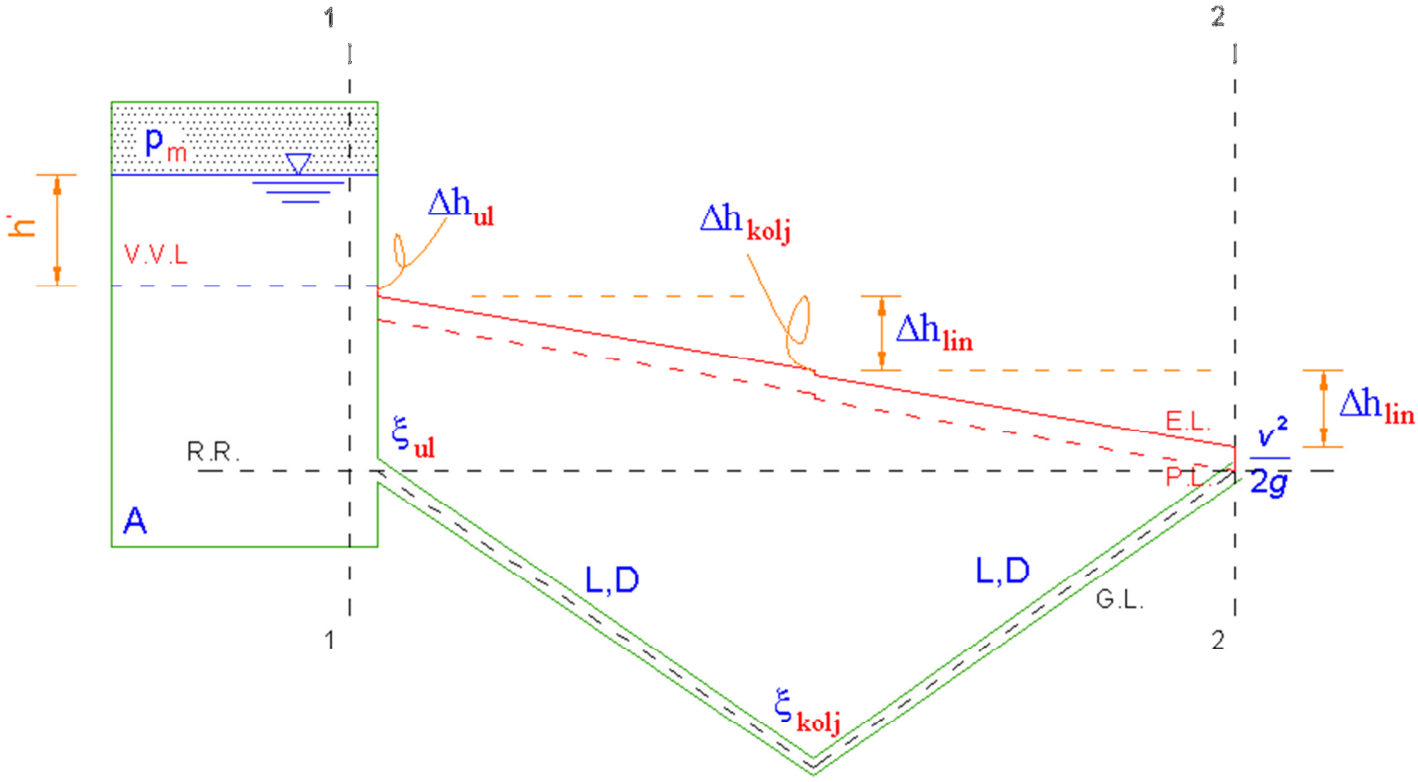
$$\Delta h_{UL}=\frac{v^2}{2g}\cdot\xi_{UL}=0,054\cdot 0,5=0,027\ m$$

$$\Delta h_{LIN}=\frac{v^2}{2g}\cdot\lambda\frac{L}{D}=0,054\cdot 0,025\cdot\frac{200}{0,2}=1,35\ m$$

$$\Delta h_{KOLJ}=\frac{v^2}{2g}\cdot\xi_{KOLJ}=0,054\cdot 0,2=0,011\ m$$

$$h'=\frac{p_m}{\rho g}=\frac{20}{1\cdot g}=2,04\ m \qquad (\textit{sniženje do virtualnog vodnog lica})$$

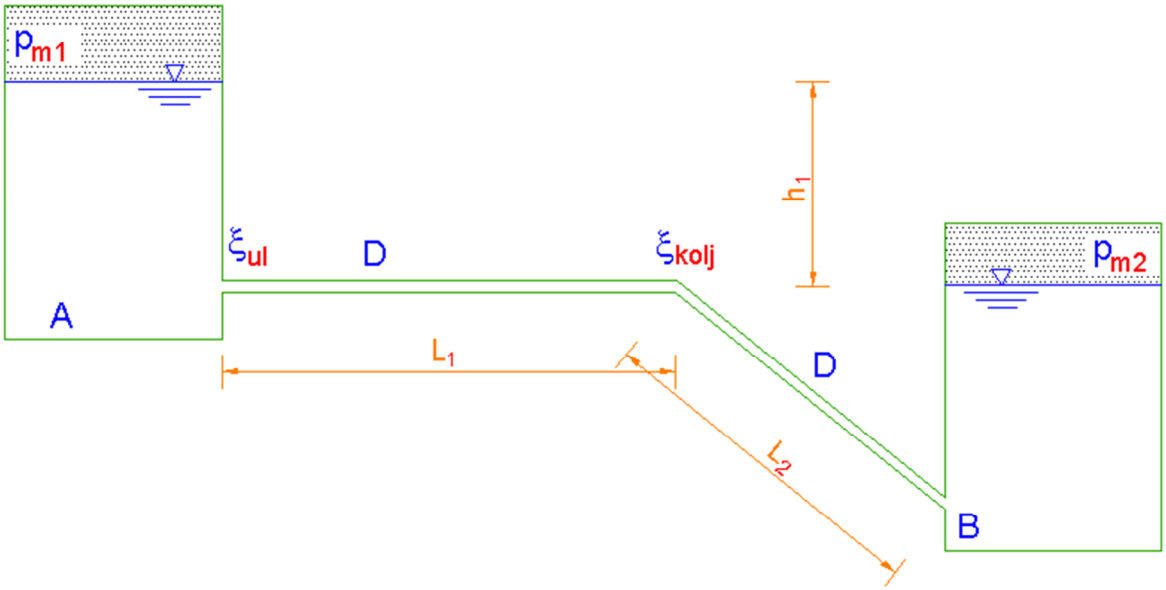
$$Q=v\cdot A=v\cdot\frac{D^2\pi}{4}=1,06\cdot\frac{0,2^2\pi}{4}=0,033\ m^3/s$$



Napomena: dimenzije u horizontalnom i vertikalnom smjeru nisu proporcionalne. E.L. i P.L. treba što je više moguće crtati u mjerilu, ali do dovoljne razlučivosti pojedinih linija.

2. Za cjevovod prema slici potrebno je odrediti brzinu strujanja v iz rezervoara A u rezervoar B i nacrtati energetsku i piezometarsku liniju. Označiti najnižu vrijednost potencijalne energije tlaka u sustavu.

Zadano je: $L_1 = 150$ m; $L_2 = 100$m; $h_1 = 5$ m; $D = 250$ mm;
$\varepsilon = 0{,}5$ mm; $\lambda = f(\varepsilon/D)$; $p_{m1} = 49{,}05$ kPa; $p_{m2} = 49{,}05$ kPa;
$\xi_{KOLJ} = 0{,}2$; $\xi_{UL} = 0{,}5$; $\rho = 1000$ kg/m^3



$$\frac{\varepsilon}{D} = \frac{0{,}5}{250} = 0{,}002 \qquad \rightarrow \qquad \lambda = 0{,}024$$

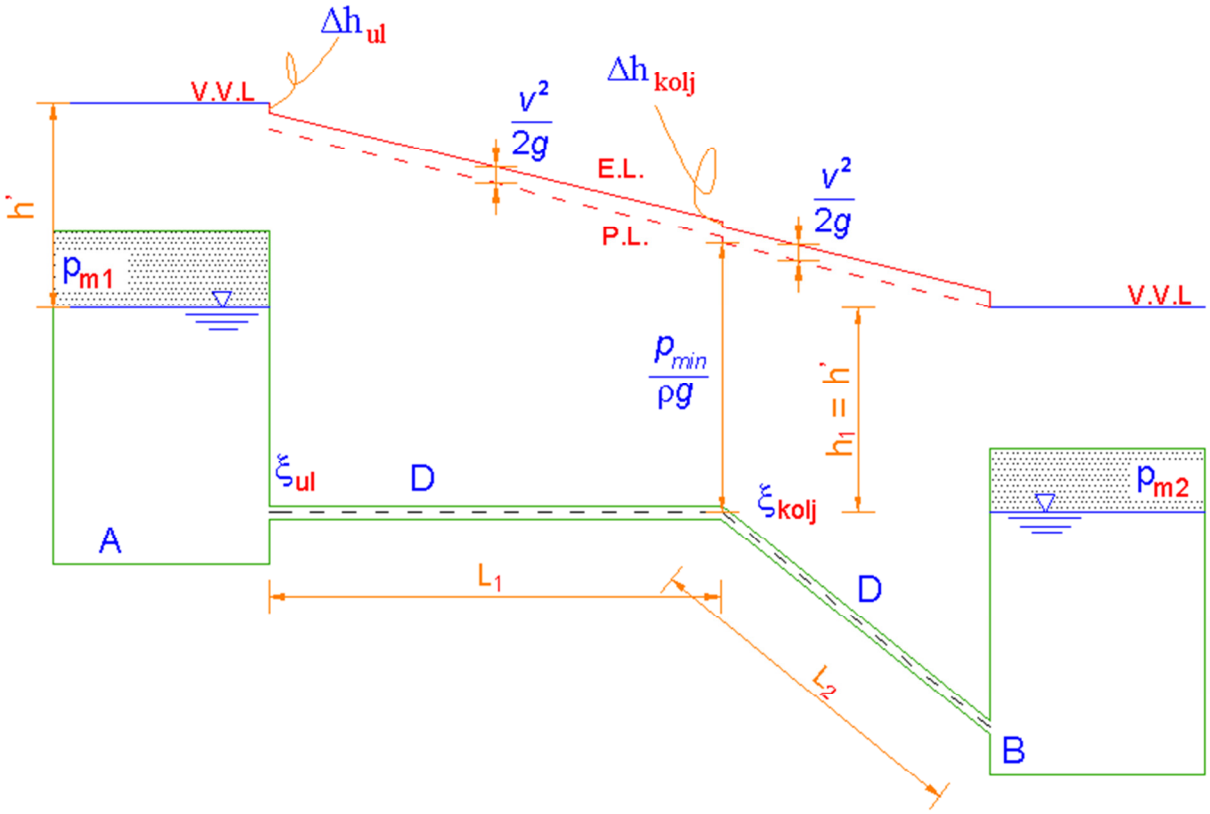
$$h_1 + \frac{p_{m1}}{\rho g} = \frac{p_{m2}}{\rho g} + \frac{v^2}{2g}\left(\xi_{UL} + \xi_{KOLJ} + \lambda \cdot \frac{L_1 + L_2}{D} + 1\right)$$

$$5 + \frac{49{,}05}{1 \cdot g} = \frac{49{,}05}{1 \cdot g} + \frac{v^2}{2g}\left(0{,}5 + 0{,}2 + 0{,}024 \cdot \frac{150 + 100}{0{,}25} + 1\right)$$

$$5 + \frac{49{,}05}{1 \cdot g} = \frac{49{,}05}{1 \cdot g} + \frac{v^2}{2g}(0{,}5 + 0{,}2 + 24 + 1)$$

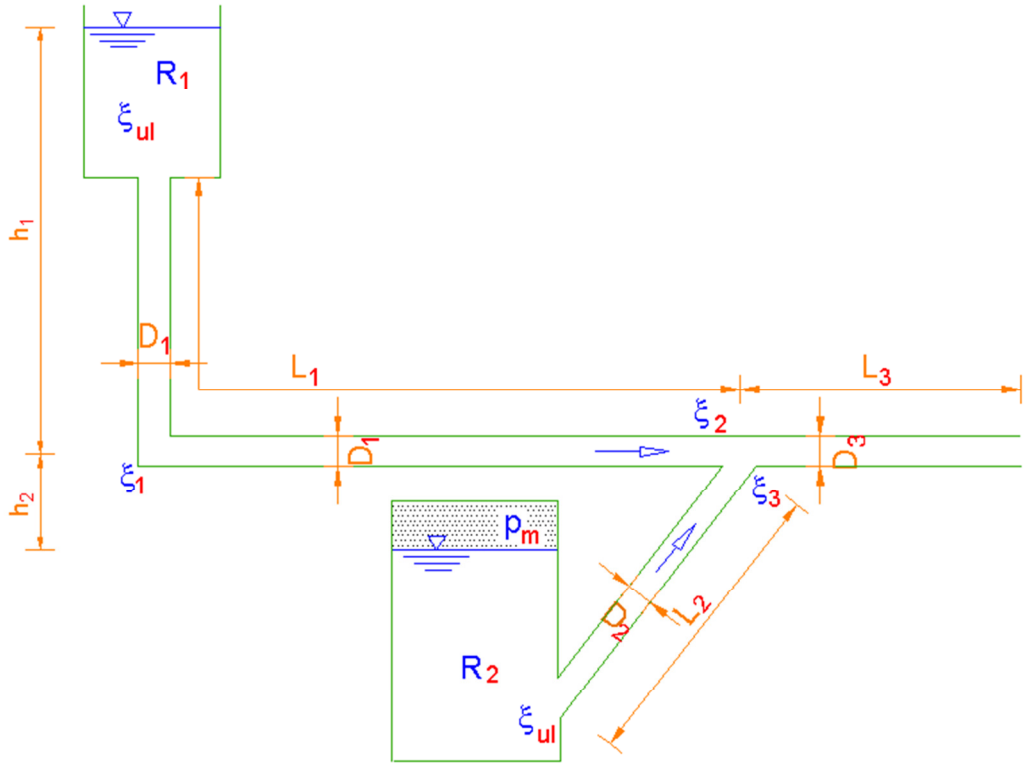
$$v = 1{,}95 \text{ m/s}$$

$$\frac{v^2}{2g} = 0{,}19\, m \qquad h' = \frac{p_{m1}}{\rho g} = \frac{p_{m2}}{\rho g} = \frac{49{,}05}{1 \cdot g} = 5{,}0\, m$$


Δh ul
V.V.L
v²/2g
Δh kolj
E.L.
P.L.
v²/2g
V.V.L
h'
p m1
p min/ρg
h1 = h'
ξ ul
D
ξ kolj
p m2
A
L1
D
L2
B

3. Rezervoar R_1 ima slobodno vodno lice, a rezervoar R_2 nalazi se pod tlakom p_m. Potrebno je odrediti protoke u cijevi 2 (Q_2) i izlazni protok u cijevi 3 (Q_3) te koeficijent lokalnog gubitka na spoju za cijev 3 $\xi_3 = f(v_2)$. Potrebno je nacrtati energetsku i piezometarsku liniju za sve cijevi.

Zadano je: $Q_1 = 0{,}146$ m^3/s; $p_m = 325$ kPa; $h_1 = 25$ m; $h_2 = 5$ m;
$D_1 = D_3 = 200$ mm; $D_2 = 100$ mm; $\lambda = 0{,}03$ (za sve cijevi);
$L_1 = 65$ m; $L_2 = 14$ m; $L_3 = 30$ m;
$\xi_{UL} = 0{,}5$; $\xi_1 = 0{,}8$; $\xi_2 = f(v_1) = 0{,}72$; $\rho = 1000$ kg/m^3



$$Q_1 = v_1 \frac{D_1^2 \pi}{4} \quad \rightarrow \quad v_1 = \frac{4Q_1}{D_1^2 \pi} = \frac{4 \cdot 0{,}146}{0{,}2^2 \pi} = 4{,}65\ m/s$$

B.J. $1-3$

$$h_1 = \frac{v_1^2}{2g}\left(\xi_{UL} + \lambda\frac{L_1}{D_1} + \xi_1 + \xi_2\right) + \frac{v_3^2}{2g}\left(\lambda\frac{L_3}{D_3} + 1\right)$$

$$25 = \frac{4{,}65^2}{2g}\left(0{,}5 + 0{,}03 \cdot \frac{65}{0{,}2} + 0{,}8 + 0{,}72\right) + \frac{v_3^2}{2g}\left(0{,}03 \cdot \frac{30}{0{,}2} + 1\right)$$

$$25 = \frac{4{,}65^2}{2g}\left(0{,}5 + 0{,}03 \cdot \frac{65}{0{,}2} + 0{,}8 + 0{,}72\right) + \frac{v_3^2}{2g}\left(0{,}03 \cdot \frac{30}{0{,}2} + 1\right)$$

$$25 = 1.1\left(0{,}5 + 9{,}75 + 0{,}8 + 0{,}72\right) + \frac{v_3^2}{2g}\left(4{,}5 + 1\right)$$

$$\frac{v_3^2}{2g} = 2{,}19\ m \quad \rightarrow \quad v_3 = 6{,}55\ m/s$$

$$Q_1 + Q_2 = Q_3$$

$$v_1 \frac{D_1^2 \pi}{4} + v_2 \frac{D_2^2 \pi}{4} = v_3 \frac{D_3^2 \pi}{4}$$

$$4,65 \cdot 0,2^2 + v_2 \cdot 0,1^2 = 6,55 \cdot 0,2^2$$

$$v_2 = 7,6\, m/s \qquad \rightarrow \qquad \frac{v_2^2}{2g} = 2,94\, m$$

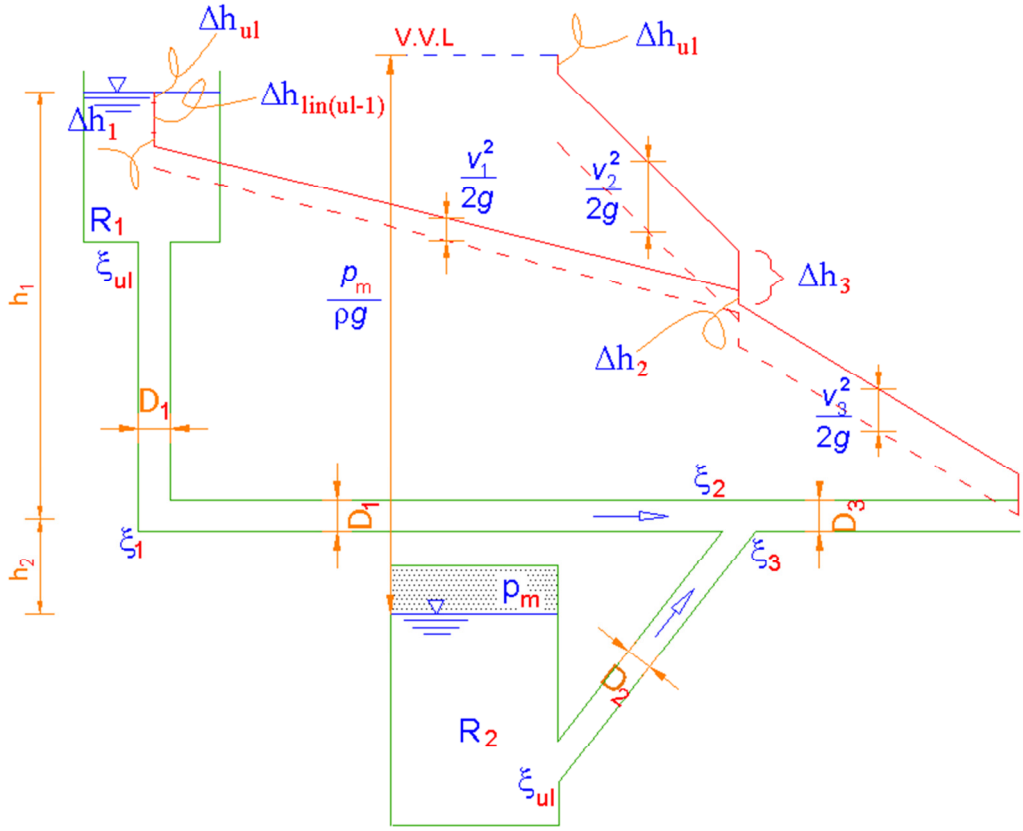
B.J. $2-3$

$$-h_2 + \frac{p_m}{\rho g} = \frac{v_2^2}{2g}\left(\xi_{UL} + \lambda \frac{L_2}{D_2} + \xi_3\right) + \frac{v_3^2}{2g}\left(\lambda \frac{L_3}{D_3} + 1\right)$$

$$-5 + \frac{325}{1 \cdot g} = 2,94\left(0,5 + 0,03 \cdot \frac{14}{0,1} + \xi_3\right) + 2,19\left(0,03 \cdot \frac{30}{0,2} + 1\right)$$

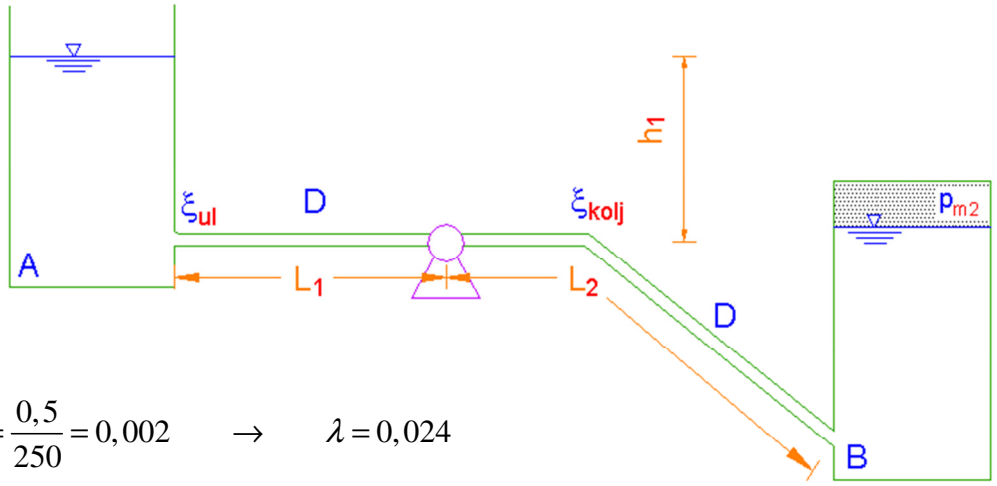
$$28,13 = 13,82 + 2,94\xi_3 + 12,05$$

$$\xi_3 = 0,77$$



4. Za cjevovod prema slici potrebno je odrediti brzinu strujanja iz rezervoara *A* u rezervoar *B* i nacrtati energetsku i piezometarsku liniju za sustav s pumpom.

Zadano je : $H_{pumpa} = 10$ m; $h_1 = 5$ m; $L_1 = 100$ m; $L_2 = 150$ m;
$p_m = 49{,}05$ kPa; $\varepsilon = 0{,}5$ mm; $D = 250$ mm; $\lambda = f(\varepsilon/D)$;
$\xi_{KOLJ} = 0{,}2$; $\xi_{UL} = 0{,}5$; $\rho = 1000$ kg/m^3

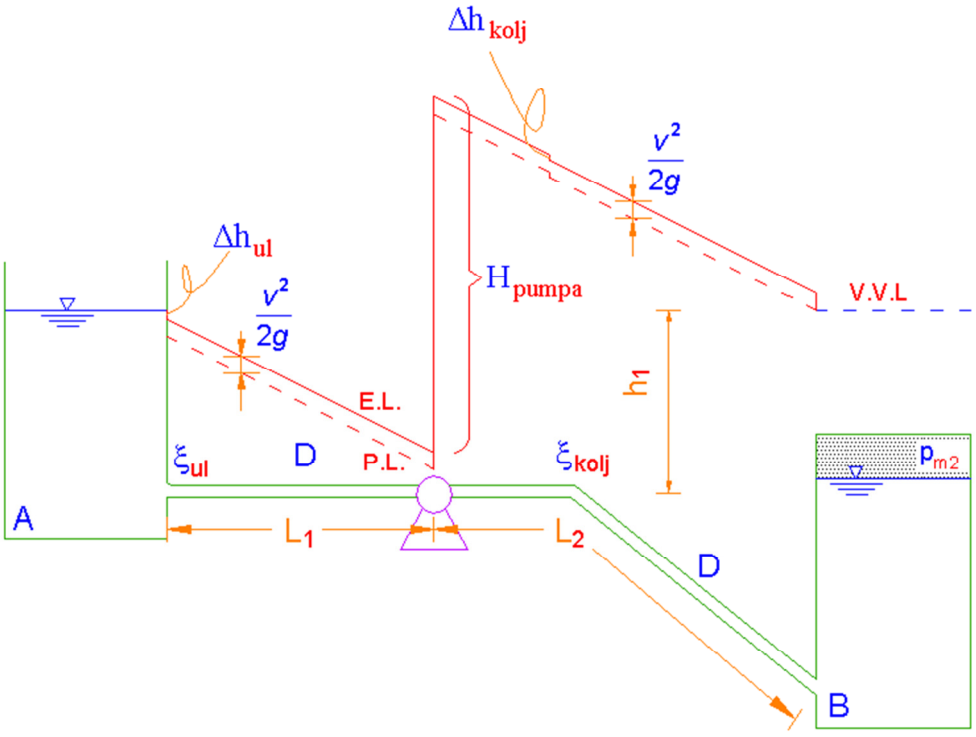


$$\frac{\varepsilon}{D} = \frac{0{,}5}{250} = 0{,}002 \quad \rightarrow \quad \lambda = 0{,}024$$

$$h_1 + H_{pumpa} = \frac{p_m}{\rho g} + \frac{v^2}{2g}\left(\xi_{UL} + \lambda \cdot \frac{L_1}{D} + \xi_{KOLJ} + \lambda \cdot \frac{L_2}{D} + 1\right)$$

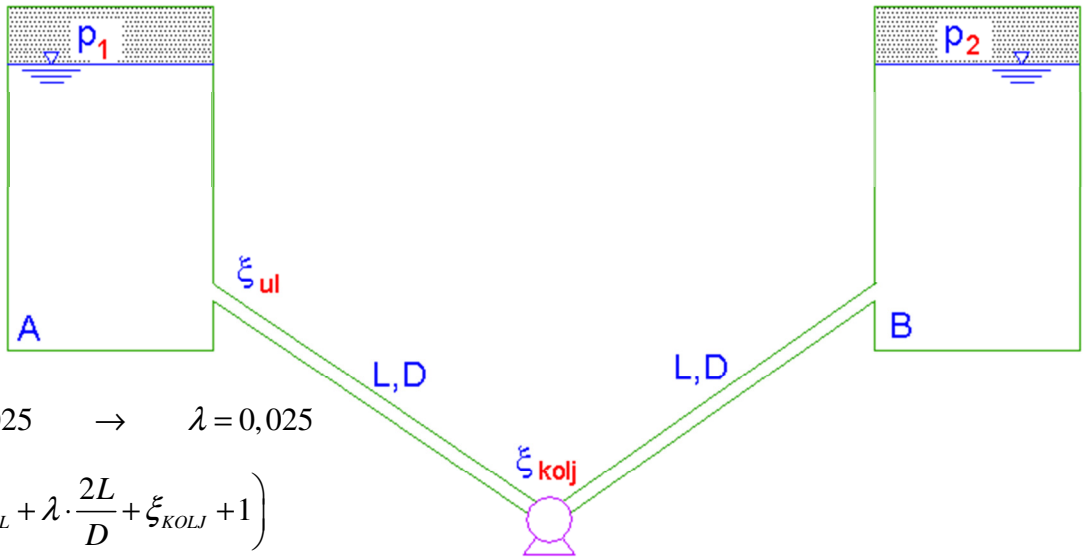
$$5 + 10 = 5 + \frac{v^2}{2g}\left(0{,}5 + 9{,}6 + 0{,}2 + 14{,}4 + 1\right)$$

$$\frac{v^2}{2g} = 0{,}39\ m \quad \rightarrow \quad v = 2{,}76\ m/s$$



5. Potrebno je odrediti protok za sistem kao na slici te nacrtati piezometarsku i energetsku liniju. Smjer strujanja fluida je iz komore *A* u komoru *B*.

Zadano je : $H_{pumpa} = 10$ m; $L = 200$ m; $\varepsilon = 0{,}5$ mm;
$D = 200$ mm; $\lambda = f(\varepsilon/D)$; $p_{1,2} = 20$ kPa;
$\xi_{KOLJ} = 0{,}2$; $\xi_{UL} = 0{,}5$; $\rho = 1000$ kg/m^3



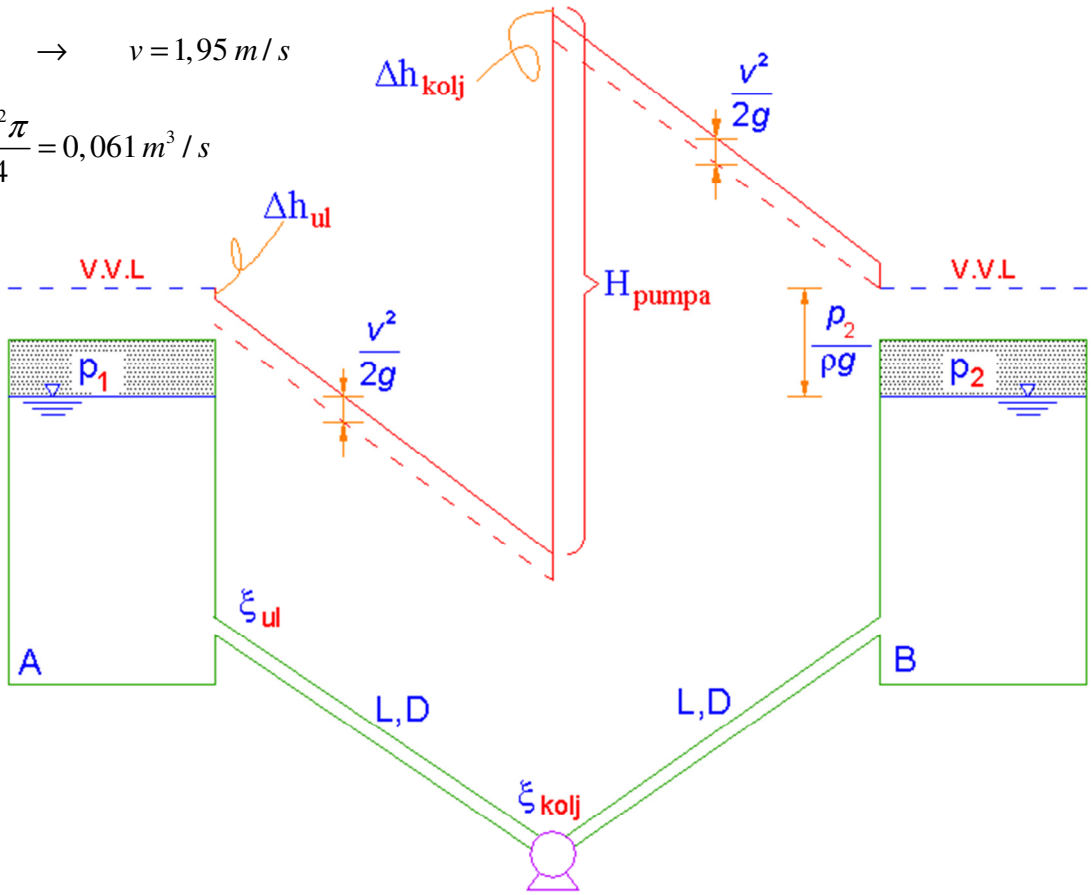
$$\frac{\varepsilon}{D} = \frac{0{,}5}{200} = 0{,}0025 \quad \rightarrow \quad \lambda = 0{,}025$$

$$H_{pumpa} = \frac{v^2}{2g}\left(\xi_{UL} + \lambda \cdot \frac{2L}{D} + \xi_{KOLJ} + 1\right)$$

$$10 = \frac{v^2}{2g}(0{,}5 + 50 + 0{,}2 + 1)$$

$$\frac{v^2}{2g} = 0{,}193\,m \quad \rightarrow \quad v = 1{,}95\,m/s$$

$$Q = v \cdot A = v \cdot \frac{D^2\pi}{4} = 0{,}061\,m^3/s$$

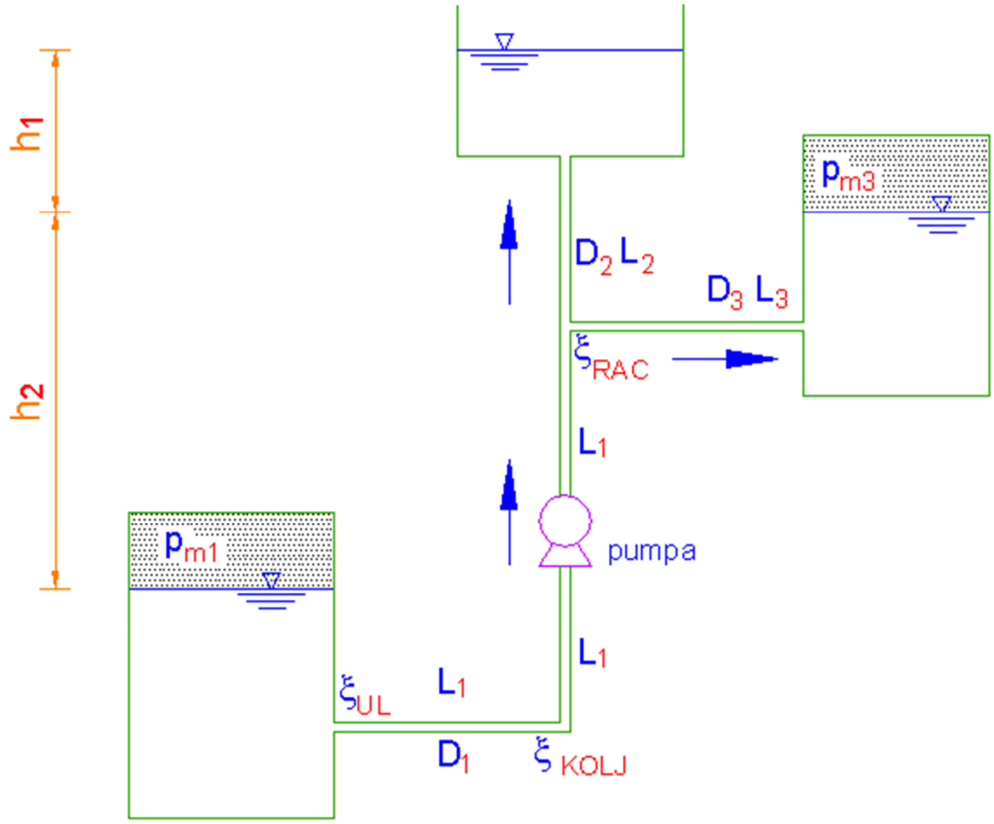


6. Za sistem kao na slici potrebno je odrediti protoke Q_2 i Q_3. Nacrtati piezometarsku i energetsku liniju.

Zadano je:

$Q_1 = 0{,}2\ m^3/s$;	$p_{m1} = 39{,}25$ kPa;	$D_1 = 250$ mm;	$D_2 = 150$ mm;
$D_3 = 200$ mm;	$L_1 = 20$ m;	$L_2 = 10$ m;	$L_3 = 20$ m;
$\xi_{UL} = 0{,}5$;	$\xi_{KOLJ} = 0{,}3$;	$\xi_{RAČ} = f(v_1) = 0{,}3$;	$h_2 = 40$ m;
$h_1 = 10$ m;	$H_{pumpa} = 60$ m;	$\lambda = 0{,}02$;	$\rho = 1000\ kg/m^3$

Napomena: izraz $\xi_{RAČ} = f(v_1)$ znači da lokalni gubitak energije u račvi ovisi o kinetičkoj energiji u cijevi 1 (uzvodnoj kinetičkoj energiji) i koeficijentu lokalnog gubitka na račvi $\xi_{RAČ}$.



B.J. $1-2$

$$\frac{p_{m1}}{\rho g} + H_{pumpa} = h_1 + h_2 + \frac{Q_1^2}{2gA_1^2}\left(\xi_{UL} + \lambda\frac{3L_1}{D_1} + \xi_{KOLJ} + \xi_{RAC}\right) + \frac{Q_2^2}{2gA_2^2}\left(\lambda\frac{L_2}{D_2} + 1\right)$$

$$\frac{39{,}25}{9{,}81} + 60 = 10 + 40 + \frac{0{,}2^2}{2\cdot 9{,}81\cdot 0{,}049^2}\left(0{,}5 + 0{,}02\cdot\frac{60}{0{,}25} + 0{,}3 + 0{,}3\right) + \frac{Q_2^2}{2\cdot 9{,}81\cdot 0{,}0177^2}\left(0{,}02\frac{10}{0{,}15} + 1\right)$$

$$4 + 60 = 10 + 40 + 0{,}85\left(0{,}5 + 4{,}8 + 0{,}3 + 0{,}3\right) + 162{,}69Q_2^2\left(1{,}33 + 1\right)$$

$$Q_2 = 0{,}154\ m^3/s \quad \rightarrow \quad v_2 = \frac{Q_2}{A_2} = \frac{0{,}154}{0{,}0177} = 8{,}7\ m/s \quad \rightarrow \quad \frac{v_2^2}{2g} = 3{,}86\ m$$

Jednadžba kontinuiteta

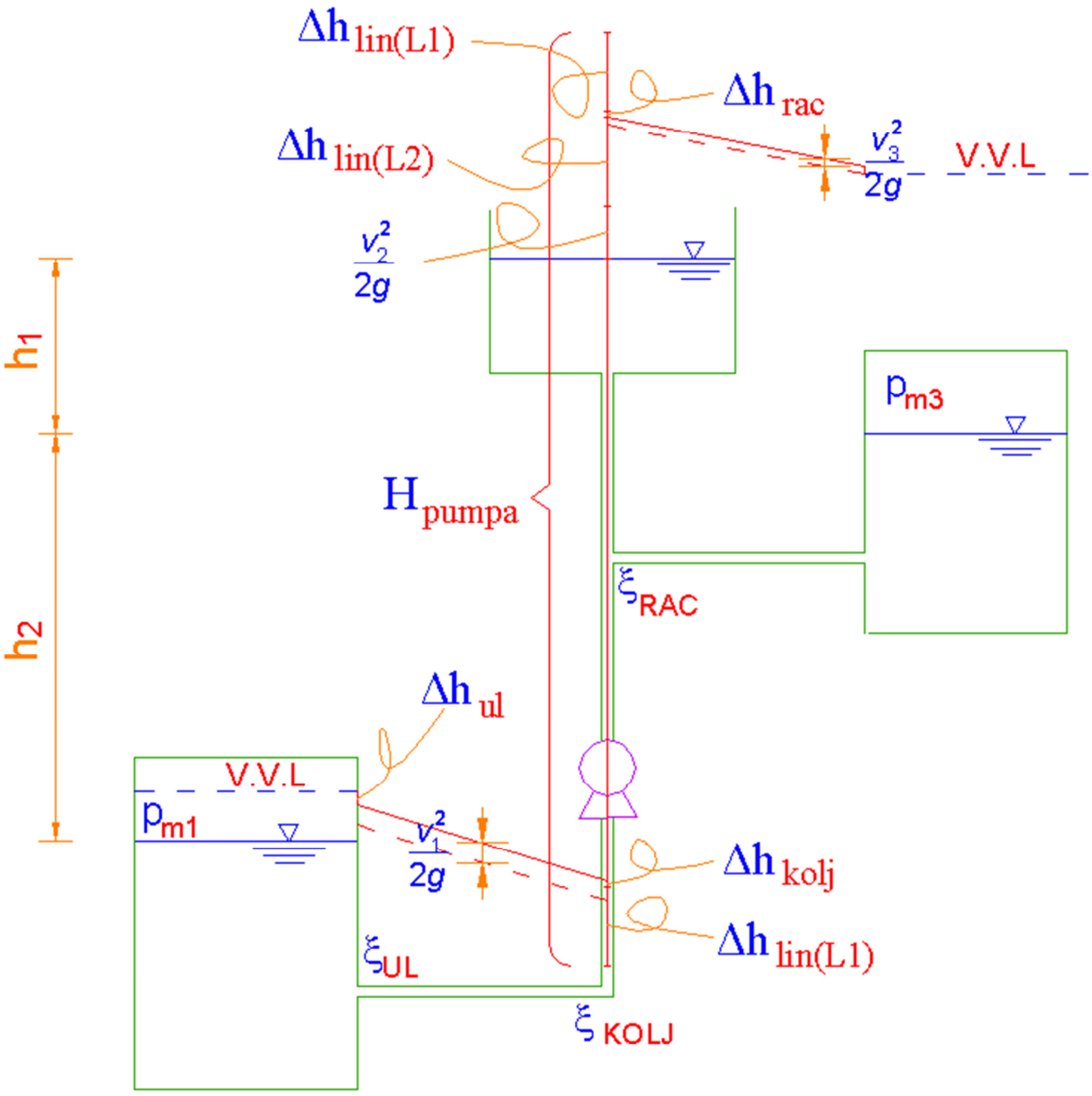
$$Q_3 = Q_1 - Q_2 = 0,2 - 0,154 = 0,046\ m^3/s \quad \rightarrow \quad v_3 = \frac{Q_3}{A_3} = \frac{0,046}{0,0314} = 1,46\ m/s \quad \rightarrow \quad \frac{v_3^2}{2g} = 0,11\ m$$

B.J. $1-3$

$$\frac{p_{m1}}{\rho g} + H_{pumpa} = h_2 + \frac{p_{m3}}{\rho g} + \frac{Q_1^2}{2gA_1^2}\left(\xi_{UL} + \lambda\frac{L_1}{D_1} + \xi_{KOLJ} + \xi_{RAC}\right) + \frac{Q_3^2}{2gA_3^2}\left(\lambda\frac{L_3}{D_3} + 1\right)$$

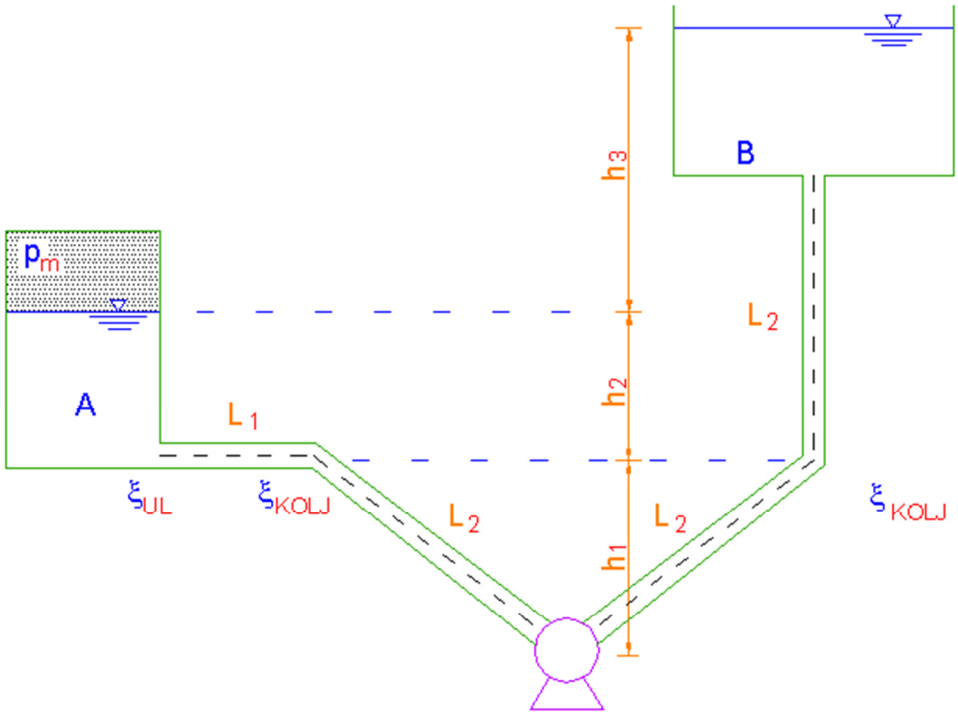
$$4 + 60 = 40 + \frac{p_{m3}}{\rho g} + 0,85(0,5 + 4,8 + 0,3 + 0,3) + 0,11(2+1)$$

$$\frac{p_{m3}}{\rho g} = 18,65\ m \quad \rightarrow \quad p_{m3} = 183,0\ kPa$$



7. Za sistem kao na slici potrebno je odrediti protok Q koji će se postići ako je zadana snaga pumpe N_{pumpa} = 500 kW uz koeficijent iskoristivosti pumpe η = 0,75 i ako je poznat koeficijent linijskih gubitaka (Darcyjev koeficijent trenja u cijevima) λ = 0,025. Tlak u gornjem dijelu komore A koji je ispunjen zrakom iznosi p_m = 500 kPa. Predviđen smjer kretanja vode je iz vodospreme A u vodospremu B. Promjer D je isti za sve cijevi u sustavu. Nacrtati piezometarsku i energetsku liniju.

Zadano je: D = 200 mm; h_1 = 30 m; h_2 = 20 m; h_3 = 200 m;
ξ_{UL} = 0,5; ξ_{KOLJ} = 0,3; L_1 = 120 m; L_2 = 200 m;
ρ = 1000 kg/m^3



$$N_{pumpa} = \frac{\rho g Q H_{pumpa}}{\eta} = 500\,kW \quad \rightarrow \quad H_{pumpa} = \frac{500 \cdot 0,75}{9,81 \cdot Q} = \frac{38,23}{Q}$$

$$A = \frac{D^2\pi}{4} = \frac{0,2^2\pi}{4} = 0,0314\,m^2$$

$$\frac{p_m}{\rho g} + H_p = h_3 + \frac{Q^2}{2gA^2}\left(\xi_{UL} + 2\xi_{KOLJ} + \lambda\frac{L_1 + 3L_2}{D} + 1\right)$$

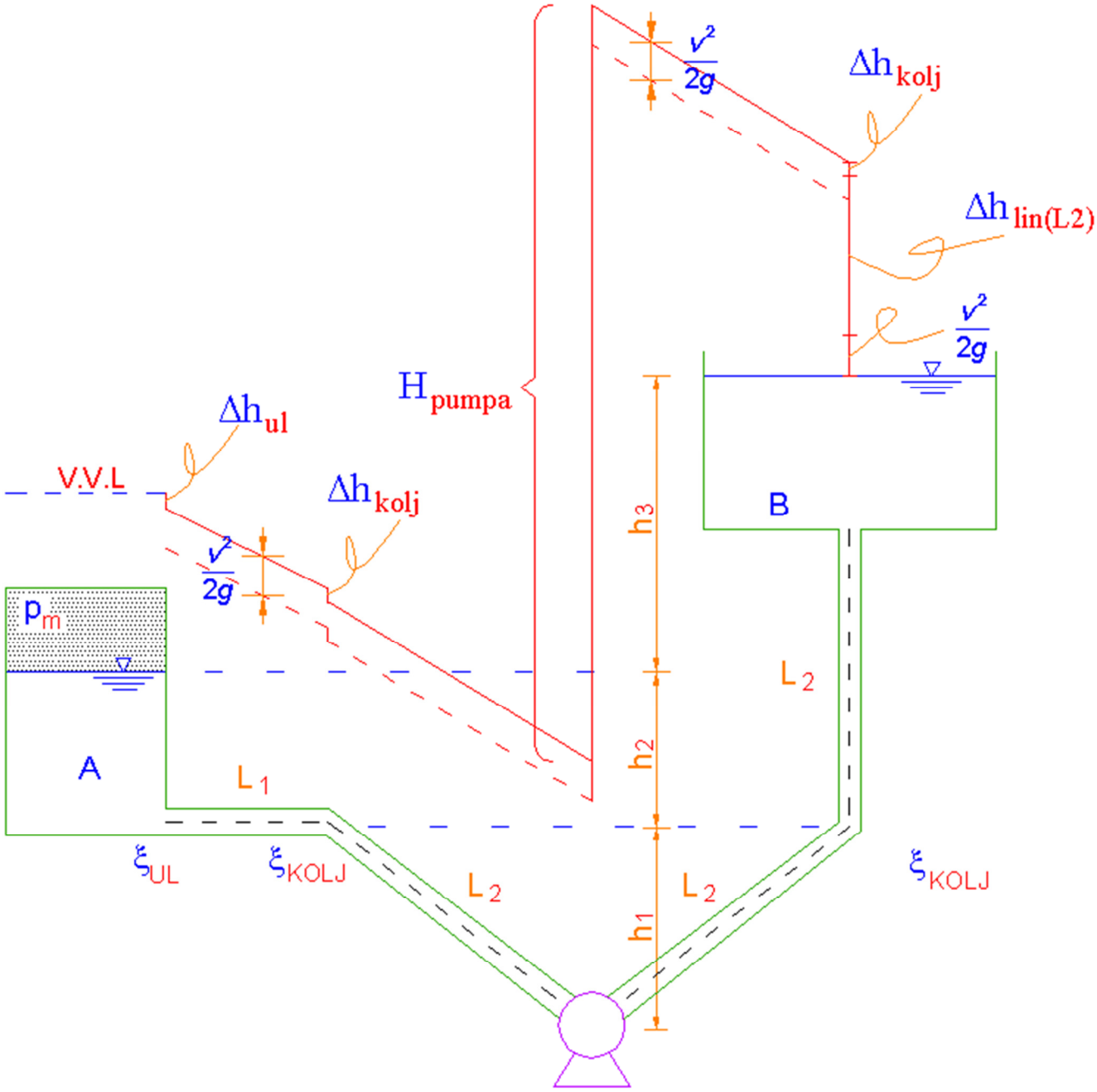
$$\frac{500}{9,81} + \frac{38,23}{Q} = 200 + \frac{Q^2}{2 \cdot 9,81 \cdot 0,0314^2}\left(0,5 + 2 \cdot 0,3 + 0,025 \cdot \frac{120 + 3 \cdot 200}{0,2} + 1\right)$$

$$50,97 + \frac{38,23}{Q} = 200 + 5169,42Q^2$$

$$5169,42Q^2 - \frac{38,23}{Q} + 149,03 = 0$$

Iteracijski postupak...

$$Q = 0,1468\,m^3/s$$

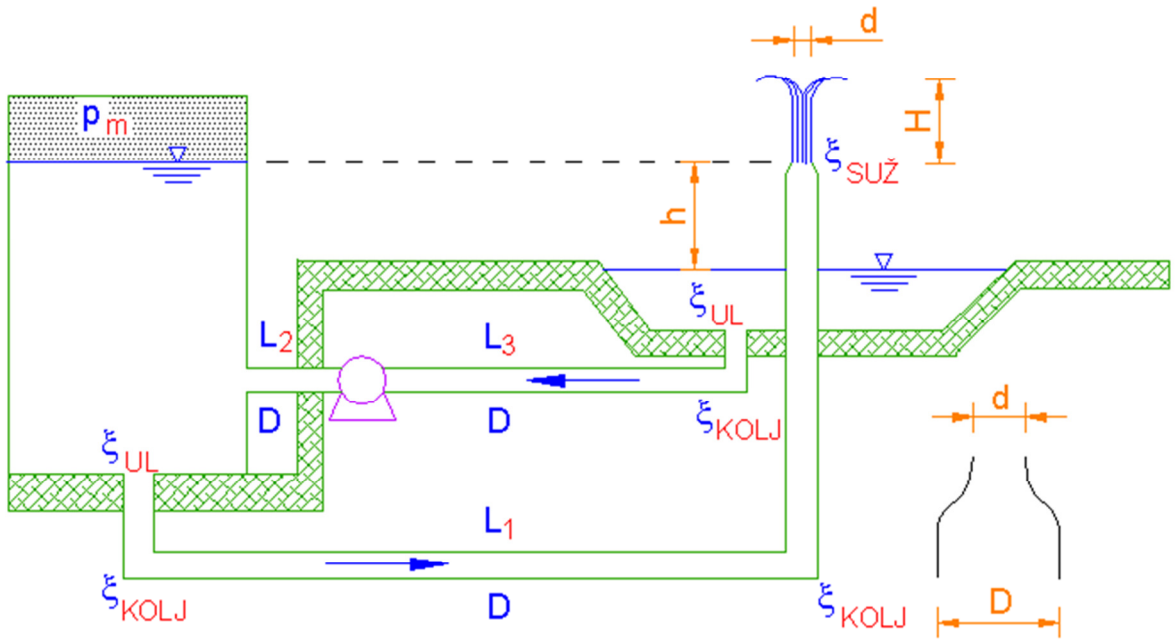

$\frac{v^2}{2g}$
Δh_{kolj}
$\Delta h_{lin(L2)}$
$\frac{v^2}{2g}$
H_{pumpa}
Δh_{ul}
V.V.L
Δh_{kolj}
$\frac{v^2}{2g}$
B
h_3
p_m
L_2
A
L_1
h_2
ξ_{UL}
ξ_{KOLJ}
L_2
L_2
h_1
ξ_{KOLJ}

8. Za sistem kao na slici treba odrediti tlak u komori p_m i snagu pumpe N_{pumpa} da bi se ostvarilo stacionarno tečenje kao na slici. Nacrtati energetsku i piezometarsku liniju.

Zadano je:

$H = 5$ m;	$h = 6$ m;	$\rho = 1000$ kg/m^3;	$\nu = 1\cdot10^{-6}$ m^2/s;
$D = 50$ mm;	$d = 15$ mm;	$\varepsilon = 0{,}5$ mm;	$\eta_{pumpa} = 0{,}7$;
$L_1 = 30$ m;	$L_2 = 5$ m;	$L_3 = 10$ m;	$\xi_{UL} = 0{,}5$;
$\xi_{KOLJ} = 0{,}2$;	$\xi_{SUŽ} = f(v_D) = 0{,}05$;		$\rho = 1000$ kg/m^3

Napomena: H je visina koju doseže mlaz; ν je kinematski koeficijent viskoznosti vode i kad je zadan pretpostavlja se da u cijevima vlada prijelazni režim (između laminarnog i turbulentnog) tečenja.



$$v_d = \sqrt{2gH} = \sqrt{2\cdot 9{,}81\cdot 5} = 9{,}9\ m/s$$

$$Q = v_d \cdot A_d = v_d \cdot \frac{d^2\pi}{4} = 9{,}9\cdot\frac{0{,}015^2\cdot\pi}{4} = 0{,}00175\ m^3/s$$

$$v_D = \frac{Q}{A_D} = \frac{4Q}{D^2\pi} = \frac{4\cdot 0{,}00175}{0{,}05^2\cdot\pi} = 0{,}891\ m/s$$

$$\left.\begin{aligned} \mathrm{Re} &= \frac{v_D\cdot D}{\upsilon} = \frac{0{,}891\cdot 0{,}05}{1\cdot10^{-6}} = 4{,}5\cdot10^4 \\ \frac{\varepsilon}{D} &= \frac{0{,}5}{50} = 0{,}01 \end{aligned}\right\} \quad \lambda = 0{,}04$$

$$\frac{p_m}{\rho g} = H + \frac{v_D^2}{2g}\left(\xi_{UL} + 2\xi_{KOLJ} + \lambda\frac{L_1}{D} + \xi_{SUŽ}\right)$$

$$\frac{p_m}{\rho g} = 5 + \frac{0,891^2}{2g}\left(0,5 + 2\cdot 0,2 + 0,04\cdot\frac{30}{0,05} + 0,05\right)$$

$$\frac{p_m}{\rho g} = 5 + 0,04\left(0,5 + 2\cdot 0,2 + 24 + 0,05\right)$$

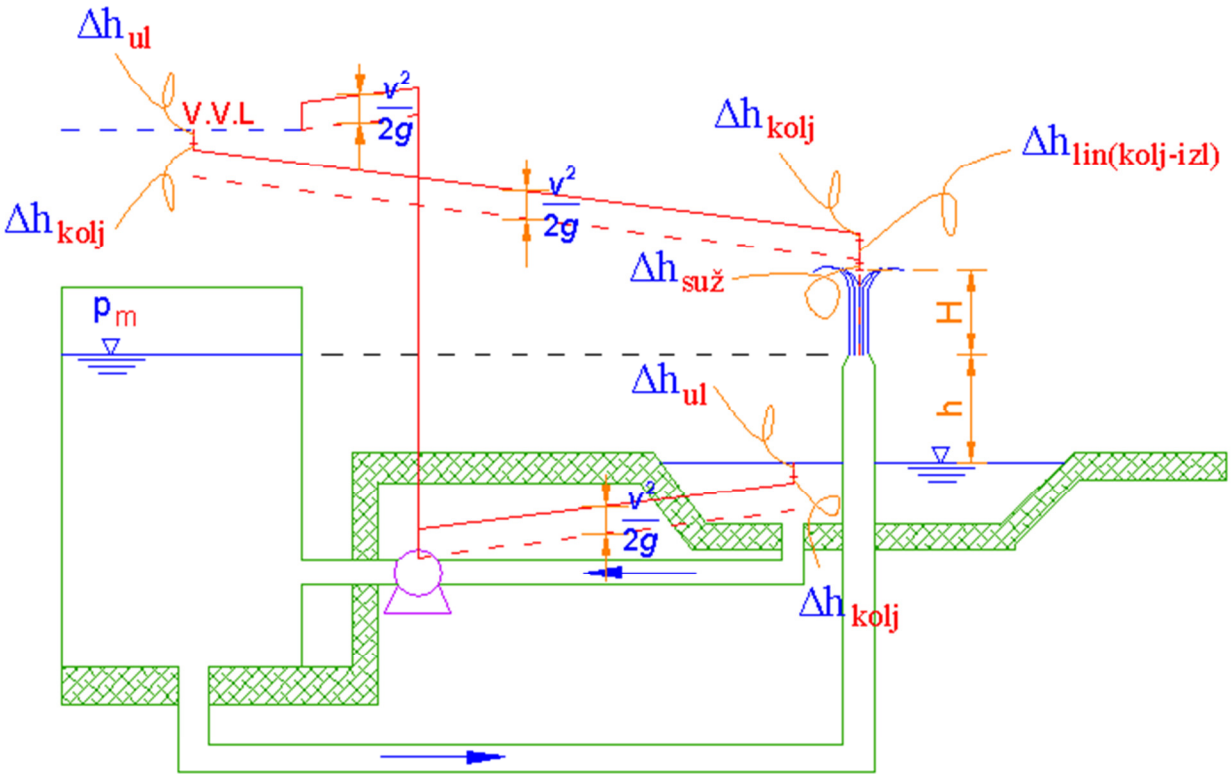
$$\frac{p_m}{\rho g} = 6,0 \qquad \rightarrow \qquad p_m = 58,84\ kPa$$

$$H_{pumpa} = h + \frac{p_m}{\rho g} + \frac{v_D^2}{2g}\left(\xi_{UL} + \xi_{KOLJ} + \lambda\frac{L_3 + L_2}{D} + 1\right)$$

$$H_{pumpa} = 6 + 6 + 0,04\left(0,5 + 0,2 + 0,04\cdot\frac{10+5}{0,05} + 1\right)$$

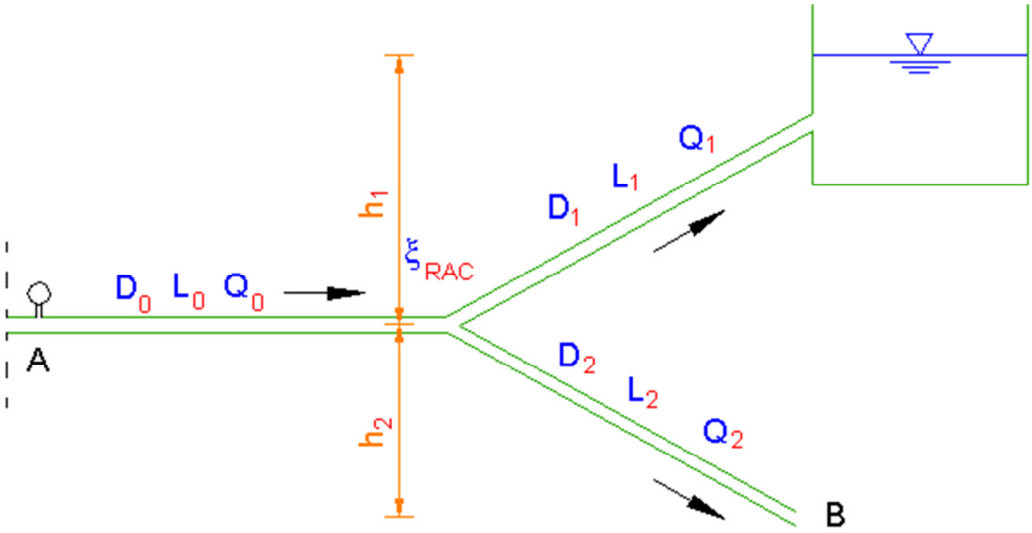
$$H_{pumpa} = 12,55\ m$$

$$N_{pumpa} = \frac{\rho g H_{pumpa} Q}{\eta} = \frac{1\cdot 9,81\cdot 12,55\cdot 0,00175}{0,7} = 0,308\ kW$$



9. Za cjevovod kao na slici treba odrediti protoke Q_0, Q_1 i Q_2 te nacrtati piezometarsku i energetsku liniju za sve cijevi. U cijevi 1 izmjerena je brzina v_1 = 5,06 m/s, a na manometru (tlakomjeru) u točki A je izmjeren tlak od p_A = 2 bar. U točki B je slobodno istjecanje u atmosferu. Lokalni gubitak energije (tlaka) na račvi je funkcija brzine uzvodno od račve.

Zadano je: $\rho = 1000$ kg/m^3; $D_0 = 300$ mm; $D_1 = D_2 = 200$ mm; $\lambda = 0{,}02$ (za sve cijevi); $L_0 = L_1 = L_2 = 80$ m; $h_1 = 3$ m; $h_2 = 2$ m; $\xi_{RAČ} = 0{,}2$



$$Q_1 = v_1 \frac{D_1^2 \pi}{4} = 5{,}06 \frac{0{,}2^2 \pi}{4} = 0{,}159\ m^3/s$$

$$\frac{p_A}{\rho g} + \frac{v_0^2}{2g} = h_1 + \frac{v_0^2}{2g}\left(\lambda \frac{L_0}{D_0} + \xi_{RAC}\right) + \frac{v_1^2}{2g}\left(\lambda \frac{L_1}{D_1} + 1\right)$$

$$\frac{200}{9{,}81} + \frac{v_0^2}{2 \cdot 9{,}81} = 3 + \frac{v_0^2}{2 \cdot 9{,}81}\left(0{,}02 \frac{80}{0{,}3} + 0{,}2\right) + \frac{5{,}06^2}{2 \cdot 9{,}81}\left(0{,}02 \frac{80}{0{,}2} + 1\right)$$

$$20{,}387 + 0{,}051 v_0^2 = 3 + 0{,}282 v_0^2 + 11{,}745$$

$$v_0 = 4{,}94\ m/s$$

$$Q_0 = v_0 \frac{D_0^2 \pi}{4} = 4{,}94 \frac{0{,}3^2 \pi}{4} = 0{,}349\ m^3/s$$

$$Q_2 = Q_0 - Q_1 = 0{,}349 - 0{,}159 = 0{,}19\ m^3/s$$

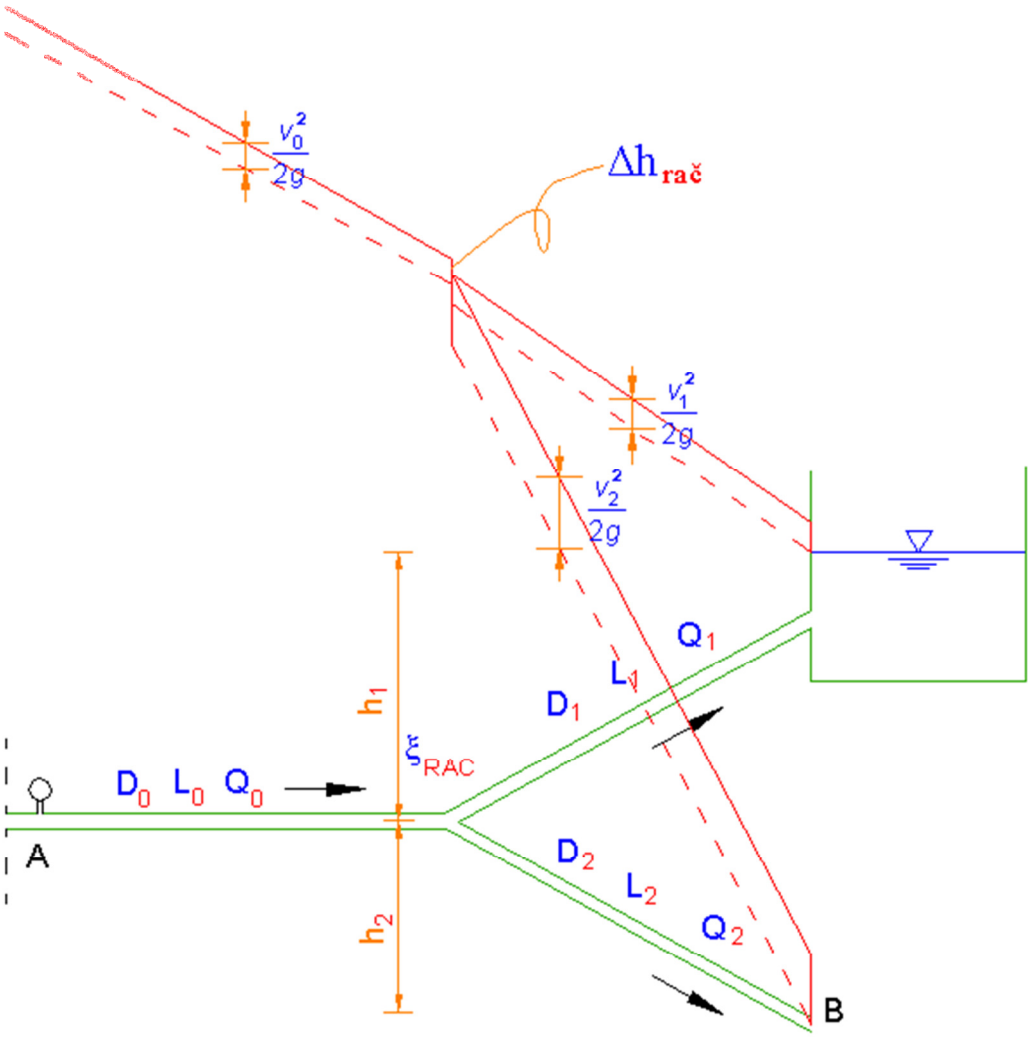
$$v_2 = \frac{Q_2 \cdot 4}{D_2^2 \pi} = \frac{0{,}19 \cdot 4}{0{,}2^2 \pi} = 6{,}05\ m/s$$

provjera :

$$\frac{p_A}{\rho g}+\frac{v_0^2}{2g}=h_2+\frac{v_0^2}{2g}\left(\lambda\frac{L_0}{D_0}+\xi_{RAC}\right)+\frac{v_2^2}{2g}\left(\lambda\frac{L_1}{D_1}+1\right)$$

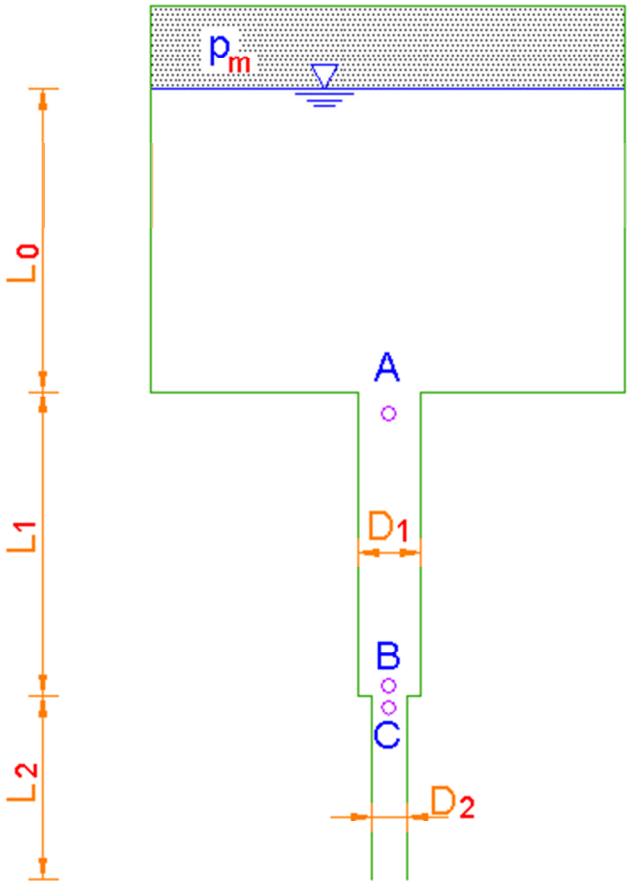
$$\frac{200}{9,81}+\frac{4,94^2}{2\cdot 9,81}=-2+\frac{4,94^2}{2\cdot 9,81}\left(0,02\frac{80}{0,3}+0,2\right)+\frac{6,05^2}{2\cdot 9,81}\left(0,02\frac{80}{0,2}+1\right)$$

$21,63=21,67$ (*zadovoljavajuće točno*!)



10. Iz vodospreme istječe voda kroz vertikalnu cijev. Odredite izlaznu brzinu iz cjevovoda i tlak u točkama A, B i C te nacrtajte energetsku i piezometarsku liniju.

Zadano je: $\rho = 1000$ kg/m^3; $p_m = 19{,}62$ kPa; $\lambda = 0{,}025$ (za sve cijevi);
$D_1 = 200$ mm; $D_2 = 150$ mm; $\xi_{UL} = 0{,}5$; $\xi_{SUŽ} = 0{,}3 = f(v_2)$;
$L_0 = 5$ m; $L_1 = 5$ m; $L_2 = 3$ m



$$v_1 \cdot A_1 = v_2 \cdot A_2$$

$$v_1 = v_2 \cdot \frac{D_2^2}{D_1^2} = v_2 \cdot \frac{0{,}15^2}{0{,}2^2} = 0{,}563 v_2$$

$$(L_2 + L_1 + L_0) + \frac{p_m}{\rho g} = \frac{v_2^2}{2g}\left(\lambda \frac{L_2}{D_2} + \xi_{SUŽ} + 1\right) + \frac{v_1^2}{2g}\left(\xi_{UL} + \lambda \frac{L_1}{D_1}\right)$$

$$(3+5+5) + 2 = \frac{v_2^2}{2g}(0{,}5 + 0{,}3 + 1) + \frac{(0{,}563 v_2)^2}{2g}(0{,}5 + 0{,}625)$$

$$15 = 0{,}0917 v_2^2 + 0{,}0182 v_2^2$$

$$v_2 = 11{,}68\ m/s \quad \rightarrow \quad \frac{v_2^2}{2g} = 6{,}96\ m$$

$$v_1 = 0{,}563 v_2 = 6{,}58\ m/s \quad \rightarrow \quad \frac{v_1^2}{2g} = 2{,}2\ m$$

$$L_2 + \frac{p_C}{\rho g} + \frac{v_2^2}{2g} = \frac{v_2^2}{2g} + \lambda \cdot \frac{L_2}{D_2} \cdot \frac{v_2^2}{2g}$$

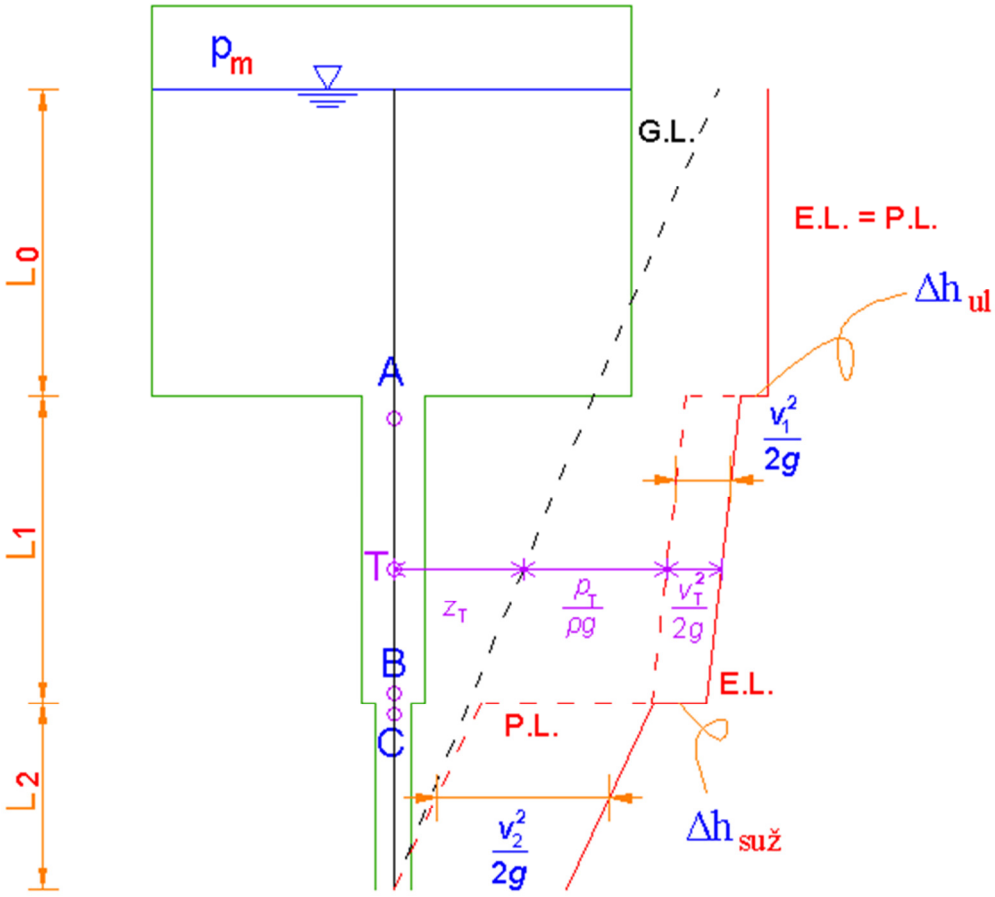
$$\frac{p_C}{\rho g} = 0,48\,m \qquad \rightarrow \qquad p_C = 4,71\,kPa$$

$$L_2 + \frac{p_B}{\rho g} + \frac{v_1^2}{2g} = L_2 + \frac{p_C}{\rho g} + \frac{v_2^2}{2g} + \frac{v_2^2}{2g} \cdot \xi_{SUŽ}$$

$$\frac{p_B}{\rho g} = 7,33\,m \qquad \rightarrow \qquad p_B = 71,89\,kPa$$

$$L_2 + L_1 + \frac{p_A}{\rho g} + \frac{v_1^2}{2g} = L_2 + \frac{p_B}{\rho g} + \frac{v_1^2}{2g} + \frac{v_1^2}{2g} \cdot \lambda \frac{L_1}{D_1}$$

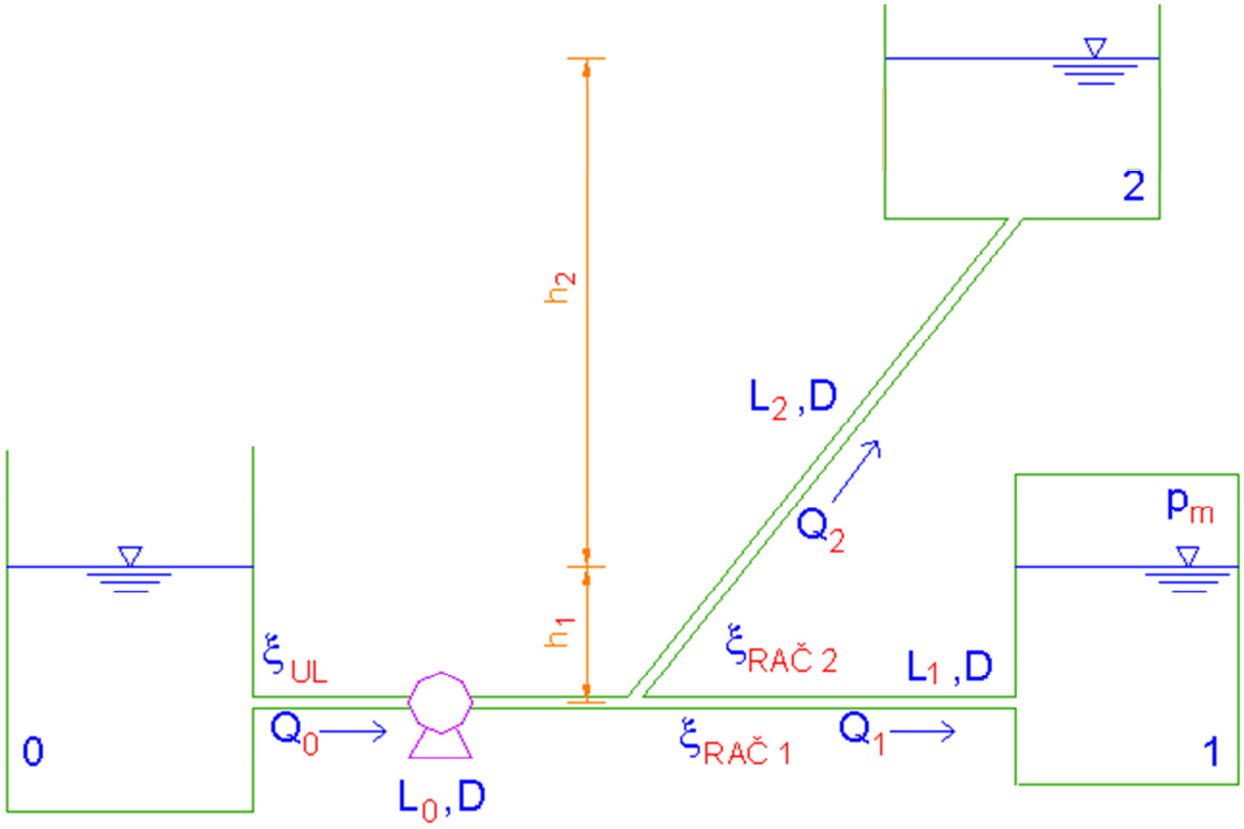
$$\frac{p_A}{\rho g} = 3,71\,m \qquad \rightarrow \qquad p_A = 36,35\,kPa$$



11. Za cjevovod prema slici potrebno je odrediti protoke Q_1, Q_2 i razliku vodnih lica h_2 između komore 2 i komore 1, odnosno 0. Nacrtati energetsku i piezometarsku liniju.

Zadano je:

$Q_0 = 0{,}2$ m^3/s;	$H_{pumpa} = 35$ m;	$p_m = 98{,}1$ kPa;
$L_0 = 200$ m;	$L_1 = 200$ m;	$L_2 = 300$ m;
$D = 250$ mm;	$\varepsilon = 0{,}5$ mm;	$\lambda = f(\varepsilon/D)$;
$\xi_{RAČ1} = f(v_1) = 0{,}3$;	$\xi_{RAČ\,2} = f(v_2) = 0{,}4$;	$\xi_{UL} = 0{,}5$;
$h_1 = 4$ m;	$\rho = 1000$ kg/m^3	



$$A = \frac{D^2\pi}{4} = 0{,}049\ m^2$$

$$\frac{\varepsilon}{D} = \frac{0{,}5}{250} = 0{,}002 \qquad \rightarrow \qquad \lambda = 0{,}024$$

$$H_{pumpa} = \frac{p_m}{\rho g} + \frac{Q_0^2}{2gA^2}\left(\xi_{UL} + \lambda\frac{L_0}{D}\right) + \frac{Q_1^2}{2gA^2}\left(\xi_{RAČ1} + \lambda\frac{L_1}{D} + 1\right)$$

$$35 = \frac{98{,}1}{1\cdot g} + \frac{0{,}2^2}{2g\cdot 0{,}049^2}\left(0{,}5 + 0{,}024\cdot\frac{200}{0{,}25}\right) + \frac{Q_1^2}{2g\cdot 0{,}049^2}\left(0{,}3 + 0{,}024\cdot\frac{200}{0{,}25} + 1\right)$$

$$35 = 10 + 16{,}73 + 435{,}17Q_1^2$$

$$Q_1 = 0{,}138\ m^3/s \qquad \rightarrow \qquad v_1 = \frac{0{,}138}{0{,}049} = 2{,}82\ m/s \qquad \rightarrow \qquad \frac{v_1^2}{2g} = 0{,}4\ m$$

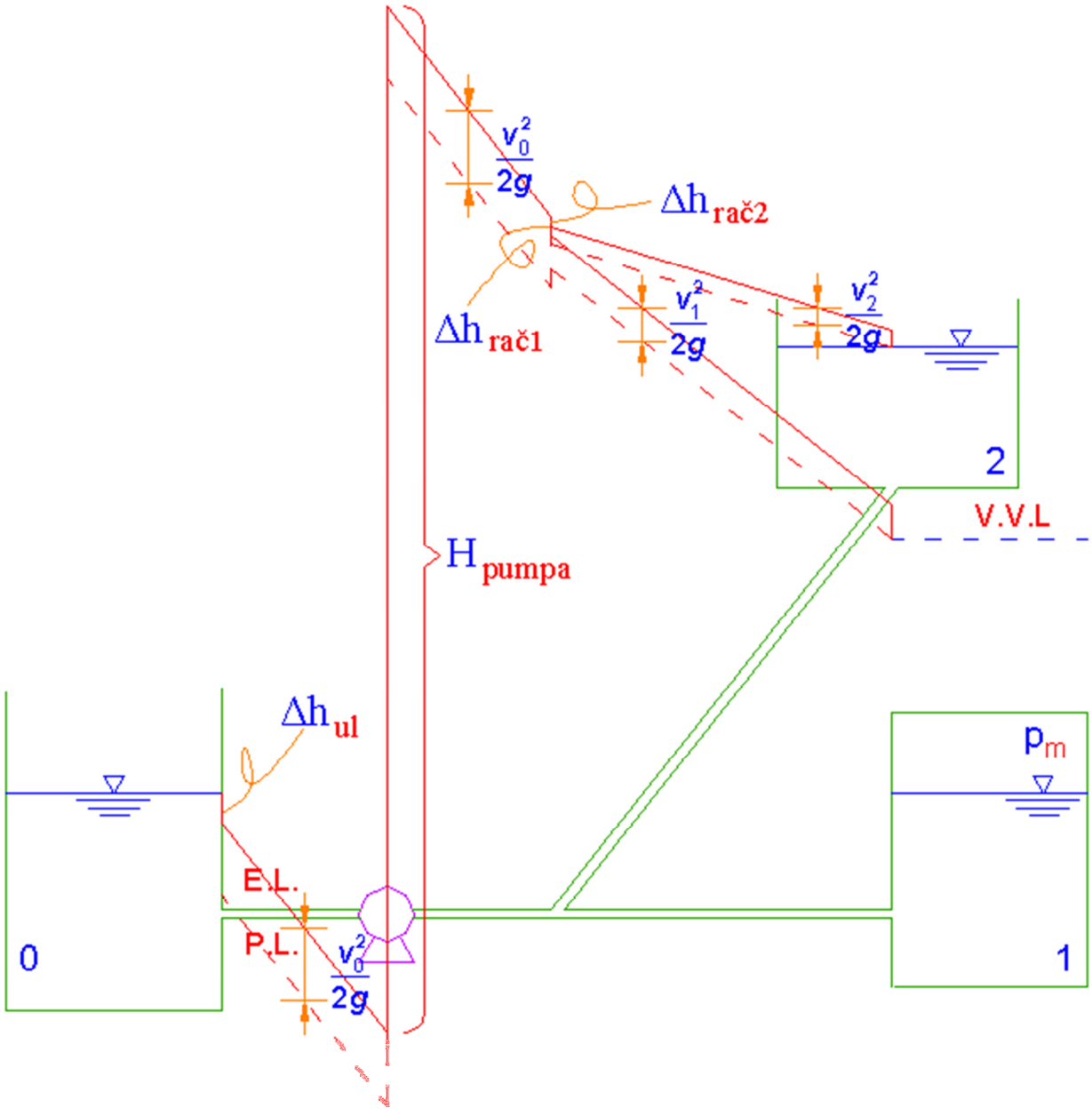
$$Q_2 = Q_0 - Q_1 = 0{,}2 - 0{,}138$$

$$Q_2 = 0{,}062\ m^3/s \qquad \rightarrow \qquad v_2 = \frac{0{,}062}{0{,}049} = 1{,}27\ m/s \qquad \rightarrow \qquad \frac{v_2^2}{2g} = 0{,}08\ m$$

$$H_{pumpa} = h_2 + \frac{v_0^2}{2g}\left(\xi_{UL} + \lambda\frac{L_0}{D}\right) + \frac{v_2^2}{2g}\left(\xi_{RAČ2} + \lambda\frac{L_2}{D} + 1\right)$$

$$35 = h_2 + 0,85\left(0,5 + 19,2\right) + 0,08\left(0,4 + 28,8 + 1\right)$$

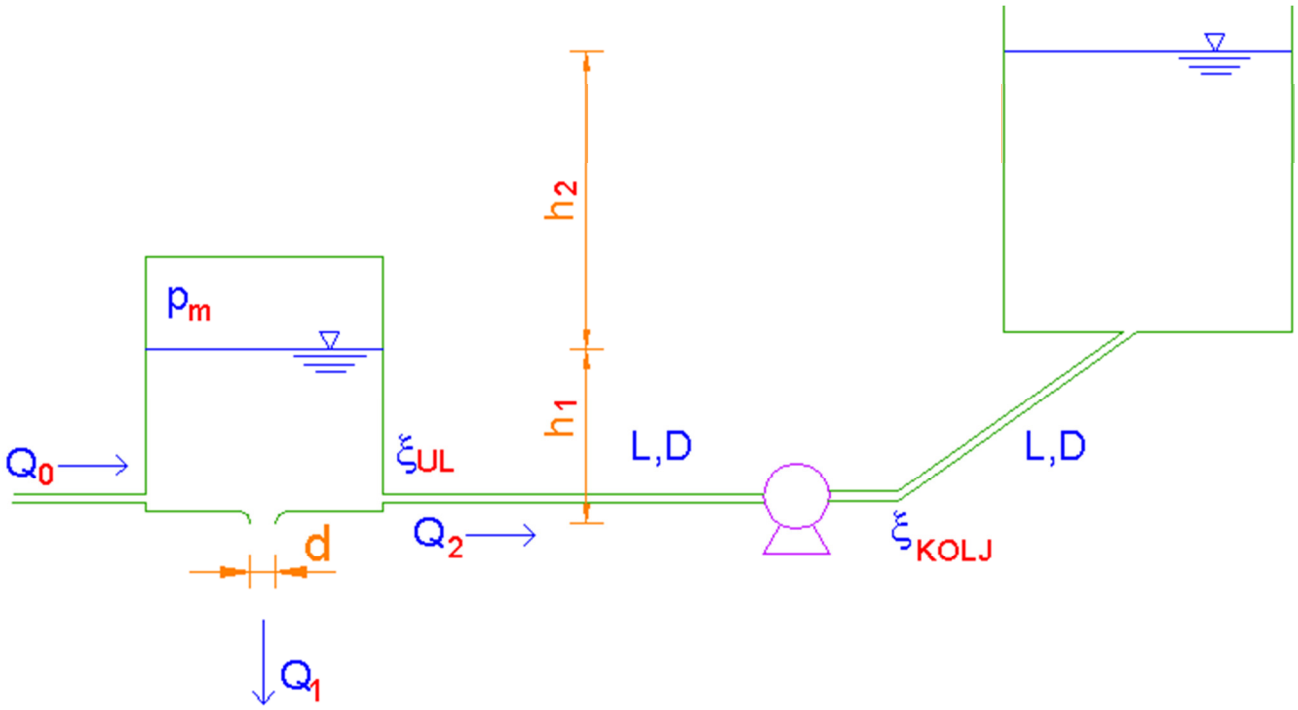
$$h_2 = 15,84\ m$$



Napomena: Iz skice piezometarske linije se vidi da dolazi do podtlaka (negativnog tlaka) na dijelu cjevovoda ispred pumpe. Takva situacija se ne dozvoljava u hidrotehničkoj praksi.

12. Za stacionarno strujanje u cjevovodu prema slici potrebno je odrediti protoke Q_1 i Q_2, snagu pumpe N_{pumpa} i nacrtati energetsku i piezometarsku liniju.

Zadano je: $Q_0 = 0{,}04$ m^3/s; $p_m = -29{,}43$ kPa; $\eta_{pumpa} = 0{,}7$; $\nu = 1\cdot10^{-6}$ m^2/s
$d = 50$ mm; $D = 200$ mm; $\varepsilon = 0{,}2$ mm; $\lambda = f\,(Re, \varepsilon/D)$
$L = 100$ m; $\xi_{UL} = 0{,}5$; $\xi_{KOLJ} = 0{,}3$;
$\rho = 1000$ kg/m^3; $h_1 = 5$ m; $h_2 = 10$ m;



$$v_1 = \sqrt{2g\left(h + \frac{p_m}{\rho g}\right)} = \sqrt{2g\left(5 - \frac{29{,}43}{1\cdot g}\right)} = 6{,}26\ m/s$$

$$Q_1 = v_1 \frac{d^2\pi}{4} = 6{,}26 \cdot \frac{0{,}05^2\pi}{4} = 0{,}012\ m^3/s$$

$$Q_2 = Q_0 - Q_1 = 0{,}04 - 0{,}012 = 0{,}028\ m^3/s$$

$$v_2 = \frac{Q_2}{A_2} = \frac{4Q_2}{D^2\pi} = \frac{0{,}028\cdot 4}{0{,}2^2\cdot\pi} = 0{,}89\ m/s \quad \rightarrow \quad \frac{v_2^2}{2g} = 0{,}04\ m$$

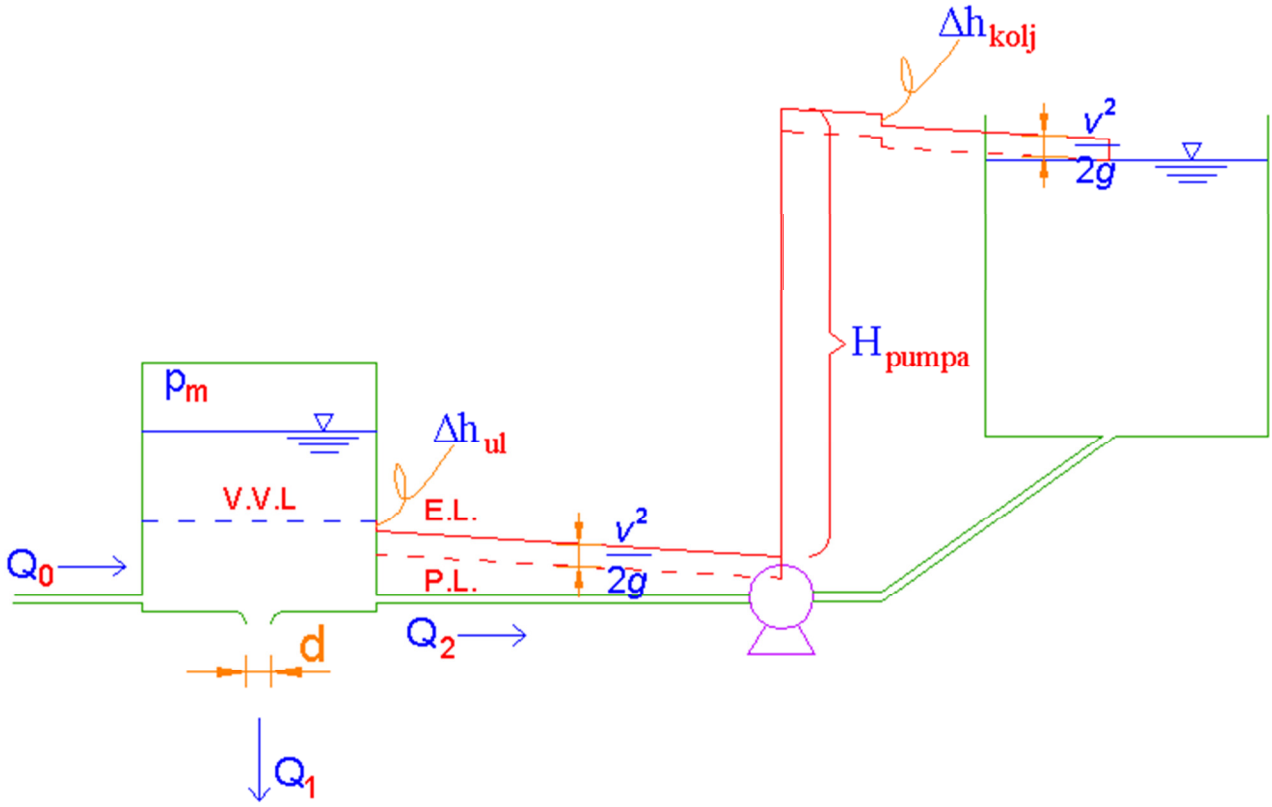
$$\frac{\varepsilon}{D} = \frac{0{,}2}{200} = 0{,}001 \qquad \mathrm{Re} = \frac{v_2 \cdot D}{\upsilon} = \frac{0{,}89\cdot 0{,}2}{1\cdot10^{-6}} = 1{,}78\cdot10^5 \quad \rightarrow \quad \lambda = 0{,}022$$

$$\frac{p_m}{\rho g} + H_{pumpa} = h_2 + \frac{v_2^2}{2g}\left(\xi_{UL} + \xi_{KOLJ} + \lambda\frac{2L}{D} + 1\right)$$

$$-3 + H_{pumpa} = 10 + 0{,}04\,(0{,}5 + 0{,}3 + 22 + 1)$$

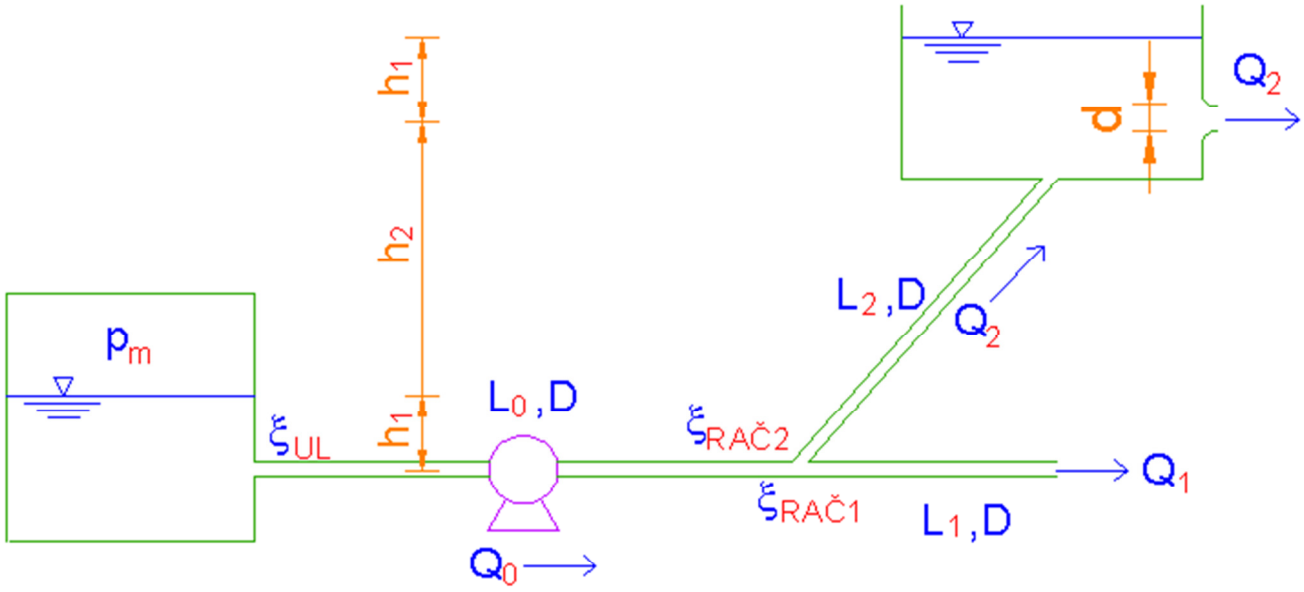
$$H_{pumpa} = 13{,}95\ m$$

$$N_{pumpa} = \frac{\rho g Q_2 H_{pumpa}}{\eta_{pumpa}} = \frac{1\cdot 9{,}81\cdot 0{,}028\cdot 13{,}95}{0{,}7} = 5{,}47\ kW$$


Δh kolj
v²
2g
H pumpa
p m
Δh ul
V.V.L
E.L.
P.L.
v²
2g
Q0
Q2
d
Q1

13. Za stacionarno strujanje u cjevovodu prema slici potrebno je odrediti protoke Q_0 , Q_1 i koeficijent lokalnog gubitka $\xi_{\text{RAČ 1}} = f(v_1)$. Nacrtati energetsku i piezometarsku liniju.

Zadano je: $H_{pumpa} = 15$ m; $p_m = 19{,}62$ kPa; $h_1 = 2$ m;
$h_2 = 6$ m; $L_0 = 200$ m; $L_1 = 200$ m;
$L_2 = 300$ m; $d = 50$ mm; $D = 250$ mm ;
$\xi_{UL} = 0{,}5$; $\xi_{RAČ2} = f(v_2) = 0{,}4$; $\lambda = 0{,}025$;
$\rho = 1000$ kg/m^3



$$Q_2 = \sqrt{2g \cdot h_1} \cdot A_d = \sqrt{2g \cdot 2} \cdot \frac{0{,}05^2 \pi}{4} = 0{,}012 \text{ m}^3/\text{s}$$

$$v_2 = \frac{4Q_2}{D^2 \pi} = 0{,}24\, m/s \quad \rightarrow \quad \frac{v_2^2}{2g} = 0{,}003\, m$$

$$\frac{p_m}{\rho g} + H_{pumpe} = h_1 + h_2 + \frac{v_0^2}{2g}\left(\xi_{\text{UL}} + \lambda \frac{L_0}{D}\right) + \frac{v_2^2}{2g}\left(\xi_{\text{RAČ2}} + \lambda \frac{L_2}{D} + 1\right)$$

$$2 + 15 = 2 + 6 + \frac{v_0^2}{2g}\left(0{,}5 + 0{,}025 \cdot \frac{200}{0{,}25}\right) + 0{,}003\left(0{,}4 + 0{,}025 \cdot \frac{300}{0{,}25} + 1\right)$$

$$17 = 8 + \frac{v_0^2}{2g}(0{,}5 + 20) + 0{,}003(0{,}4 + 30 + 1)$$

$$\frac{v_0^2}{2g} = 0{,}43\, m \quad \rightarrow \quad v_0 = 2{,}92\, m/s \quad \rightarrow \quad Q_0 = 0{,}143\, m^3/s$$

$$Q_1 = Q_0 - Q_2 = 0{,}143 - 0{,}012$$

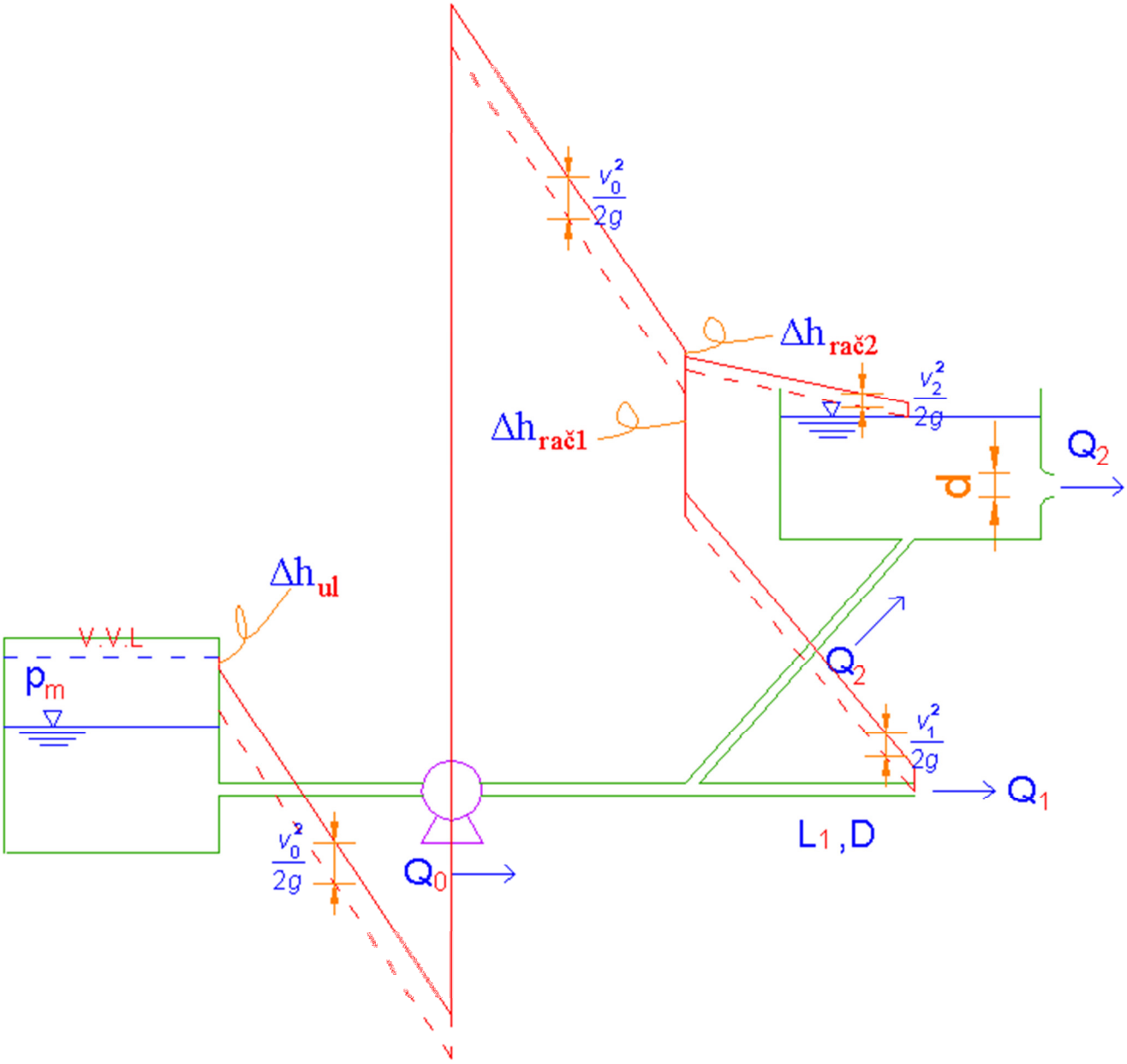
$$Q_1 = 0{,}131 \text{ m}^3/\text{s} \quad \rightarrow \quad v_1 = 2{,}67\, m/s \quad \rightarrow \quad \frac{v_1^2}{2g} = 0{,}36\, m$$

$$h_1+\frac{p_m}{\rho g}+H_{pumpa}=\frac{v_0^2}{2g}\left(\xi_{UL}+\lambda\cdot\frac{L_0}{D}\right)+\frac{v_1^2}{2g}\left(\xi_{RAČ1}+\lambda\cdot\frac{L_1}{D}+1\right)$$

$$19=0,43\left(0,5+20\right)+0,36\left(\xi_{RAČ1}+20+1\right)$$

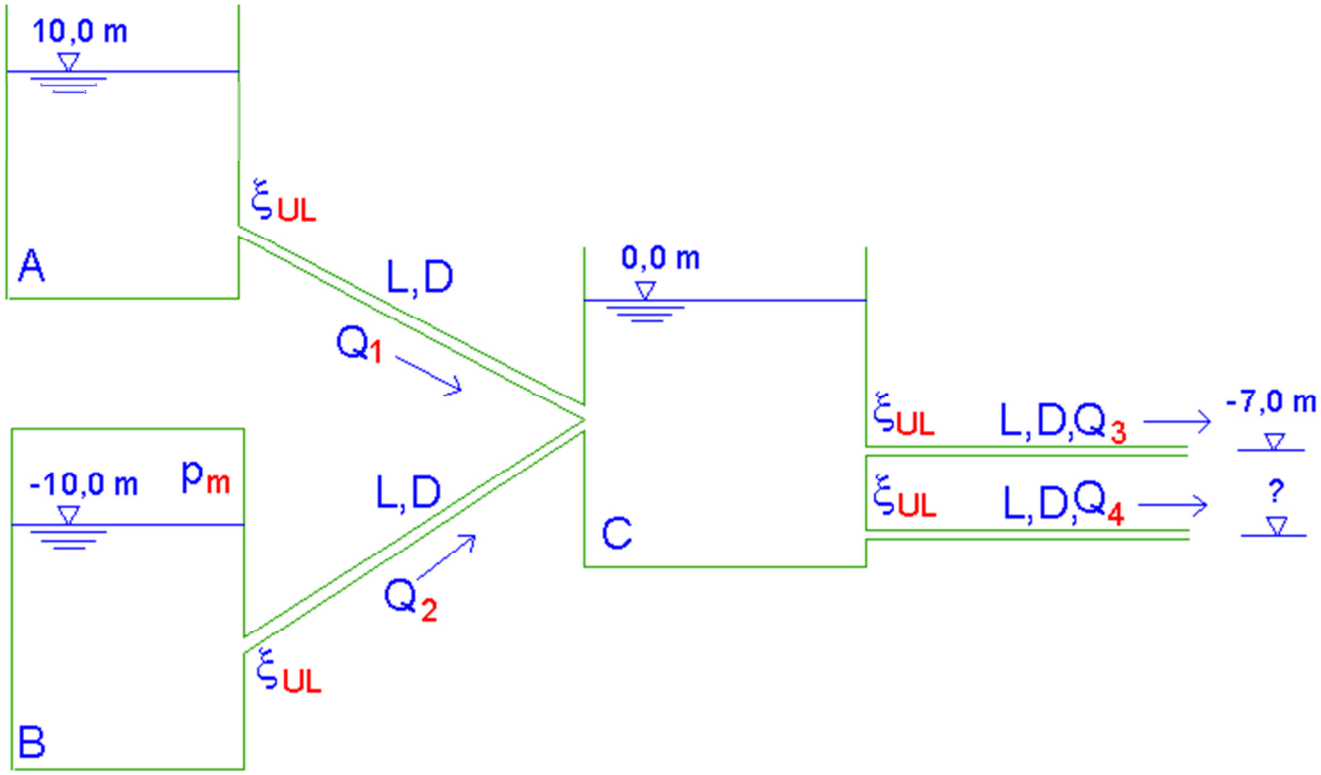
$$19=8,82+0,36\xi_{RAČ1}+7,56$$

$$\xi_{RAČ1}=7,28$$



14. Za cjevovod prema slici potrebno je odrediti protoke Q_1 ,Q_2 ,Q_3, Q_4 i kotu osi donje horizontalne cijevi (cijev 4) koja izlazi iz komore *C* kako bi razina vode u komori *C* bila konstantna. Potrebno je nacrtati energetsku i piezometarsku liniju za sve cijevi.

Zadano je: $p_m = 147{,}15$ kPa; $L = 200$ m (sve cijevi); $\rho = 1000$ kg/m^3;
$D = 250$ mm (sve cijevi); $\lambda = 0{,}025$ (sve cijevi); $\xi_{UL} = 0{,}5$



$$A = \frac{D^2\pi}{4} = 0{,}05\ m^2$$

$$10 = \frac{Q_1^2}{2gA^2}\left(\xi_{UL} + \lambda\cdot\frac{L}{D} + 1\right)$$

$$Q_1 = 0{,}151\ m^3/s$$

$$-10 + \frac{p_m}{\rho g} = \frac{Q_2^2}{2gA^2}\left(\xi_{UL} + \lambda\cdot\frac{L}{D} + 1\right)$$

$$Q_2 = 0{,}107\ m^3/s$$

$$Q_1 + Q_2 = 0{,}257\ m^3/s$$

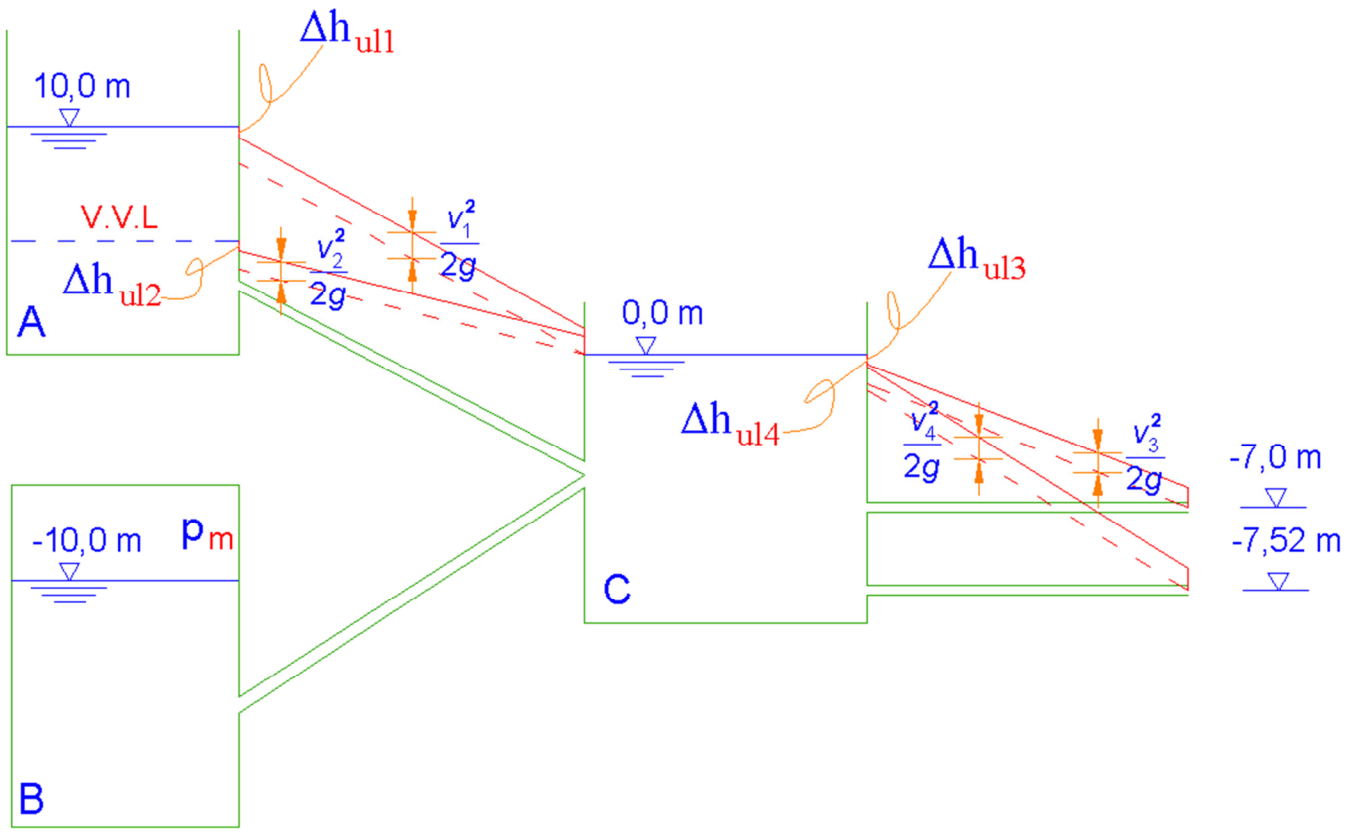
$$7 = \frac{Q_3^2}{2gA^2}\left(\xi_{UL} + \lambda\cdot\frac{L}{D} + 1\right)$$

$$Q_3 = 0{,}126\ m^3/s$$

$$Q_4 = (Q_1 + Q_2) - Q_3 = 0{,}257 - 0{,}126 = 0{,}131\ m^3/s$$

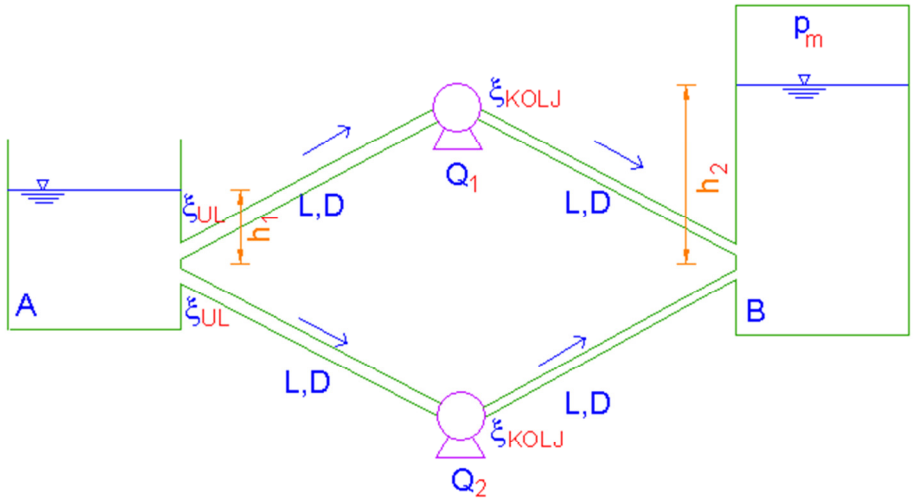
$$x = \frac{Q_4^2}{2gA^2}\left(\xi_{UL} + \lambda\cdot\frac{L}{D} + 1\right)$$

$x = 7{,}52\ m \quad \rightarrow \quad$ *kota osi cijevi* 4 *je na* $-7{,}52\ m$


Δh ul1
10,0 m
V.V.L
Δh ul2
A
v1²/2g
v2²/2g
0,0 m
Δh ul3
Δh ul4
v4²/2g
v3²/2g
-7,0 m
-7,52 m
-10,0 m
pm
B
C

15. Za stacionarno strujanje u cjevovodu prema slici potrebno je odrediti ukupni protok Q iz komore A u komoru B ako je predtlak u komori B p_m = 49,05 kPa. Nacrtati energetsku i piezometarsku liniju za cijeli sustav.

Zadano je:
L = 100 m (sve cijevi); D = 200 mm (sve cijevi); h_1 = 2 m;
h_2 = 5 m; λ = 0,02 (sve cijevi); ξ_{UL} = 0,5;
ξ_{KOLJ} = 0,2; N_{pumpa} = 23,5 kW; η_{pumpa} = 0,7;
ρ = 1000 kg/m^3



$$Q_1 = Q_2 = Q$$

$$h_1 + H_{pumpa} = h_2 + \frac{p_m}{\rho g} + \frac{Q^2}{2gA^2}\left(\xi_{UL} + \xi_{KOLJ} + \lambda\frac{2L}{D} + 1\right)$$

$$h_1 + \frac{N_{pumpa}\cdot\eta}{\rho g Q} = h_2 + \frac{p_m}{\rho g} + \frac{Q^2}{2gA^2}\left(\xi_{UL} + \xi_{KOLJ} + \lambda\frac{2L}{D} + 1\right)$$

$$2 + \frac{23,5\cdot 0,7}{1\cdot 9,81\cdot Q} = 5 + 5 + \frac{Q^2}{2\cdot 9,81\cdot 0,0314^2}\left(0,5 + 0,2 + 0,02\cdot\frac{200}{0,2} + 1\right)$$

$$1121,76Q^2 - \frac{1,677}{Q} + 8 = 0$$

$$1121,76Q^3 + 8Q - 1,677 = 0$$

$$Q = \frac{1,677 - 1121,76Q^3}{8}$$ *implicitna jednadžba (potrebno rješiti iteracijskim postupkom)*

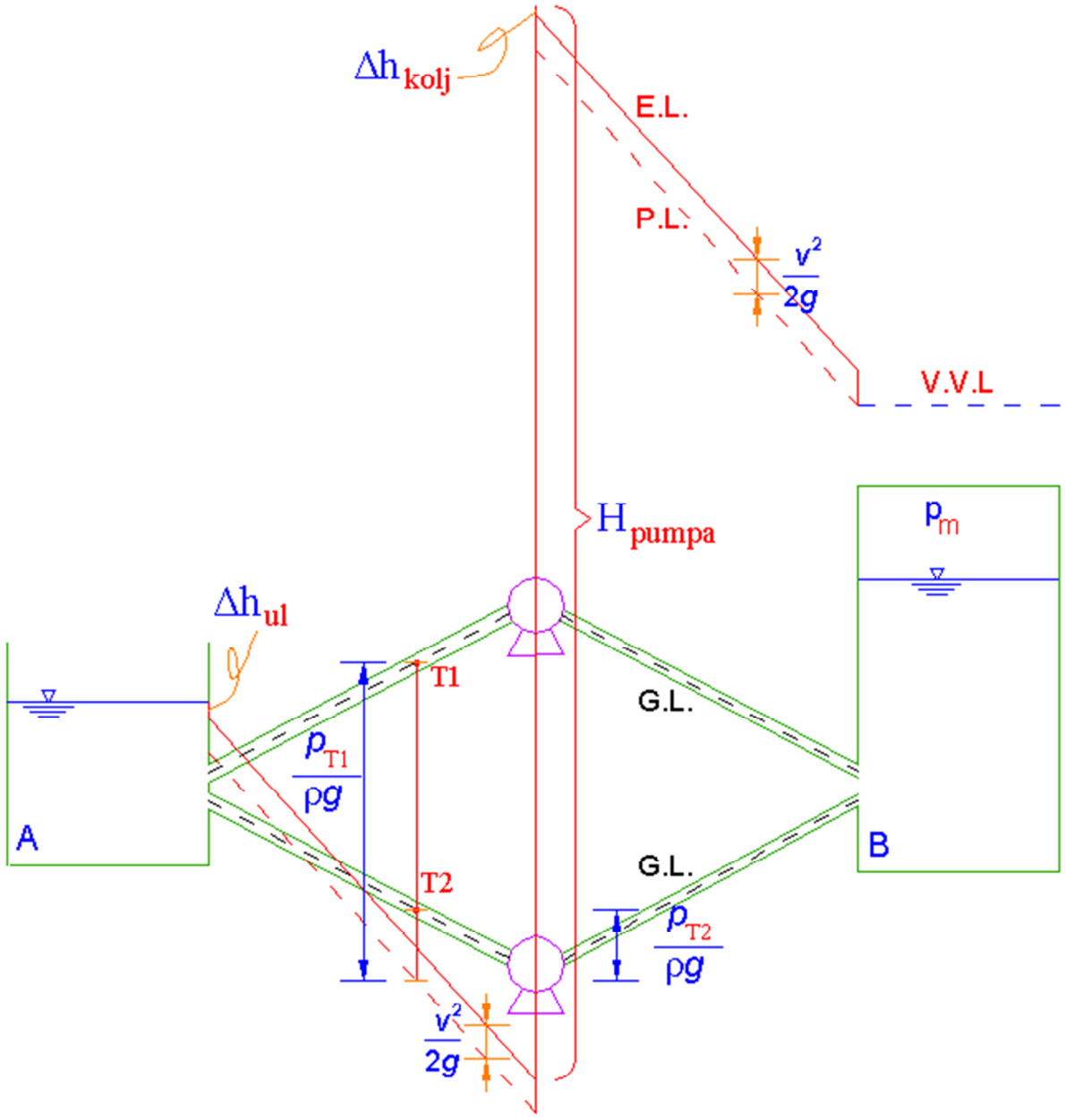
Q pretp (desna strana)	Q dobiv (lijeva strana)
1	-140,01
0,1	0,07
0,05	0,19
0,09	0,11
0,095	0,089
0,094	0,093

$Q = 0,094\ m^3/s$ *prihvaćeno rješenje sa zadovoljavajućom točnošću*

$Q_{UK} = 2Q = 2 \cdot 0,094 = 0,188\ m^3/s$

$$\frac{v^2}{2g} = \frac{Q^2}{2gA^2} = \frac{0,094^2}{2 \cdot 9,81 \cdot 0,0314^2} = 0,46\ m$$

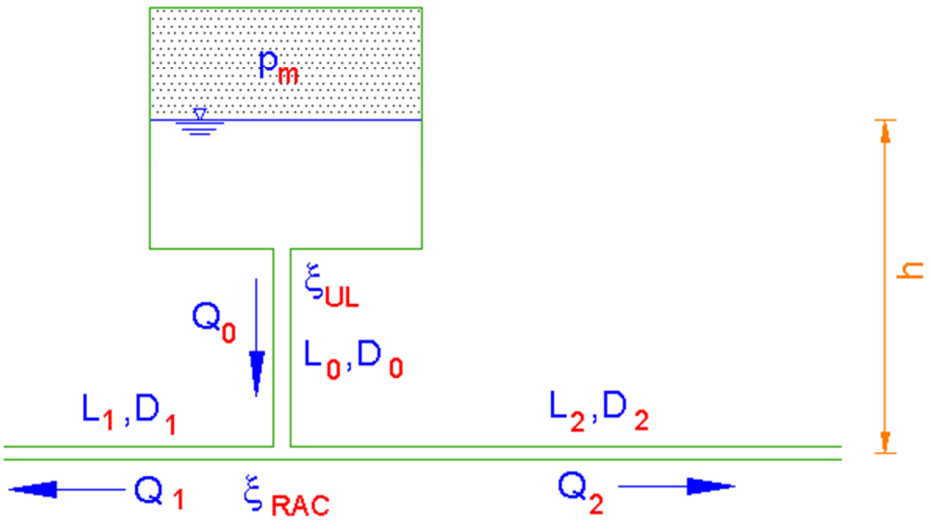
$$H_{pumpa} = \frac{N_{pumpa} \cdot \eta}{\rho g Q} = \frac{23,5 \cdot 0,7}{1 \cdot 9,81 \cdot 0,094} = 17,84\ m$$



Napomena: energetska i piezometarska linija su identične za obje cijevi. Razlika je u odnosima vrijednosti potencijalne energije položaja i tlaka u gornjoj i donjoj cijevi. Na crtežu se vidi da vrijednosti tlakova p_{T1} i p_{T2} imaju negativne vrijednosti (P.L. se nalazi ispod G.L.).

16. Za stacionarno strujanje u cjevovodu prema slici potrebno je odrediti protok Q_0, Q_2 i promjer cjevovoda D_2 . Nacrtati energetsku i piezometarsku liniju.

Zadano je: $\rho = 1000$ kg/m^3; $p_m = -39{,}24$ kPa; $Q_1 = 0{,}1$ m^3/s; $h = 8$ m;
$L_0 = 5$ m; $L_1 = 100$ m; $L_2 = 200$ m; $D_0 = D_1 = 300$ mm;
$\lambda = 0{,}02$; $\xi_{UL} = 0{,}5$; $\xi_{RAC} = f(v_0) = 0{,}5$



$$A_0 = \frac{D_0^2\pi}{4} = \frac{0{,}3^2\pi}{4} = 0{,}0707\ m^2$$

$$A_0 = A_1$$

$$v_1 = \frac{Q_1}{A_1} = \frac{0{,}1}{0{,}0707} = 1{,}41\ m/s \qquad \rightarrow \qquad \frac{v_1^2}{2g} = 0{,}1\ m$$

$$h + \frac{p_m}{\rho g} = \frac{v_0^2}{2g}\left(\xi_{UL} + \lambda\frac{L_0}{D_0} + \xi_{RAC}\right) + \frac{v_1^2}{2g}\left(\lambda\frac{L_1}{D_1} + 1\right)$$

$$8 + \frac{-39{,}24}{\rho g} = \frac{v_0^2}{2g}\left(0{,}5 + 0{,}02\cdot\frac{5}{0{,}3} + 0{,}5\right) + 0{,}1\left(0{,}02\cdot\frac{100}{0{,}3} + 1\right)$$

$$8 - 4 = \frac{v_0^2}{2g}\left(0{,}5 + 0{,}33 + 0{,}5\right) + 0{,}1\left(6{,}67 + 1\right)$$

$$\frac{v_0^2}{2g} = 2{,}43\ m \qquad \rightarrow \qquad v_0 = 6{,}9\ m/s \qquad \rightarrow \qquad Q_0 = v_0 \cdot A_0 = 0{,}488\ m^3/s$$

$$Q_2 = Q_0 - Q_1 = 0{,}488 - 0{,}1$$

$$Q_2 = 0{,}388\ m^3/s \qquad \rightarrow \qquad v_2 = \frac{Q_2}{A_2} = \frac{4\cdot 0{,}388}{D_2^2\pi} = \frac{0{,}494}{D_2^2}$$

$$\frac{v_2^2}{2g} = \left(\frac{0{,}494}{D_2^2}\right)^2 \cdot \frac{1}{2g} = \frac{0{,}012}{D_2^4}$$

$$h+\frac{p_m}{\rho g}=\frac{v_0^2}{2g}\left(\xi_{UL}+\lambda\frac{L_0}{D_0}+\xi_{RAC}\right)+\frac{v_2^2}{2g}\left(\lambda\frac{L_2}{D_2}+1\right)$$

$$8-4=2,43\left(0,5+0,33+0,5\right)+\frac{0,012}{D_2^4}\left(0,02\frac{200}{D_2}+1\right)$$

$$0,768=\frac{0,012}{D_2^4}\left(\frac{4}{D_2}+1\right)$$

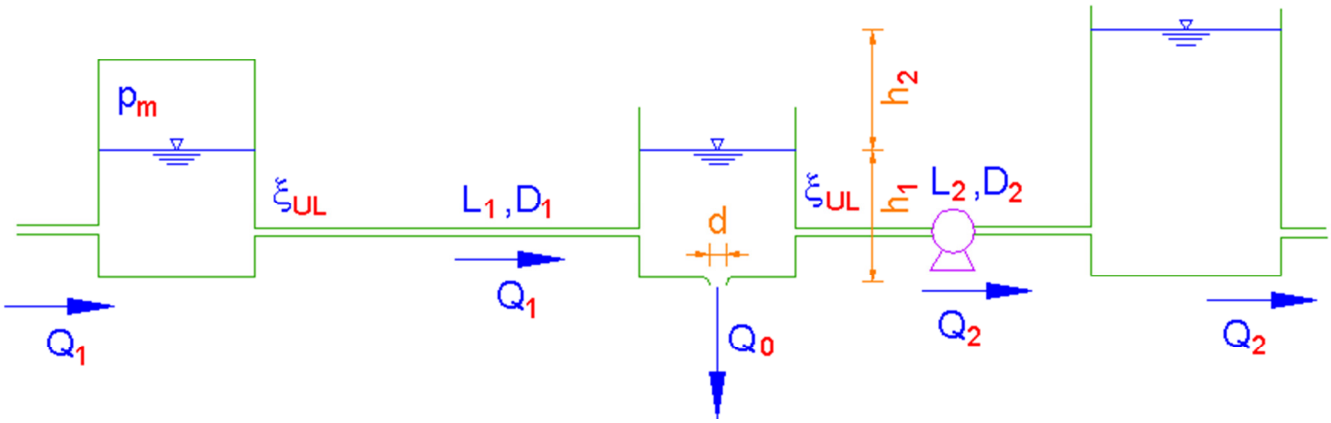
$$D_2^4=0,0156\left(\frac{4}{D_2}+1\right)\qquad\rightarrow\qquad D_2=\sqrt[4]{0,0156\left(\frac{4}{D_2}+1\right)}$$

D_2 pretp (desna strana)	D_2 dobiv (lijeva strana)
1	0,528
0,8	0,553
0,6	0,588
0,59	0,590

Odabrano: $D_2=590\,mm$

17. Za stacionarno strujanje u cjevovodu prema slici potrebno je odrediti protoke Q_1, Q_2 i snagu pumpe N_{pumpa} te nacrtati energetsku i piezometarsku liniju.

Zadano je: $h_1 = h_2 = 2$ m; $\rho = 1000$ kg/m^3; $d = 70$ mm; $D_1 = 300$ mm;
$D_2 = 200$ mm; $L_1 = 300$ m; $L_2 = 200$ m; $\varepsilon_1 = 0{,}3$ mm;
$\varepsilon_2 = 0{,}2$ mm; $\xi_{UL} = 0{,}5$; $\lambda_1 = f(\varepsilon_1/D_1)$; $\lambda_2 = f(\varepsilon_2/D_2)$;
$p_m = 49{,}05$ kPa; $\eta_{pumpa} = 0{,}7$



$$A_1 = \frac{D_1^2\pi}{4} = \frac{0{,}3^3\pi}{4} = 0{,}071\ \text{m}^2 \qquad A_2 = \frac{0{,}2^2\pi}{4} = 0{,}031\ \text{m}^2$$

$$\frac{\varepsilon_1}{D_1} = \frac{0{,}3}{300} = 0{,}001 \qquad \frac{\varepsilon_2}{D_2} = \frac{0{,}2}{200} = 0{,}001 \qquad \lambda_1 = \lambda_2 = 0{,}02$$

$$\frac{p_m}{\rho g} = \frac{v_1^2}{2g}\left(\xi_{UL} + \lambda_1\frac{L_1}{D_1} + 1\right)$$

$$5 = \frac{v_1^2}{2g}\left(0{,}5 + 0{,}02\cdot\frac{300}{0{,}3} + 1\right)$$

$$\frac{v_1^2}{2g} = 0{,}23\ m \quad \rightarrow \quad v_1 = 2{,}14\ m/s \quad \rightarrow \quad Q_1 = v_1\cdot A_1 = 2{,}14\cdot 0{,}071 = 0{,}152\ m^3/s$$

$$Q_0 = \sqrt{2\cdot g\cdot h_1}\cdot\frac{d^2\pi}{4} = \sqrt{2\cdot g\cdot 2}\cdot\frac{0{,}07^2\pi}{4} = 0{,}024\ \text{m}^3/\text{s}$$

$$Q_2 = Q_1 - Q_0 = 0{,}152 - 0{,}024 = 0{,}128\ \text{m}^3/\text{s}$$

$$v_2 = \frac{Q_2}{A_2} = \frac{0{,}128}{0{,}031} = 4{,}13\ m/s \quad \rightarrow \quad \frac{v_2^2}{2g} = 0{,}87\ m$$

$$H_{pumpa} = h_2 + \frac{v_2^2}{2g}\left(\xi_{UL} + \lambda_2 \frac{L_2}{D_2} + 1\right)$$

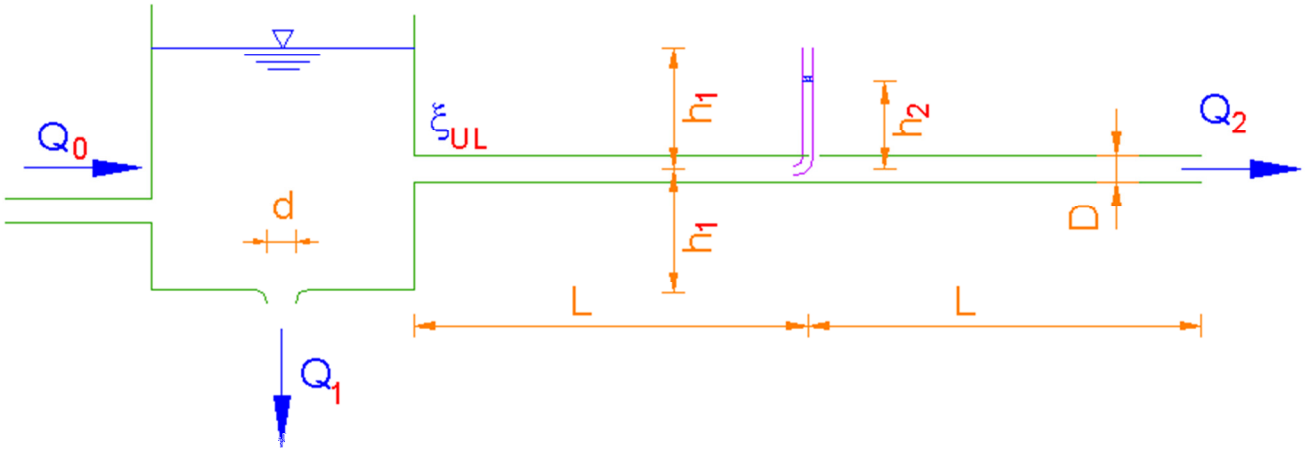
$$H_{pumpa} = 2 + 0,87\left(0,5 + 0,02 \cdot \frac{200}{0,2} + 1\right)$$

$$H_{pumpa} = 20,71\,m$$

$$N_{pumpa} = \frac{\rho g Q_2 H_{pumpa}}{\eta} = \frac{1 \cdot 9,81 \cdot 0,128 \cdot 20,71}{0,7} = 37,15\,kW$$

18. Za sistem prikazan na slici izračunati protoke Q_1 i Q_2 te promjer d pod uvjetom da je $Q_1 = 4Q_2$. Pri tome odrediti visinu vode h_2 u Pitot-ovoj cijevi postavljenoj u osi cjevovoda na polovini njezine dužine. Nacrtati E.L. i P.L.

Zadano je: $h_1 = 2$ m; $D = 120$ mm; C_C (koef. kontrakcije pri istjecanju) = 0,7; $\xi_{UL} = 0,5$; $L = 12$ m; $\lambda = 0,025$; $\rho = 1000$ kg/m^3



$$h_1 = \frac{v_2^2}{2g}\left(\xi_{UL} + \lambda\frac{2L}{D} + 1\right)$$

$$2 = \frac{v_2^2}{2g}\left(0,5 + 0,025\cdot\frac{2\cdot 12}{0,12} + 1\right)$$

$$\frac{v_2^2}{2g} = 0,308\,m \qquad \rightarrow \qquad v_2 = 2,46\,m/s$$

$$Q_2 = v_2 \cdot A_2 = v_2 \cdot \frac{D^2\pi}{4} = 2,46\cdot\frac{0,12^2\pi}{4} = 0,028\,m^3/s$$

$$Q_1 = 4\cdot Q_2 = 0,112\,m^3/s$$

$$v_1 = \sqrt{2\cdot g\cdot 2h_1} = \sqrt{2\cdot g\cdot 2\cdot 2} = 8,86\,m/s$$

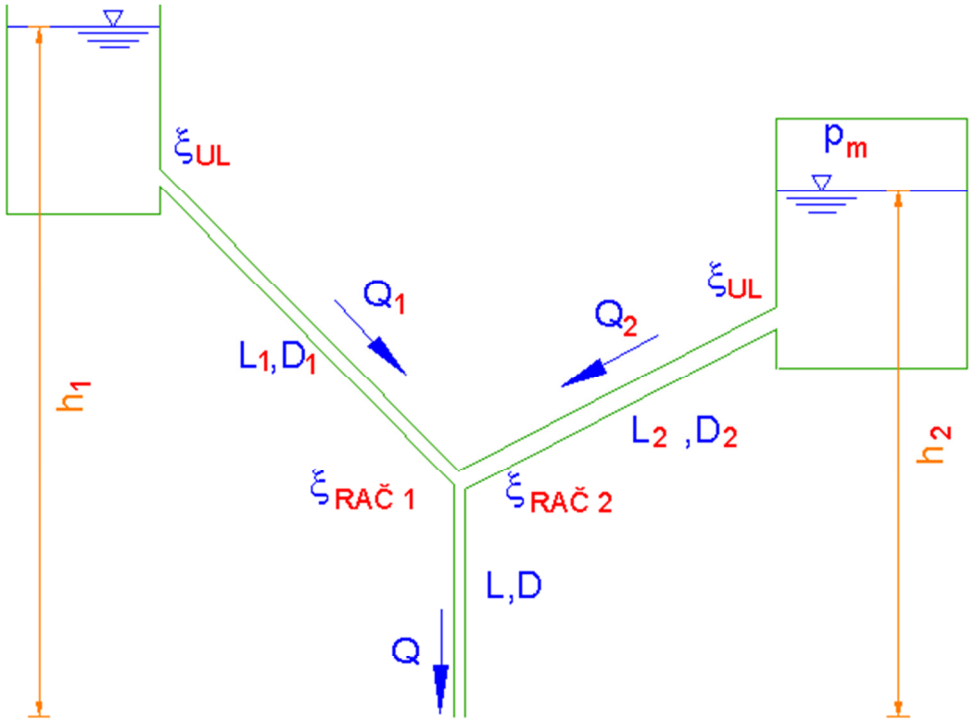
$$Q_1 = v_1 \cdot A_1 \cdot C_C = v_1 \cdot \frac{d^2\pi}{4}\cdot C_C \qquad \rightarrow \qquad d = \sqrt{\frac{Q_1\cdot 4}{v_1\cdot C_C\cdot\pi}} = \sqrt{\frac{0,112\cdot 4}{8,86\cdot 0,7\cdot\pi}} = 0.152\,m$$

$$h_2 = h_1 - \frac{v_2^2}{2g}\left(\xi_{UL} + \lambda\frac{L}{D}\right) = 2 - 0,308\left(0,5 + 0,025\cdot\frac{12}{0,12}\right) = 1,08\,m$$

19. Za sistem kao na slici potrebno je odrediti tlak p_m i protoke Q_1 i Q_2. Lokalni gubitci energije vezeni su za nizvodnu brzinu. Nacrtajte energetsku i piezometarsku liniju. Voda slobodno istječe u atmosferu.

Zadano je:

$\rho = 1000$ kg/m^3;	$h_1 = 8$ m;	$h_2 = 6$ m;
$Q = 50$ l/s;	$\lambda = \lambda_1 = 0{,}025$;	$\lambda_2 = 0{,}03$;
$D = 100$ mm;	$D_1 = 100$ mm;	$D_2 = 80$ mm;
$\xi_{UL} = 0{,}5$;	$\xi_{RAČ\ 1} = 0{,}7$;	$\xi_{RAČ\ 2} = 0{,}8$;
$L = 3$ m;	$L_1 = 10$ m;	$L_2 = 6$ m



$$Q = v \cdot A \quad \rightarrow \quad v = \frac{4Q}{D^2\pi} = \frac{4 \cdot 0{,}05}{0{,}1^2\pi} = 6{,}37\ m/s \quad \rightarrow \quad \frac{v^2}{2g} = 2{,}07\ m$$

$$Q_1 + Q_2 = Q$$

$$h_1 = \frac{v_1^2}{2g}\left(\xi_{UL} + \lambda_1 \frac{L_1}{D_1}\right) + \frac{v^2}{2g}\left(\xi_{RAČ1} + \lambda \frac{L}{D} + 1\right)$$

$$8 = \frac{v_1^2}{2g}\left(0{,}5 + 0{,}025 \cdot \frac{10}{0{,}1}\right) + 2{,}07\left(0{,}7 + 0{,}025 \cdot \frac{3}{0{,}1} + 1\right)$$

$$\frac{v_1^2}{2g} = 0{,}976\ m \quad \rightarrow \quad v_1 = 4{,}38\ m/s \quad \rightarrow \quad Q_1 = v_1 \cdot \frac{D_1^2\pi}{4} = 0{,}034\ m^3/s$$

$$Q_2 = Q - Q_1 = 0{,}05 - 0{,}034 = 0{,}016\ m^3/s$$

$$v_2 = \frac{4Q_2}{D_2^2\pi} = \frac{4 \cdot 0{,}016}{0{,}08^2\pi} = 3{,}18\ m/s \quad \rightarrow \quad \frac{v_2^2}{2g} = 0{,}52\ m$$

$$h_2 + \frac{p_m}{\rho g} = \frac{v_2^{\,2}}{2g}(\xi_{UL} + \lambda_2 \frac{L_2}{D_2}) + \frac{v^2}{2g}(\xi_{RAČ\,2} + \lambda \frac{L}{D} + 1)$$

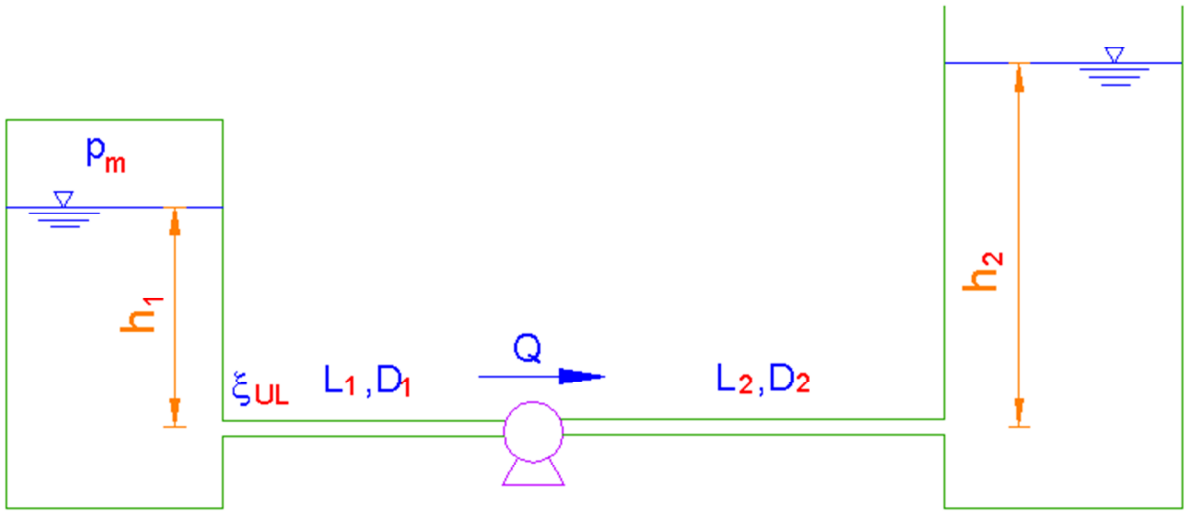
$$6 + \frac{p_m}{\rho g} = 0,52\left(0,5 + 0,03 \cdot \frac{6}{0,08}\right) + 2,07\left(0,8 + 0,025 \cdot \frac{3}{0,1} + 1\right)$$

$$\frac{p_m}{\rho g} = 0,71\,m \qquad \rightarrow \qquad p_m = 6,95\,kPa$$

20. Odredi brzine v_1 i v_2 koje se formiraju za slučaj strujanja kroz cjevovod kao na slici. Potrebno je nacrtati E.L i P.L.

Zadano je:

$D_1 = 0{,}1$ m;	$D_2 = 0{,}2$ m;	$p_m = 19{,}62$ kPa;
$h_1 = 10$ m;	$h_2 = 12$ m;	$H_{pumpa} = 2$ m;
$L_1 = 5$ m;	$L_2 = 7$ m;	$\rho = 1000$ kg/m^3;
$\lambda_1 = 0{,}015$;	$\lambda_2 = 0{,}02$;	$\xi_{UL} = 0{,}5$



$$v_1 \cdot A_1 = v_2 \cdot A_2$$

$$v_2 = v_1 \cdot \frac{A_1}{A_2} = v_1 \cdot \frac{D_1^2}{D_2^2} = v_1 \cdot \frac{0{,}1^2}{0{,}2^2} = 0{,}25 v_1$$

$$h_1 + \frac{p_m}{\rho g} + H_{pumpa} = h_2 + \frac{v_1^2}{2g}\left(\xi_{UL} + \lambda_1 \frac{L_1}{D_1}\right) + \frac{v_2^2}{2g}\left(\lambda_2 \frac{L_2}{D_2} + 1\right)$$

$$10 + 2 + 2 = 12 + \frac{v_1^2}{2g}\left(0{,}5 + 0{,}015 \cdot \frac{5}{0{,}1}\right) + \frac{0{,}0625 v_1^2}{2g}\left(0{,}02 \cdot \frac{7}{0{,}2} + 1\right)$$

$$2 = \frac{v_1^2}{2g}(0{,}5 + 0{,}75) + \frac{0{,}0625 v_1^2}{2g}(0{,}7 + 1)$$

$$2 = \frac{v_1^2}{2g}(0{,}5 + 0{,}75) + \frac{0{,}0625 v_1^2}{2g}(0{,}7 + 1)$$

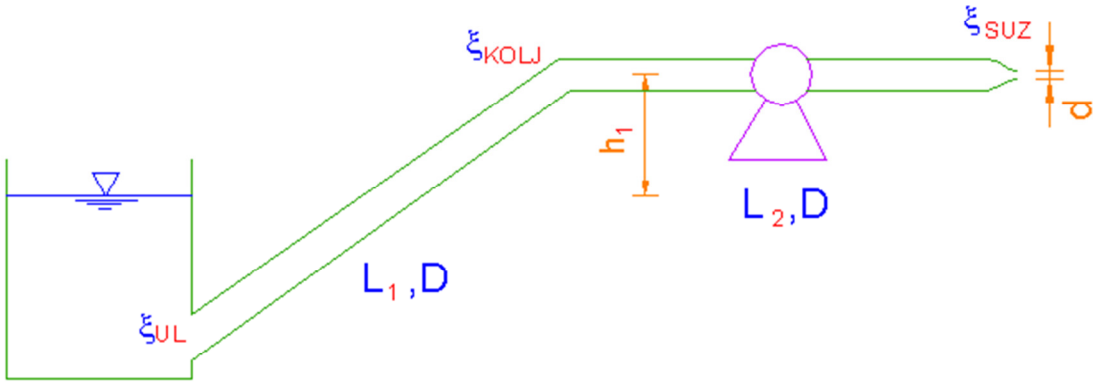
$$\frac{v_1^2}{2g} = 1{,}47\ m \qquad \rightarrow \qquad v_1 = 5{,}38\ m/s$$

$$v_2 = 0{,}25 \cdot 5{,}38 = 1{,}34\ m/s \qquad \rightarrow \qquad \frac{v_2^2}{2g} = 0{,}09\ m$$

21. Odredi potrebnu snagu pumpe N_{pumpa} koja će omogućiti istjecanje kroz sapnicu v_d = 20 m/s. Potrebno je nacrtati E.L. i P.L.

Zadano je:

D = 80 mm;	d = 30 mm;	ε = 0,0003 m;
L_1 = 20 m;	L_2 = 10 m;	$\nu = 1\cdot10^{-6}$ m^2/s;
ξ_{UL} = 0,5;	ξ_{KOLJ} = 0,2;	$\xi_{SAP} = f(v_D)$ = 0,3;
η = 0,7;	h_1 = 3 m;	ρ = 1000 kg/m^3



$$Q = v_d \cdot \frac{d^2\pi}{4} = 20 \cdot \frac{0,03^2\pi}{4} = 0,014\ m^3/s$$

$$v_D = \frac{4Q}{D^2\pi} = \frac{4\cdot0,014}{0,08^2\pi} = 2,81\ m/s$$

$$\left.\begin{aligned} \frac{\varepsilon}{D} &= \frac{0,3}{80} = 0,0038 \\ \mathrm{Re}_D &= \frac{v_D \cdot D}{\upsilon} = \frac{2,81\cdot0,08}{1\cdot10^{-6}} = 2,2\cdot10^5 \end{aligned}\right\} \lambda = 0,029$$

$$H_{pumpa} = h_1 + \frac{v_d^2}{2g} + \frac{v_D^2}{2g}\left(\xi_{ul} + \lambda\frac{L_1+L_2}{D} + \xi_{kolj} + \xi_{sap}\right)$$

$$H_{pumpa} = 3 + \frac{20^2}{2g} + \frac{2,81^2}{2g}\left(0,5 + 0,029\frac{20+10}{0,08} + 0,2 + 0,3\right)$$

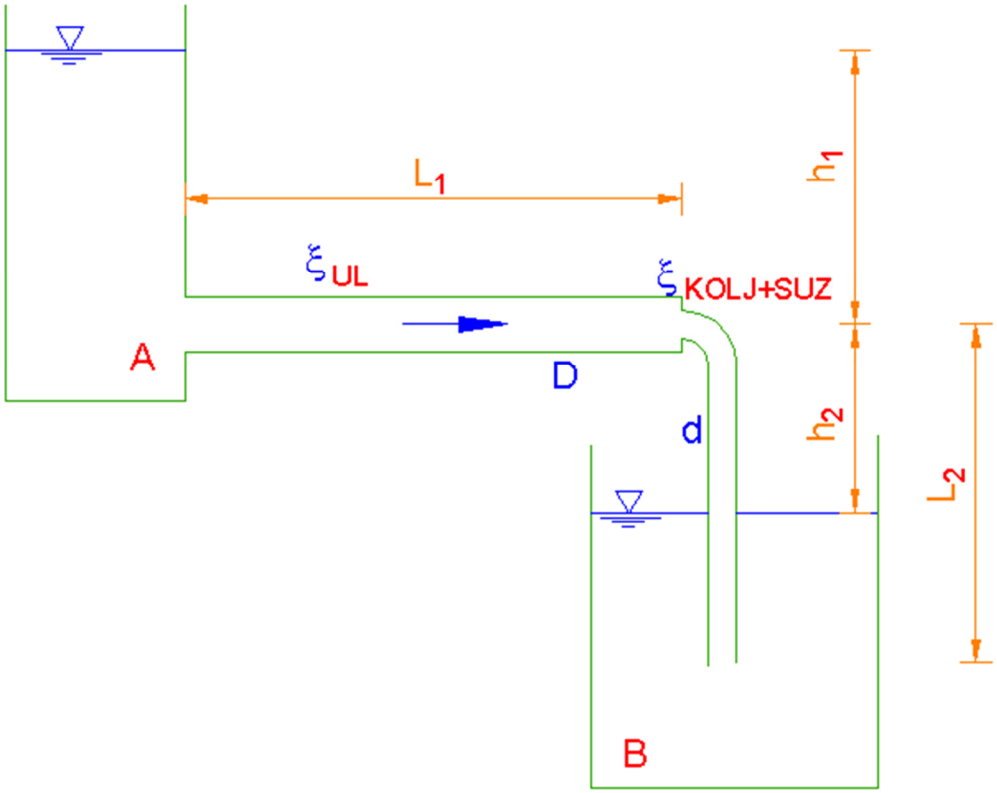
$$H_{pumpa} = 3 + 20,39 + 0,43(0,5 + 10,88 + 0,2 + 0,3)$$

$$H_{pumpa} = 28,5\ m$$

$$N_{pumpa} = \frac{\rho g H_{pumpa}\cdot Q}{\eta} = \frac{1\cdot9,81\cdot28,5\cdot0,014}{0,7} = 5,19\ kW$$

22. Protok vode iz spremnika *A* u spremnik *B* iznosi Q = 170 l/s. Odredi promjer horizontalnog dijela cjevovoda *D*. Treba nacrtati energetsku i piezometarsku liniju za zadani sistem.

Zadano je: L_1 = 65 m; L_2 = 35 m; h_1= 35 m; h_2 = 19 m;
d = 150 mm; $\nu = 1\cdot10^{-6}$ m^2/s; ε = 0,046 mm; ξ_{UL} = 0,4;
$\xi_{KOLJ+SUŽ} = f(v_d)$ = 0,9; ρ = 1000 kg/m^3



$$Q = v_d \cdot A_d = v_D \cdot A_D \qquad v_d = \frac{4Q}{d^2\pi} = \frac{4\cdot 0{,}17}{0{,}15^2\pi} = 9{,}62\ m/s \qquad \frac{v_d^2}{2g} = 4{,}72\ m$$

$$\frac{\varepsilon}{d} = \frac{0{,}046}{150} = 0{,}0003 \qquad \mathrm{Re}_d = \frac{v_d \cdot d}{\upsilon} = \frac{9{,}62\cdot 0{,}15}{1\cdot10^{-6}} = 1{,}4\cdot10^6 \qquad \lambda_d = 0{,}016$$

$$h_1 + h_2 = \frac{v_D^2}{2g}\left(\xi_{UL} + \lambda_D \frac{L_1}{D}\right) + \frac{v_d^2}{2g}\left(\xi_{KOLJ+SUZ} + \lambda_d \frac{L_2}{d} + 1\right)$$

$$35 + 19 = \frac{v_D^2}{2g}\left(0{,}4 + \lambda_D \frac{65}{D}\right) + 4{,}72\left(0{,}9 + 3{,}73 + 1\right)$$

$$27{,}41 = \frac{v_D^2}{19{,}62}\left(0{,}4 + \lambda_D \frac{65}{D}\right)$$

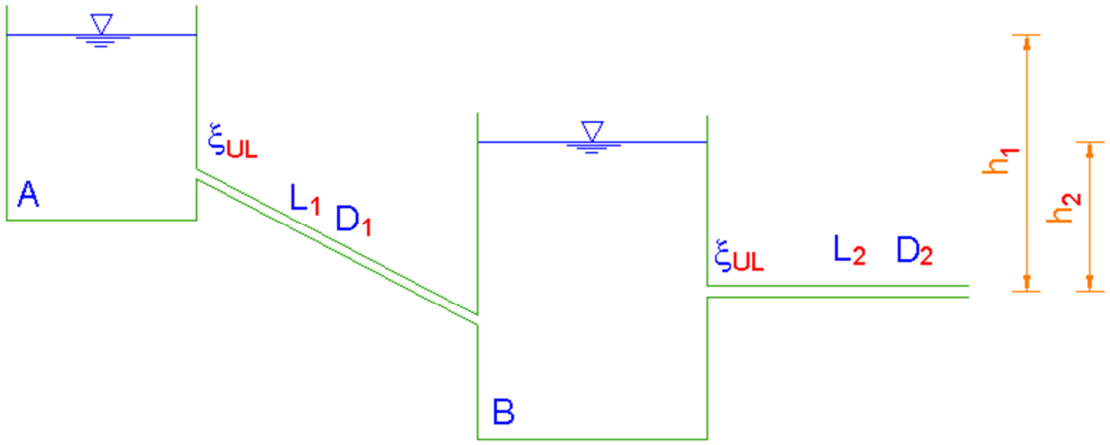
iteracijski postupak :

D	$v_D = \frac{4Q}{D^2\pi}$	$\mathrm{Re}_D = \frac{v_D \cdot D}{\upsilon}$	$\frac{\varepsilon}{D}$	λ_D		
0,5	0,866	$4{,}3\cdot10^5$	0,00009	0,0145	27,41 ≠	0,09
0,2	5,411	$1{,}1\cdot10^6$	0,00023	0,0150	27,41 ≠	7,87
0,157	8,781	$1{,}4\cdot10^6$	0,00029	0,0155	27,41 ≈	26,79

Odabrano: $D = 157\ mm$

23. Odredite razine vode u rezervoarima h_1 i h_2 te nacrtajte energetsku i piezometarsku liniju za stacionarno strujanje.

Zadano je : $Q = 0{,}15$ m^3/s; $\xi_{UL} = 0{,}5$; $L_1 = 20$ m; $L_2 = 10$ m;
$D_1 = 200$ mm; $D_2 = 150$ mm; $\lambda_1 = 0{,}03$; $\lambda_2 = 0{,}034$;
$\rho = 1000$ kg/m^3



$$Q = v_1 A_1 = v_2 A_2$$

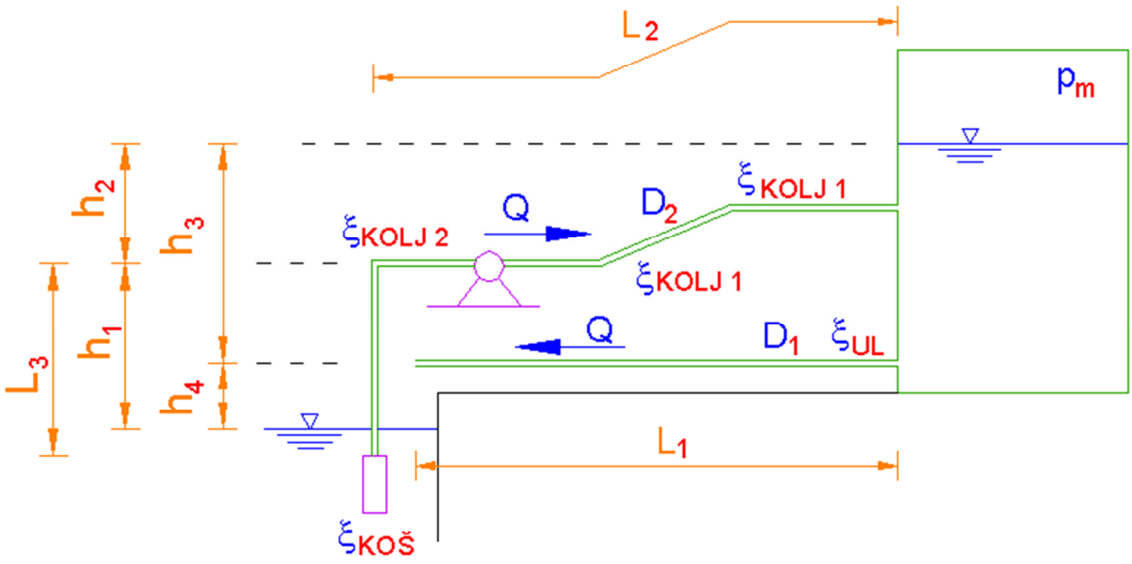
$$v_1 = \frac{Q}{A_1} = \frac{4 \cdot Q}{D_1^2 \cdot \pi} = \frac{4 \cdot 0{,}15}{0{,}2^2 \cdot \pi} = 4{,}77\ m/s \quad \rightarrow \quad \frac{v_1^2}{2g} = 1{,}16\ m$$

$$v_2 = \frac{Q}{A_2} = \frac{4 \cdot Q}{D_2^2 \cdot \pi} = \frac{4 \cdot 0{,}15}{0{,}15^2 \cdot \pi} = 8{,}49\ m/s \quad \rightarrow \quad \frac{v_2^2}{2g} = 3{,}67\ m$$

$$h_2 = \frac{v_2^2}{2g}(\xi_{UL} + \lambda_2 \frac{L_2}{D_2} + 1) = 3{,}67(0{,}5 + 0{,}034 \cdot \frac{10}{0{,}15} + 1) = 13{,}82\ m$$

$$h_1 = h_2 + \frac{v_1^2}{2g}\left(\xi_{UL} + \lambda_1 \frac{L_1}{D_1} + 1\right) = 13{,}82 + 1{,}16\left(0{,}5 + 0{,}03 \cdot \frac{20}{0{,}2} + 1\right) = 19{,}04\ m$$

24. Zadan je sistem s visinama h_1 = 3,5 m, h_2 = 2,5 m, h_3 = 5 m i h_4 = 1 m te horizontalnim dužinama L_1 = 600 m, L_2 = 620 m i L_3 = 4 m. Promjeri cjevovoda su D_1 = 200 mm i D_2 = 300 mm, a koeficijenti lokalnih gubitaka iznose $\xi_{KOŠ}$ = 0,1, $\xi_{KOLJ\,1}$ = 0,15 , $\xi_{KOLJ\,2}$ = 0,12 i ξ_{UL} = 0,5. Koeficijent iskoristivosti pumpe je η = 0,75. Potrebno je izračunati protok u sistemu ako je problem stacionaran te snagu pumpe koja će omogućiti tu stacionarnost. Tlak u komori je p_m = 9,81 kPa, a koeficijent trenja je jednak za sve dijelove sistema i iznosi λ = 0,012. Nacrtati piezometarsku i energetsku liniju za dovodni i odvodni cjevovod. Gustoća vode iznosi ρ = 1000 kg/m^3.



$$h_3+\frac{p_m}{\rho g}=\frac{v_1^2}{2g}\left(\xi_{ul}+\lambda\frac{L_1}{D_1}+1\right)$$

$$5+1=\frac{v_1^2}{2g}(0,5+36+1)$$

$$\frac{v_1^2}{2g}=0,16\,m \qquad \rightarrow \qquad v_1=1,77\,m/s$$

$$Q=v_1\cdot\frac{D_1^2\pi}{4}=1,77\cdot\frac{0,2^2\pi}{4}=0,056\,m^3/s$$

$$v_2=\frac{4Q}{D_2^2\pi}=\frac{4\cdot0,056}{0,3^2\pi}=0,79\,m/s \qquad \rightarrow \qquad \frac{v_2^2}{2g}=0,03\,m$$

$$H_{pumpa}=h_1+h_2+\frac{p_m}{\rho g}+\frac{v_2^2}{2g}\left(\xi_{KOŠ}+\lambda\frac{L_3}{D_2}+\xi_{KOLJ\,2}+2\xi_{KOLJ\,1}+\lambda\frac{L_2}{D_2}+1\right)$$

$$H_{pumpa}=3,5+2,5+1+0,03\left(0,1+0,012\cdot\frac{4}{0,3}+0,12+2\cdot0,15+0,012\cdot\frac{620}{0,3}+1\right)$$

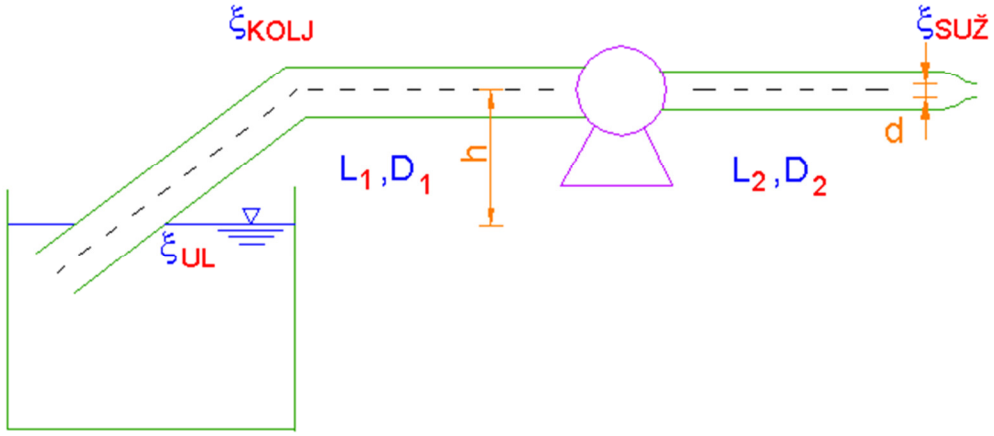
$$H_{pumpa}=7,79\,m$$

$$N_{pumpa}=\frac{\rho g Q H_{pumpa}}{\eta}=\frac{1\cdot9,81\cdot0,056\cdot7,79}{0,75}=5,71\,kW$$

25. Za sistem na slici treba proračunati snagu pumpe i nacrtati energetsku i piezometarsku liniju. Zbog kvalitetnog oblikovanja sapnice ne dolazi do kontrakcije mlaza na izlazu, odnosno koeficijent kontrakcije mlaza ima vrijednost $C_C = 1$.

Zadano je:

$v_d = 30$ m/s;	$\upsilon = 1\cdot10^{-6}$ m^2/s;	$L_1 = 10$ m;	$L_2 = 5$ m;
$D_1 = 100$ mm;	$D_2 = 60$ mm;	$d = 25$ mm;	$h = 2$ m;
$\xi_{UL} = 0{,}7$;	$\xi_{KOLJ} = 0{,}15$;	$\xi_{SUŽ} = f(v_2)$=0,2;	$\varepsilon = 0{,}2$ mm;
$\eta = 0{,}8$;	$\rho = 1000$ kg/m^3		



$$Q = v_d \cdot A_d = v_d \cdot \frac{d^2\pi}{4} = 30 \cdot \frac{0{,}025^2\pi}{4} = 0{,}0147\ m^3/s$$

$$v_1 = \frac{Q}{A_1} = \frac{4\cdot Q}{D_1^2\cdot\pi} = \frac{4\cdot 0{,}0147}{0{,}1^2\cdot\pi} = 1{,}875\ m/s \quad \rightarrow \quad \frac{v_1^2}{2g} = 0{,}18\ m$$

$$v_2 = \frac{Q}{A_2} = \frac{4\cdot Q}{D_2^2\cdot\pi} = \frac{4\cdot 0{,}0147}{0{,}06^2\cdot\pi} = 5{,}199\ m/s \quad \rightarrow \quad \frac{v_2^2}{2g} = 1{,}38\ m$$

$$\left.\begin{aligned} &\frac{\varepsilon}{D_1} = \frac{0{,}2}{100} = 0{,}002 \\ &\mathrm{Re}_1 = \frac{v_1 D_1}{\upsilon} = \frac{1{,}875\cdot 0{,}1}{1\cdot10^{-6}} = 1{,}9\cdot10^5 \end{aligned}\right\} \lambda_1 = 0{,}025$$

$$\left.\begin{aligned} &\frac{\varepsilon}{D_2} = \frac{0{,}2}{60} = 0{,}0033 \\ &\mathrm{Re}_2 = \frac{v_2 D_2}{\upsilon} = \frac{5{,}199\cdot 0{,}06}{1\cdot10^{-6}} = 3{,}1\cdot10^5 \end{aligned}\right\} \lambda_2 = 0{,}028$$

$$H_{pumpa} = h + \frac{v_d^2}{2g} + \frac{v_1^2}{2g}\left(\xi_{UL} + \xi_{KOLJ} + \lambda_1\frac{L_1}{D_1}\right) + \frac{v_2^2}{2g}\left(\xi_{SUŽ} + \lambda_2\frac{L_2}{D_2}\right)$$

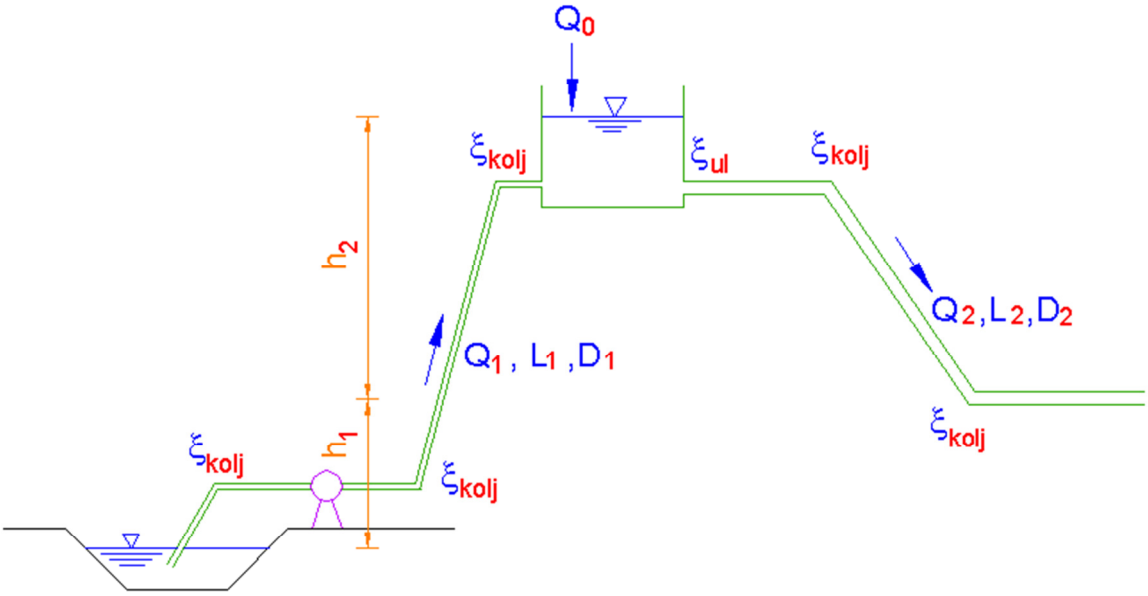
$$H_{pumpa} = 2 + 45{,}87 + 0{,}18\left(0{,}7 + 0{,}15 + 2{,}5\right) + 1{,}38\left(0{,}2 + 2{,}33\right)$$

$$H_{pumpa} = 51{,}96\ m$$

$$N_{pumpa} = \frac{\rho g H_{pumpa} Q}{\eta} = \frac{1\cdot 9{,}81\cdot 51{,}96\cdot 0{,}0147}{0{,}8} = 9{,}37\ \ kW$$

26. Za cjevovod prema slici i za stacionarne uvjete strujanja i razina potrebno je odrediti protoke Q_0, Q_1, Q_2 i koeficijent iskoristivosti pumpe η_p. Potrebno je nacrtati energetsku i piezometarsku liniju.

Zadano je: $\rho = 1000$ kg/m^3; $D_1 = 150$ mm; $D_2 = 200$ mm; $\varepsilon = 0{,}5$ mm;
$L_1 = 100$ m; $L_2 = 200$ m; $h_1 = 5$ m; $h_2 = 10$ m;
$\lambda = f(\varepsilon/D)$; $\xi_{UL} = 0{,}5$; $\xi_{KOLJ} = 0{,}3$; $H_{pumpe} = 20$ m;
$N_{pumpe} = 10$ kW



$$\frac{\varepsilon}{D_2} = \frac{0{,}5}{200} = 0{,}0025 \quad \rightarrow \quad \lambda_2 = 0{,}025$$

$$h_2 = \frac{v_2^2}{2g}\left(\xi_{UL} + 2\cdot\xi_{KOLJ} + \lambda_2\frac{L_2}{D_2} + 1\right)$$

$$10 = \frac{v_2^2}{2g}\left(0{,}5 + 2\cdot 0{,}3 + 25 + 1\right)$$

$$\frac{v_2^2}{2g} = 0{,}37\ m \quad \rightarrow \quad v_2 = 2{,}69\ m/s \quad \rightarrow \quad Q_2 = v_2\cdot\frac{D_2^2\pi}{4} = 0{,}0845\ m^3/s$$

$$\frac{\varepsilon}{D_1} = \frac{0{,}5}{150} = 0{,}0033 \quad \rightarrow \quad \lambda_2 = 0{,}027$$

$$H_{pumpa} = h_1 + h_2 + \frac{v_1^2}{2g}\left(3\cdot\xi_{KOLJ} + \lambda_1\frac{L_1}{D_1} + 1\right)$$

$$20 = 5 + 10 + \frac{v_1^2}{2g}\left(3\cdot 0{,}3 + 18 + 1\right)$$

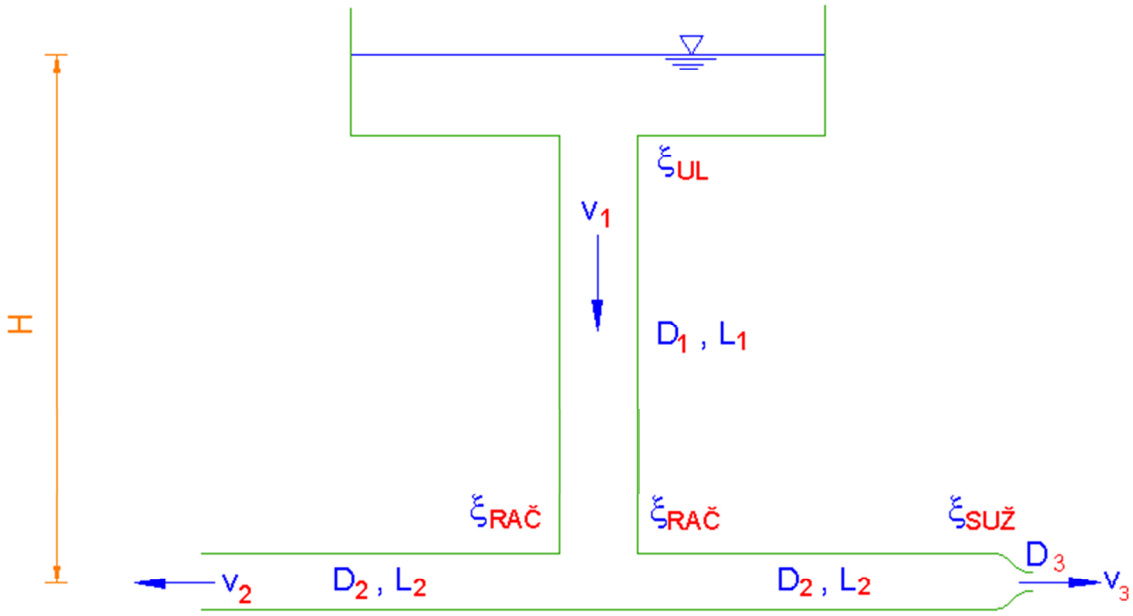
$$\frac{v_1^2}{2g} = 0{,}25\ m \quad \rightarrow \quad v_1 = 2{,}22\ m/s \quad \rightarrow \quad Q_1 = v_1\cdot\frac{D_1^2\pi}{4} = 0{,}0392\ m^3/s$$

$$Q_o = Q_2 - Q_1 = 0,0845 - 0,0392 = 0,0453 \text{ m}^3/\text{s}$$

$$N_{pumpa} = \frac{\rho \cdot g \cdot H_{pumpa} \cdot Q_1}{\eta} \qquad \rightarrow \qquad \eta = \frac{\rho \cdot g \cdot H_{pumpa} \cdot Q_1}{N_{pumpa}} = \frac{1 \cdot 9,81 \cdot 20 \cdot 0,0392}{10} = 0,77$$

27. Odredi međusobni odnos protoka Q_2 i Q_3 za cjevovod sa slike i nacrtaj energetsku i piezometarsku liniju.

Zadano je: $\rho = 1000$ kg/m^3; $L_1 = L_2 = 100$ m; $\lambda = 0{,}02$;
$D_1 = 0{,}2$ m; $D_2 = 0{,}3$ m; $D_3 = 0{,}1$ m;
$\xi_{UL} = 0{,}5$; $\xi_{RAČ} = f(v_{nizvodno}) = 0{,}3$; $\xi_{SUŽ} = f(v_{nizvodno}) = 0{,}6$



$$Q_1 = Q_2 + Q_3$$

$$\frac{D_1^2 \cdot \pi}{4} \cdot v_1 = \frac{D_2^2 \cdot \pi}{4} \cdot v_2 + \frac{D_3^2 \cdot \pi}{4} \cdot v_3$$

B.J. $1-2$

$$H = \frac{v_1^2}{2g}\left(\xi_{UL} + \lambda \cdot \frac{L_1}{D_1}\right) + \frac{v_2^2}{2g}\left(\xi_{RAČ} + \lambda \cdot \frac{L_2}{D_2} + 1\right)$$

B.J. $1-3$

$$H = \frac{v_1^2}{2g}\left(\xi_{UL} + \lambda \cdot \frac{L_1}{D_1}\right) + \frac{(v_2')^2}{2g}\left(\xi_{RAČ} + \lambda \cdot \frac{L_2}{D_2}\right) + \frac{v_3^2}{2g}\left(\xi_{SUŽ} + 1\right)$$

Napomena : protoci u lijevoj i desnoj horizontalnoj cijevi nisu isti.
Iz toga slijedi da ni brzine u tim cijevima nisu iste.

$$Q_2' = Q_3$$

$$v_2' \cdot \frac{D_2^2 \pi}{4} = v_3 \cdot \frac{D_3^2 \pi}{4} \qquad \rightarrow \qquad v_2' = v_3 \cdot \frac{D_3^2}{D_2^2} = 0,111 v_3$$

Izjednačavanjem dviju B.J. se dobije izraz:

$$v_2^2 \left(\xi_{RAČ} + \lambda \cdot \frac{L_2}{D_2} + 1 \right) = \left(0,111 v_3 \right)^2 \left(\xi_{RAČ} + \lambda \cdot \frac{L_2}{D_2} \right) + v_3^2 \left(\xi_{SUŽ} + 1 \right)$$

$$v_2^2 \left(0,3 + 6,67 + 1 \right) = 0,0123 v_3^2 \left(0,3 + 6,67 \right) + v_3^2 \left(0,6 + 1 \right)$$

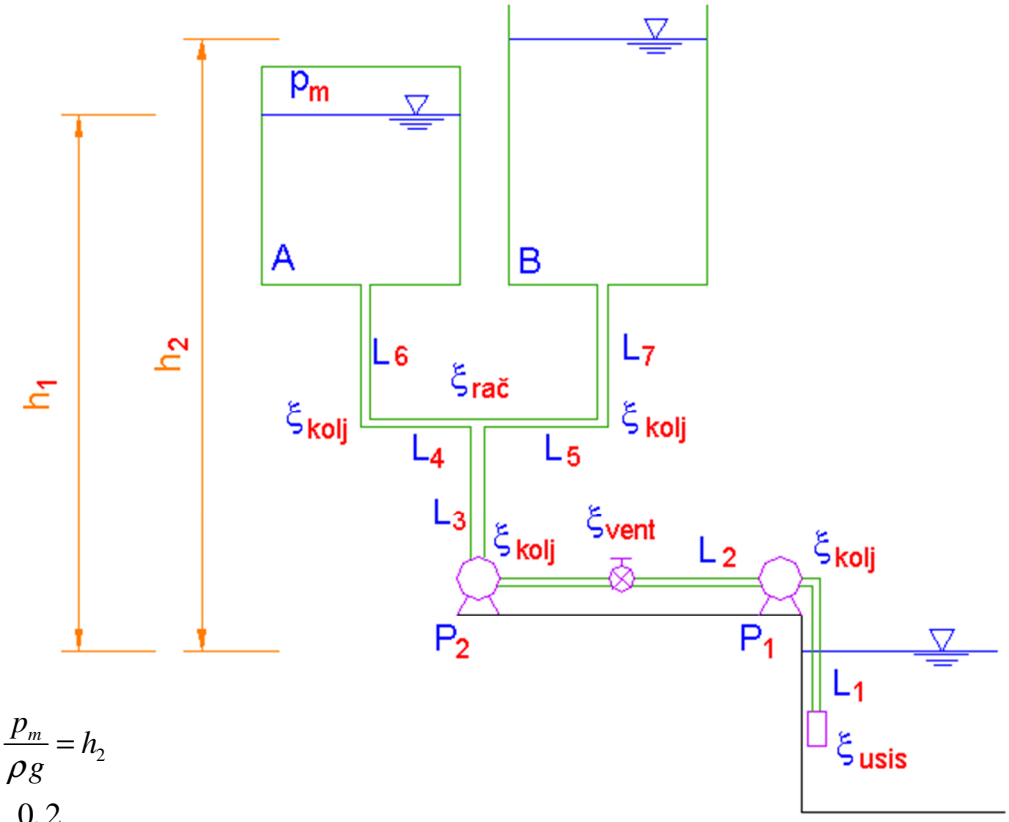
$$7,97 v_2^2 = 1,686 v_3^2 \qquad \rightarrow \qquad v_2 = 0,46\, v_3$$

$$\frac{Q_2}{Q_3} = \frac{v_2 \cdot D_2^2}{v_3 \cdot D_3^2} = \frac{0,46 v_3 \cdot 0,3^2}{v_3 \cdot 0,1^2} = 4,14$$

$$Q_2 = 4,14 \cdot Q_3$$

28. Za cjevovod prema slici potrebno je odrediti protok Q_{V0} u cijevi L_2 ukoliko je ventil potpuno otvoren (ξ_{VENT} = 0), i protok Q_{V15} ukoliko je ventil djelomično zatvoren i predstavlja koeficijent gubitka od ξ_{VENT} = 15. Potrebno je nacrtati energetsku i piezometarsku liniju za slučaj sa otvorenim zatvaračem. Tečenje se odvija turbulentno hrapavim režimom.

Zadano je: $\rho = 1000$ kg/m^3; $\varepsilon = 0{,}2$ mm; $D_{i\ (sve\ cijevi)} = 200$ mm;
$H_{pumpa1} = 15$ m; $H_{pumpa2} = 10$ m; $p_m = 19{,}62$ kPa; $h_1 = 12$ m;
$h_2 = 14$ m; $L_2\,(P_1\text{-}P_2) = 1000$ m; $L_1 = L_3 = L_4 = L_5 = L_6 = L_7 = 5$m;
$\xi_{KOLJ} = 0{,}2$; $\xi_{RAČ} = 0{,}3 = f(v_{4,5\text{-}nizvodna\ brzina})$; $\xi_{USIS} = 0$



$$h_1 + \frac{p_m}{\rho g} = h_2$$

$$\frac{\varepsilon}{D} = \frac{0{,}2}{200} = 0{,}001 \quad \rightarrow \quad \lambda = 0{,}02$$

Nakon račvanja protok se dijeli na dva jednaka dijela zbog istih karakteristika cijevi i rubnih uvjeta nizvodno od račve. Budući da se promjeri cijevi isti, brzina nakon račvanja se također prepolovljava.

$$H_{P1} + H_{P2} = h_2 + \frac{v^2}{2g}\left(2\xi_{KOLJ} + \lambda \cdot \frac{L_1 + L_2 + L_3}{D}\right) + \frac{(v/2)^2}{2g}\left(\xi_{RAČ} + \xi_{KOLJ} + \lambda \cdot \frac{L_5 + L_7}{D} + 1\right)$$

$$15 + 10 = 14 + \frac{v^2}{2g}\left(2 \cdot 0{,}2 + 0{,}02 \cdot \frac{5 + 1000 + 5}{0{,}2}\right) + \frac{1}{4} \cdot \frac{v^2}{2g}\left(0{,}3 + 0{,}2 + 0{,}02 \cdot \frac{5+5}{0{,}2} + 1\right)$$

$$\frac{v^2}{2g} = 0{,}108\,m \quad \rightarrow \quad v = 1{,}45\,m/s$$

$$Q = v \cdot \frac{D^2\pi}{4} = 0{,}046\,m^3/s \quad (u\ cijevima\ L_1, L_2\ i\ L_3)$$

$$Q/2 = 0{,}023\,m^3/s \quad (u\ cijevima\ L_4, L_5, L_6\ i\ L_7)$$

$$H_{P1} + H_{P2} = h_2 + \frac{v^2}{2g}\left(2\xi_{KOLJ} + \xi_{VENT} + \lambda \cdot \frac{L_1 + L_2 + L_3}{D}\right) + \frac{(v/2)^2}{2g}\left(\xi_{RAČ} + \xi_{KOLJ} + \lambda \cdot \frac{L_5 + L_7}{D} + 1\right)$$

$$15 + 10 = 14 + \frac{v^2}{2g}\left(2 \cdot 0,2 + 15 + 0,02 \cdot \frac{5 + 1000 + 5}{0,2}\right) + \frac{1}{4} \cdot \frac{v^2}{2g}\left(0,3 + 0,2 + 0,02 \cdot \frac{5+5}{0,2} + 1\right)$$

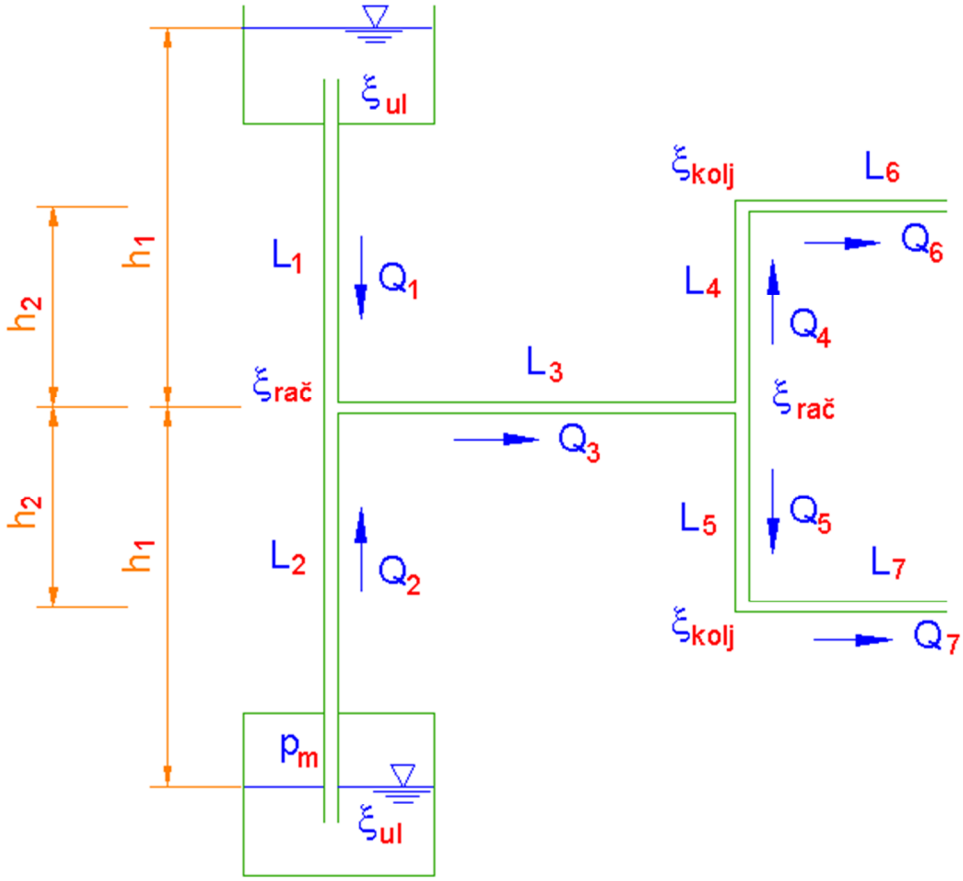
$$\frac{v^2}{2g} = 0,094\, m \quad \rightarrow \quad v = 1,36\, m/s$$

$$Q = v \cdot \frac{D^2\pi}{4} = 0,043\, m^3/s \quad \left(u\ cijevima\ L_1, L_2\ i\ L_3\right)$$

$$Q/2 = 0,021\, m^3/s \quad \left(u\ cijevima\ L_4, L_5, L_6\ i\ L_7\right)$$

29. Za cjevovod prema slici potrebno je postaviti jednadžbe za računanje protoka kroz cjevovod. Potrebno je kvalitativno nacrtati i energetsku i piezometarsku liniju. Tečenje se odvija turbulentno hrapavim režimom.

Zadano je: $D_{i\ (sve\ cijevi)} = 200$ mm; $\varepsilon = 0{,}2$ mm; $\rho = 1000$ kg/m^3;
$\xi_{KOLJ} = 0{,}4$; $\xi_{RAČ} = 0{,}3 = f(v_{nizvodna\ brzina})$; $\xi_{UL} = 0{,}5$;
$L_1 = L_2 = L_3 = 10$ m; $L_4 = L_5 = L_6 = L_7 = 5$; $p_m = 196{,}2$ kPa;
$h_1 = 10$ m; $h_2 = 5$ m



$$2h_1 = \frac{p_m}{\rho g}$$

$$Q_1 = Q_2 = Q_3 / 2$$

$$Q_4 = Q_6 \qquad Q_5 = Q_7$$

$$Q_3 = Q_4 + Q_5 \qquad (1)$$

$$\frac{\varepsilon}{D} = \frac{0{,}2}{200} = 0{,}001 \quad \rightarrow \quad \lambda = 0{,}02$$

$$A = \frac{D^2\pi}{4} = 0{,}0314\ m^2$$

$$h_1 - h_2 = \frac{(Q_3/2)^2}{2gA^2}\left(\xi_{UL} + \lambda \cdot \frac{L_1}{D}\right) + \frac{Q_3^{\,2}}{2gA^2}\left(\xi_{RAČ} + \lambda \cdot \frac{L_3}{D}\right) + \frac{Q_4^{\,2}}{2gA^2}\left(\xi_{RAČ} + \xi_{KOLJ} + \lambda \cdot \frac{L_4 + L_6}{D} + 1\right)$$

$$5 = 12,92 \cdot Q_3^{\,2} \cdot (0,5+1) + 51,69 \cdot Q_3^{\,2} \cdot (0,3+1) + 51,69 \cdot Q_4^{\,2} \cdot (0,3+0,4+1+1)$$

$$5 = 86,58 \cdot Q_3^{\,2} + 139,56 \cdot Q_4^{\,2} \qquad (2)$$

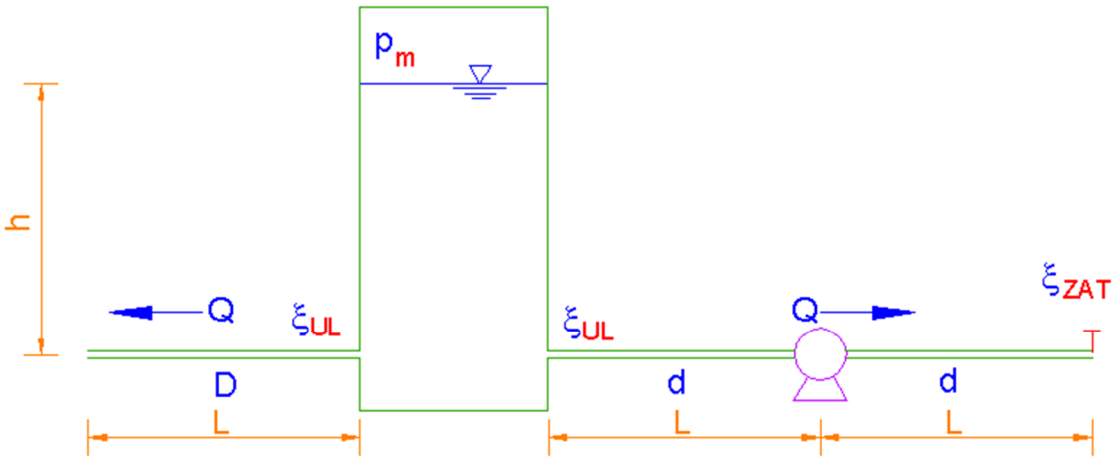
$$h_1 + h_2 = \frac{(Q_3/2)^2}{2gA^2}\left(\xi_{UL} + \lambda \cdot \frac{L_2}{D}\right) + \frac{Q_3^{\,2}}{2gA^2}\left(\xi_{RAČ} + \lambda \cdot \frac{L_3}{D}\right) + \frac{Q_5^{\,2}}{2gA^2}\left(\xi_{RAČ} + \xi_{KOLJ} + \lambda \cdot \frac{L_5 + L_7}{D} + 1\right)$$

$$15 = 86,58 \cdot Q_3^{\,2} + 139,56 \cdot Q_5^{\,2} \qquad (3)$$

Za dobivanje rezultata potrebno je riješiti gore navedeni sustav 3 *jednadžbe s* 3 *nepoznanice*

30. Za cjevovod prema slici potrebno je odrediti visinu dizanja pumpe H_{pumpa} kojom će se omogućiti isti protok u lijevoj i desnoj grani cjevovoda. Potrebno je nacrtati energetsku i piezometarsku liniju.

Zadano je: p_m = -49,05 kPa; ρ = 1000 kg/m^3; h =55 m;
ε = 0,2 mm; D = 200 mm; d = 150 mm; L = 2000 m;
λ_D = 0,02; λ_d = 0,021; ξ_{UL} = 0,5; ξ_{ZAT} = 3



lijeva grana:

$$h+\frac{p_m}{\rho g}=\frac{Q^2}{2gA_D^2}\left(\xi_{UL}+\lambda_D\cdot\frac{L}{D}+1\right)$$

$$55-5=\frac{Q^2}{2gA_D^2}(0{,}5+200+1)$$

$$\frac{Q^2}{2gA_D^2}=\frac{v_D{}^2}{2g}=0{,}248\,m \quad\rightarrow\quad v_D=2{,}21\,m/s \quad\rightarrow\quad Q=0{,}069\,m^3/s$$

desna grana:

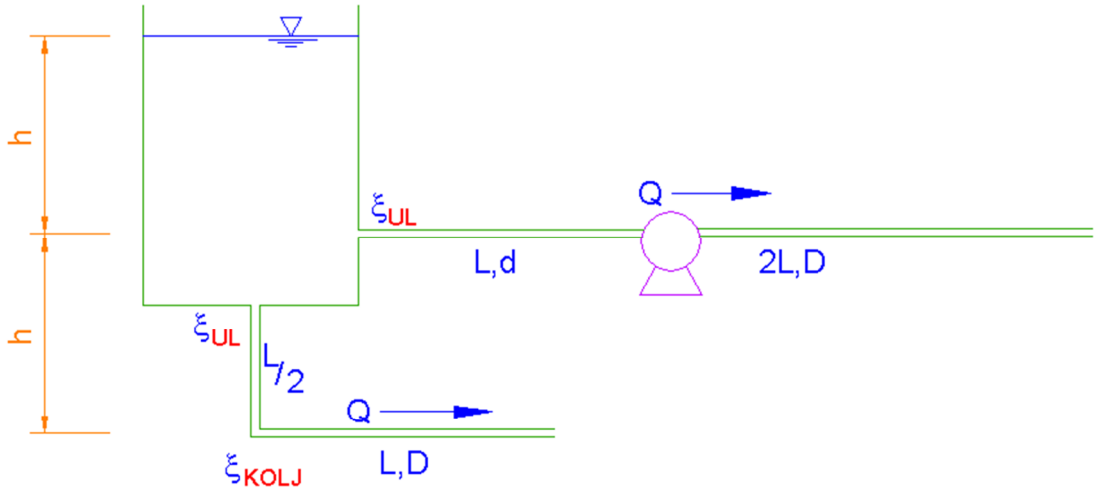
$$h+\frac{p_m}{\rho g}+H_{pumpa}=\frac{Q^2}{2gA_d^2}\left(\xi_{UL}+\lambda_{ZAT}+\lambda_d\cdot\frac{2L}{d}+1\right)$$

$$55-5+H_{pumpa}=\frac{4^2\cdot 0{,}069^2}{2\cdot 9{,}81\cdot 0{,}15^4\pi^2}(0{,}5+3+560+1)$$

$$H_{pumpa}=388{,}65\,m$$

31. Za cjevovod prema slici potrebno je odrediti visinu dizanja pumpe H_{pumpa} kojom će se omogućiti isti protok u gornjoj i donjoj grani cjevovoda. Potrebno je nacrtati energetsku i piezometarsku liniju.

Zadano je: $D = 200$ mm; $d = 150$ mm; $h = 5$ m; $\rho = 1000$ kg/m^3;
$\xi_{UL} = 0{,}5$; $\xi_{KOLJ} = 0{,}3$; $L = 500$ m; $\lambda_D = 0{,}02$; $\lambda_d = 0{,}021$



$A_D = 0{,}0314\,m^2$ $\qquad$ $A_d = 0{,}0177\,m^2$

donja cijev:

$$2h = \frac{Q^2}{2gA_D^2}\cdot\left(\xi_{UL} + \xi_{KOLJ} + \lambda_D \cdot \frac{1{,}5L}{D} + 1\right)$$

$$10 = \frac{Q^2}{2gA_D^2}\cdot\left(0{,}5 + 0{,}3 + 75 + 1\right)$$

$$\frac{Q^2}{2gA_D^2} = \frac{v_D^{\,2}}{2g} = 0{,}13\,m \quad \rightarrow \quad v_D = 1{,}6\,m/s \quad \rightarrow \quad Q = 0{,}05\,m^3/s$$

gornja cijev:

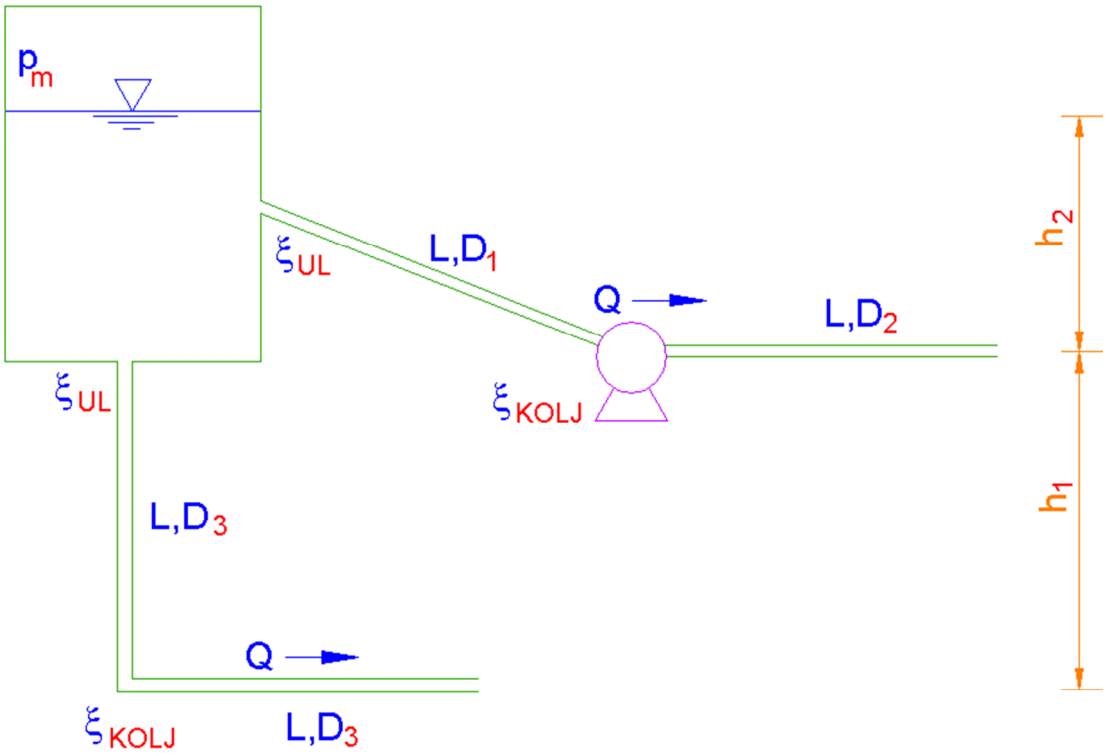
$$h + H_{pumpa} = \frac{Q^2}{2gA_d^{\,2}}\left(\xi_{UL} + \lambda_d \cdot \frac{L}{d}\right) + \frac{Q^2}{2gA_D^{\,2}}\left(\lambda_D \cdot \frac{2L}{D} + 1\right)$$

$$5 + H_{pumpa} = 0{,}407\left(0{,}5 + 70\right) + 0{,}129\left(100 + 1\right)$$

$$H_{pumpa} = 36{,}74\,m$$

32. Za cjevovod prema slici potrebno je odrediti promjer donje grane cjevovoda D_3 kojom će se omogućiti isti protok kao i u gornjoj grani cjevovoda. Potrebno je nacrtati energetsku i piezometarsku liniju. Tečenje se odvija turbulentno hrapavim režimom.

Zadano je: $p_m = -4{,}905$ kPa; $D_1 = 300$ mm; $D_2 = 250$ mm; $\varepsilon = 0{,}2$ mm;
$h_1 = 2$ m; $h_2 = 1$ m; $\xi_{UL} = 0{,}5$; $\xi_{KOLJ} = f(v_2) = 0{,}3$;
$L = 300$ m; $N_{pumpa} = 500$ W; $\eta = 0{,}8$; $\rho = 1000$ kg/m^3



gornja cijev:

$$A_1 = 0{,}071\,m^2 \qquad A_2 = 0{,}049\,m^2$$

$$\frac{\varepsilon}{D_1} = 0{,}00067 \quad \rightarrow \quad \lambda_1 = 0{,}018$$

$$\frac{\varepsilon}{D_2} = 0{,}0008 \quad \rightarrow \quad \lambda_2 = 0{,}019$$

$$N_{pumpa} = \frac{\rho \cdot g \cdot H_{pumpa} \cdot Q}{\eta} \quad \rightarrow \quad H_{pumpa} = \frac{N_{pumpa} \cdot \eta}{\rho \cdot g \cdot Q} = \frac{0{,}5 \cdot 0{,}8}{1 \cdot 9{,}81 \cdot Q} = \frac{0{,}041}{Q}$$

$$\frac{p_m}{\rho g}+h_2+H_P=\frac{Q^2}{2gA_1^2}\left(\xi_{UL}+\lambda_1\frac{L}{D_1}\right)+\frac{Q^2}{2gA_2^2}\left(\xi_{KOLJ}+\lambda_2\frac{L}{D_2}+1\right)$$

$$-0,5+1+\frac{0,041}{Q}=\frac{Q^2}{0,099}(0,5+18)+\frac{Q^2}{0,047}(0,3+22,8+1)$$

$$-0,5+1+\frac{0,041}{Q}=\frac{Q^2}{0,099}(0,5+18)+\frac{Q^2}{0,047}(0,3+22,8+1)$$

$$699.63Q^2-\frac{0,041}{Q}=0,5$$

$$0,5Q=699.63Q^3-0,041$$

$$Q=1399.26Q^3-0,082$$

Q pretp (desna strana)	Q dobiv (lijeva strana)
1	1399.178
0,1	1,317
0,05	0,093
0,045	0,046

Odabrano: $Q=0,045\ m^3/s$

donja cijev:

$$\frac{p_m}{\rho g}+h_1+h_2=\frac{Q^2}{2gA_3^2}\left(\xi_{UL}+\xi_{KOLJ}+\lambda_3\cdot\frac{2L}{D_3}+1\right)$$

$$-0,5+2+1=\frac{4^2\cdot 0,045^2}{2\cdot 9,81\cdot D_3^4\pi^2}\left(0,5+0,3+\lambda_3\cdot\frac{600}{D_3}+1\right)$$

$$2,5=\frac{0,00017}{D_3^4}\left(1,8+\lambda_3\cdot\frac{600}{D_3}\right)$$

$D_3=0,3\ m \quad\rightarrow\quad \lambda_3=0,018 \quad\rightarrow\quad 2,5\neq 0,79$

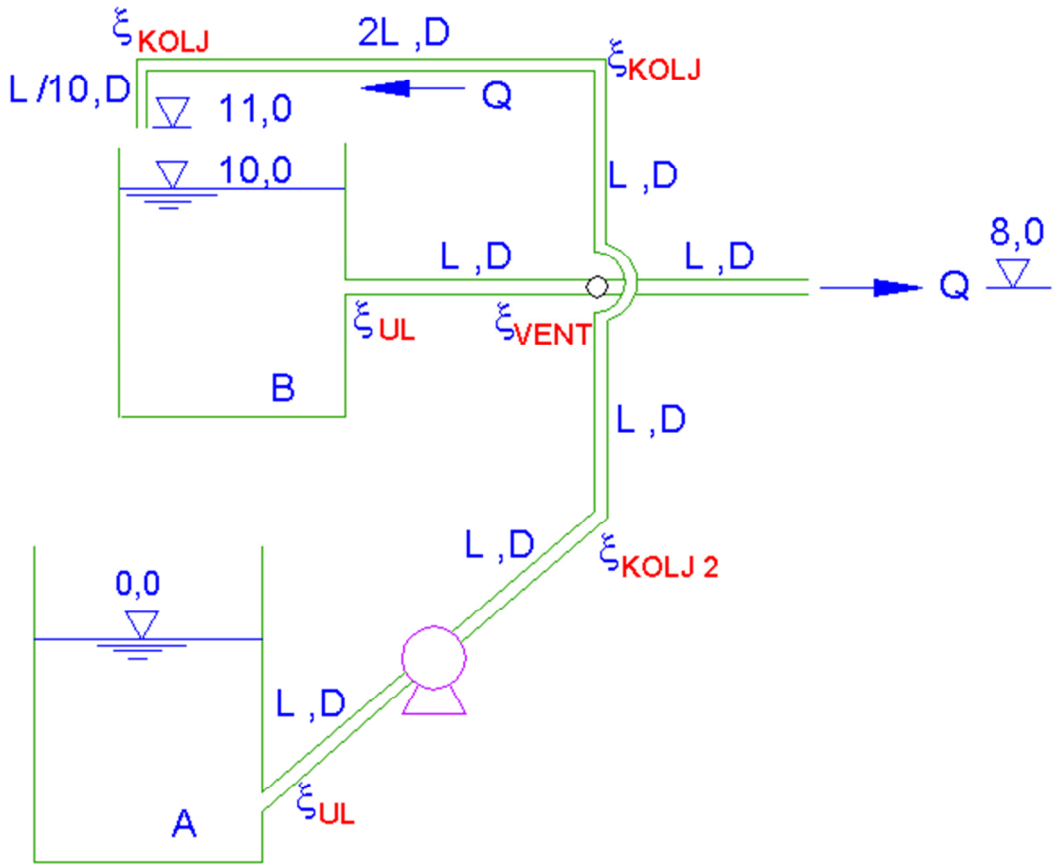
$D_3=0,2\ m \quad\rightarrow\quad \lambda_3=0,02 \quad\rightarrow\quad 2,5\neq 6,57$

$D_3=0,24\ m \quad\rightarrow\quad \lambda_3=0,019 \quad\rightarrow\quad 2,5\approx 2,53$

Odabrano: $D_3 = 240$ mm

33. Za stacionarni režim strujanja i cjevovod prema slici potrebno je odrediti snagu pumpe N_{pumpa} kojom će se omogućiti da dotok Q iz komore A u komoru B bude jednak istjecanju Q iz komore B. Potrebno je nacrtati energetsku i piezometarsku liniju za čitavi sustav.

Zadano je: $\rho_v = 1000$ kg/m^3; $D = 300$ mm; $\lambda = 0{,}02$; $L = 5$ m;
$\xi_{UL} = 0{,}5$; $\xi_{KOLJ} = 0{,}3$; $\xi_{KOLJ\,2} = 0{,}4$; $\xi_{VENT} = 1{,}5$;
$\eta_P = 0{,}7$



$$10 = 8 + \frac{v^2}{2g}\left(\xi_{UL} + \xi_{VENT} + \lambda \cdot \frac{2L}{D} + 1\right)$$

$$10 = 8 + \frac{v^2}{2g}(0{,}5 + 1{,}5 + 0{,}67 + 1)$$

$$\frac{v^2}{2g} = 0{,}545\,m \quad \rightarrow \quad v = 3{,}27\,m/s \quad \rightarrow \quad Q = 0{,}231\,m^3/s$$

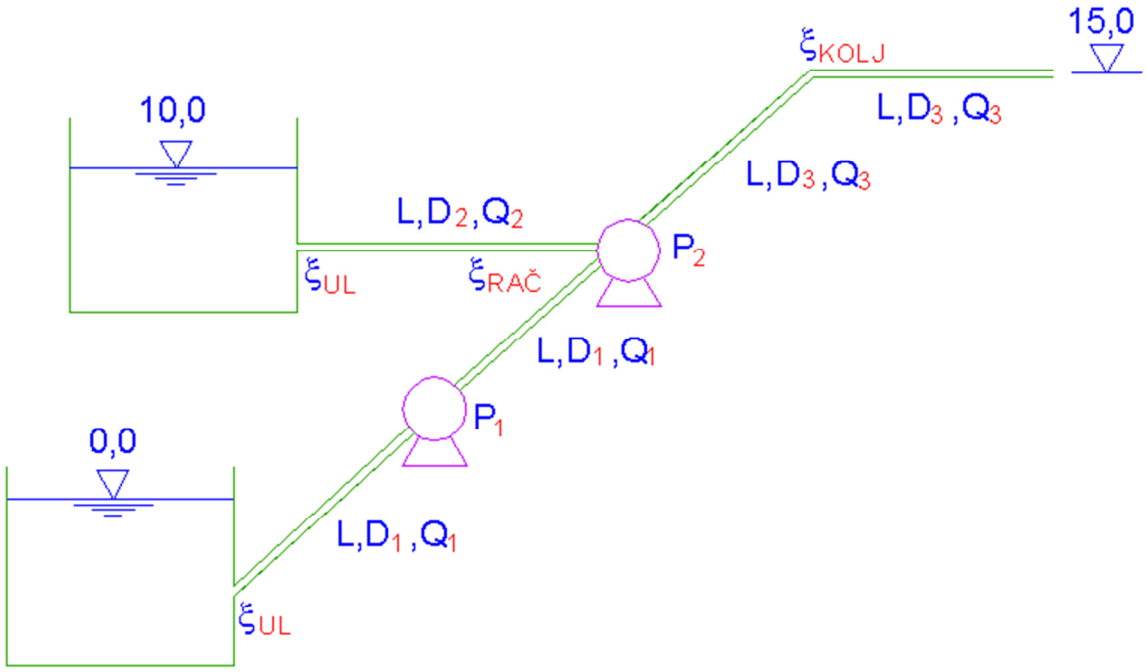
$$H_{pumpa} = 11 + \frac{v^2}{2g}\left(\xi_{UL} + \lambda \cdot \frac{6{,}1L}{D} + \xi_{KOLJ\,2} + 2\xi_{KOLJ} + 1\right)$$

$$\frac{N_{pumpa} \cdot 0{,}7}{1 \cdot 9{,}81 \cdot 0{,}231} = 11 + 0{,}545(0{,}5 + 2{,}03 + 0{,}4 + 0{,}6 + 1)$$

$$N_{pumpa} = 43{,}61\,kW$$

34. Za stacionarni režim strujanja i cjevovod prema slici potrebno je odrediti protoke Q_1, Q_2 i promjer cjevovoda D_2. Potrebno je nacrtati energetske i piezometarske linije.

Zadano je: $L = 100$ m; $\lambda = 0{,}02$; $Q_3 = 0{,}071$ m^3/s; $D_1 = 0{,}2$ m; $D_3 = 0{,}3$ m;
$\xi_{UL} = 0{,}5$; $\xi_{KOLJ} = 0{,}3$; $\xi_{RAČ} = f(v_3) = 0{,}8$; $H_{P1} = 10$ m; $H_{P2} = 6{,}8$ m;
$\rho = 1000$ kg/m^3



$$v_3 = \frac{4Q_3}{D_3^2\pi} = \frac{4\cdot 0{,}071}{0{,}3^2\pi} = 1{,}0\, m/s \quad \rightarrow \quad \frac{v_3^2}{2g} = 0{,}051\, m$$

$$H_{P1} + H_{P2} = 15 + \frac{v_1^2}{2g}\left(\xi_{UL} + \lambda\cdot\frac{2L}{D_1}\right) + \frac{v_3^2}{2g}\left(\xi_{RAČ} + \xi_{KOLJ} + \lambda\cdot\frac{2L}{D_3} + 1\right)$$

$$10 + 6{,}8 = 15 + \frac{v_1^2}{2g}(0{,}5 + \cdot 20) + 0{,}051(0{,}8 + 0{,}3 + 13{,}33 + 1)$$

$$\frac{v_1^2}{2g} = 0{,}049\, m \quad \rightarrow \quad v_1 = 0{,}98\, m/s \qquad Q_1 = v_1\cdot\frac{D_1^2\pi}{4} = 0{,}031\, m^3/s$$

$$Q_2 = Q_3 - Q_1 = 0{,}071 - 0{,}031 = 0{,}04\, m^3/s$$

$$v_2 = \frac{4Q_2}{D_2^2\pi} = \frac{4\cdot 0{,}04}{D_2^2\pi} = \frac{0{,}0509}{D_2^2} \quad \rightarrow \quad \frac{v_2^2}{2g} = \frac{1}{2g}\cdot\left(\frac{0{,}0509}{D_2^2}\right)^2 = \frac{0{,}000132}{D_2^4}$$

$$H_{P2} = 5 + \frac{v_2^2}{2g}\left(\xi_{UL} + \lambda \cdot \frac{L}{D_2}\right) + \frac{v_3^2}{2g}\left(\xi_{RAČ} + \xi_{KOLJ} + \lambda \cdot \frac{2L}{D_3} + 1\right)$$

$$6,8 = 5 + \frac{0,000132}{D_2^4}\left(0,5 + \frac{2}{D_2}\right) + 0,051\left(0,8 + 0,3 + 13,33 + 1\right)$$

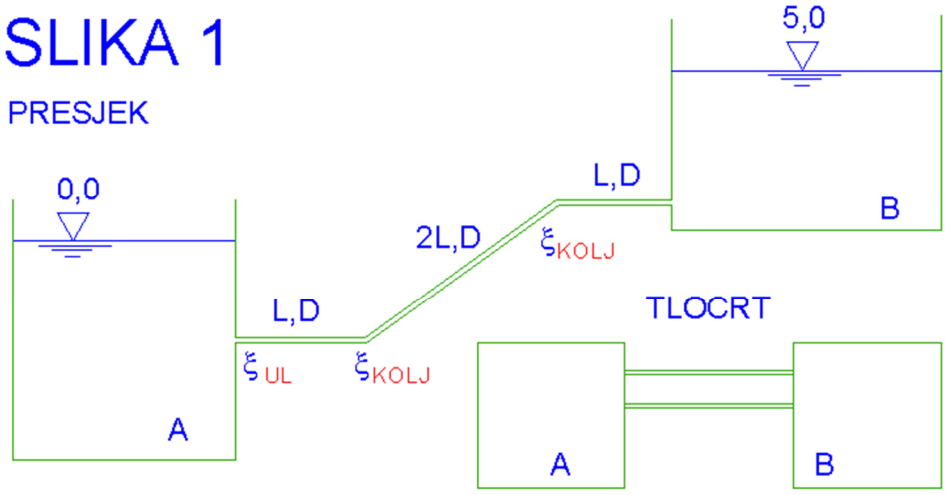
$$1,01 = \frac{0,000066}{D_2^4} + \frac{0,000264}{D_2^5}$$

Postukom iteracije: $D_2 = 0,192\ m$

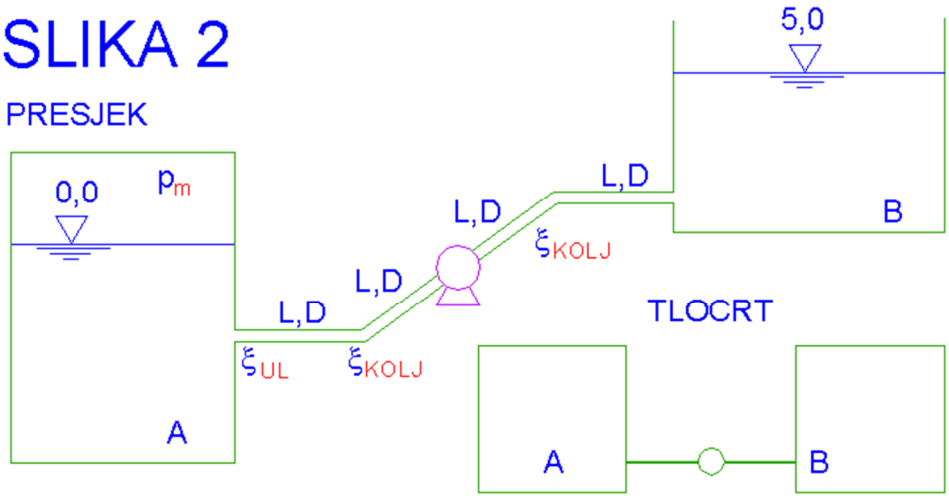
35. Komore *A* i *B* povezane su pomoću dvije cijevi kroz koje se ostvaruje protok $Q_{B\text{-}A}$ (slika 1). Ukoliko se komora *A* zatvori i stavi pod djelovanje tlaka p_m, potrebno je odrediti snagu pumpe N_P koja se postavlja u novu cijev (prve dvije se uklanjaju) da bi se ostvario protok $Q_{A\text{-}B}$ (iz komore *A* u komoru *B* na slici 2) koji je po iznosu jednak protoku $Q_{B\text{-}A}$ u prvom slučaju sa slike 1. Potrebno je nacrtati energetske i piezometarske linije. Tečenje se odvija turbulentno hrapavim režimom.

Zadano je: $L = 50$ m; $\varepsilon = 0{,}001$ m; $D = 0{,}2$ m; $p_m = 88{,}29$ kPa;
$\xi_{UL} = 0{,}5$; $\xi_{KOLJ} = 0{,}3$; $\eta = 0{,}7$; $\rho = 1000$ kg/m^3

SLIKA 1



SLIKA 2



$$5 = \frac{v_{BA}^2}{2g}\left(\xi_{UL} + \lambda \cdot \frac{4L}{D} + 2\xi_{KOLJ} + 1\right)$$

$$\frac{\varepsilon}{D} = \frac{1}{200} = 0{,}005 \qquad \rightarrow \qquad \lambda = 0{,}03$$

$$5 = \frac{v_{BA}^2}{2g}(0{,}5 + 30 + 0{,}6 + 1)$$

$$\frac{v_{BA}^2}{2g} = 0{,}156\,m \qquad \rightarrow \qquad v_{BA} = 1{,}75\,m/s \qquad \rightarrow \qquad Q_{BA} = 2 \cdot 1{,}75 \cdot \frac{0{,}2^2\pi}{4} = 0{,}11\,m^3/s$$

$$v_{AB} = 2 \cdot v_{BA} = 3,5\ m/s \qquad \rightarrow \qquad \frac{v_{AB}^2}{2g} = 0,624\ m$$

$$\frac{p_m}{\rho g} + H_P = 5 + \frac{v_{AB}^2}{2g}\left(\xi_{UL} + \lambda \cdot \frac{4L}{D} + 2\xi_{KOLJ} + 1\right)$$

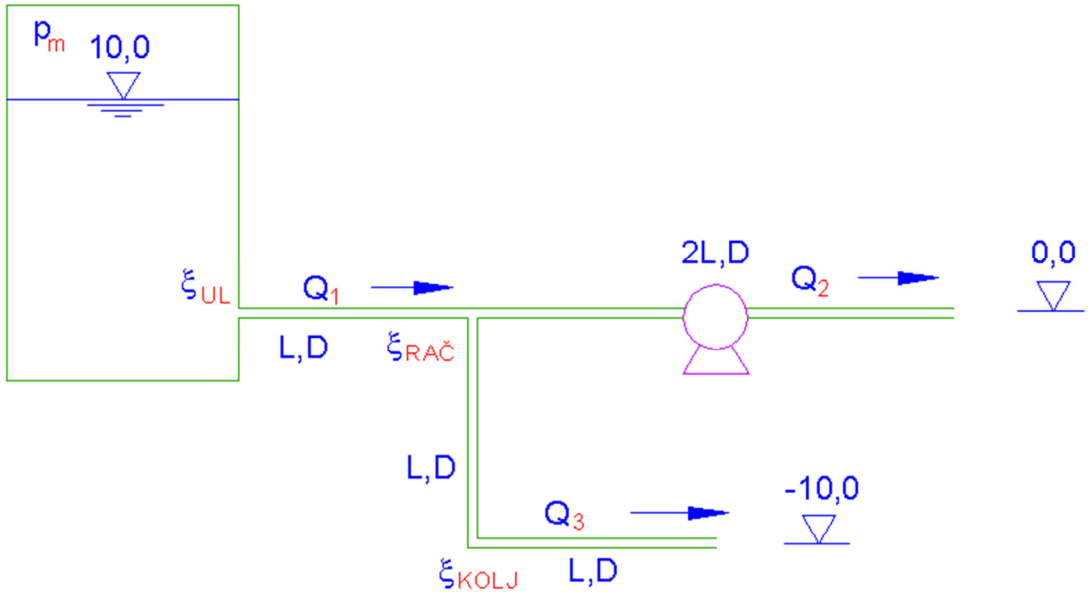
$$9 + H_P = 5 + 0,624\left(0,5 + 30 + 0,6 + 1\right)$$

$$H_P = 16,03\ m$$

$$N_P = \frac{\rho g Q_{AB} H_P}{\eta} = \frac{1 \cdot 9,81 \cdot 0,11 \cdot 16,03}{0,7} = 24,71\ kW$$

36. Potrebno je odrediti protoke Q_2, Q_3 i tlačnu visinu pumpe H_P kako bi se nakon izlaza iz komore ostvario protok $Q_1 = 0{,}1$ m^3/s. Potrebno je nacrtati energetske i piezometarske linije. Pretpostavljen je turbulentno hrapavi režim strujanja.

Zadano je: $p_m = -19{,}62$ kPa; $L = 100$ m; $\rho = 1000$ kg/m^3; $\varepsilon = 0{,}001$ m; $D = 0{,}2$ m;
$\xi_{UL} = 0{,}5$; $\xi_{KOLJ} = 0{,}3$; $\xi_{RAČ} = f(v_1) = 0{,}3$



$$10+\frac{p_m}{\rho g}=-10+\frac{v_1^2}{2g}\left(\xi_{UL}+\xi_{RAČ}+\lambda\cdot\frac{L}{D}\right)+\frac{v_3^2}{2g}\left(\lambda\cdot\frac{2L}{D}+\xi_{KOLJ}+1\right)$$

$$\frac{\varepsilon}{D}=0{,}005 \quad\rightarrow\quad \lambda=0{,}03$$

$$v_1=\frac{4\cdot Q_1}{D^2\pi}=\frac{4\cdot 0{,}1}{0{,}2^2\pi}=3{,}18\,m/s \quad\rightarrow\quad \frac{v_1^2}{2g}=0{,}516\,m$$

$$10-2=-10+0{,}516\left(0{,}5+0{,}3+15\right)+\frac{v_3^2}{2g}\left(30+0{,}3+1\right)$$

$$\frac{v_3^2}{2g}=0{,}315\,m \quad\rightarrow\quad v_3=2{,}48\,m/s \quad\rightarrow\quad Q_3=v_3\cdot\frac{D^2\pi}{4}=0{,}078\,m^3/s$$

$$Q_2=Q_1-Q_3=0{,}1-0{,}078=0{,}022\ \ m^3/s$$

$$v_2=\frac{4\cdot Q_2}{D^2\pi}=\frac{4\cdot 0{,}022}{0{,}2^2\pi}=0{,}7\,m/s \quad\rightarrow\quad \frac{v_2^2}{2g}=0{,}025\,m$$

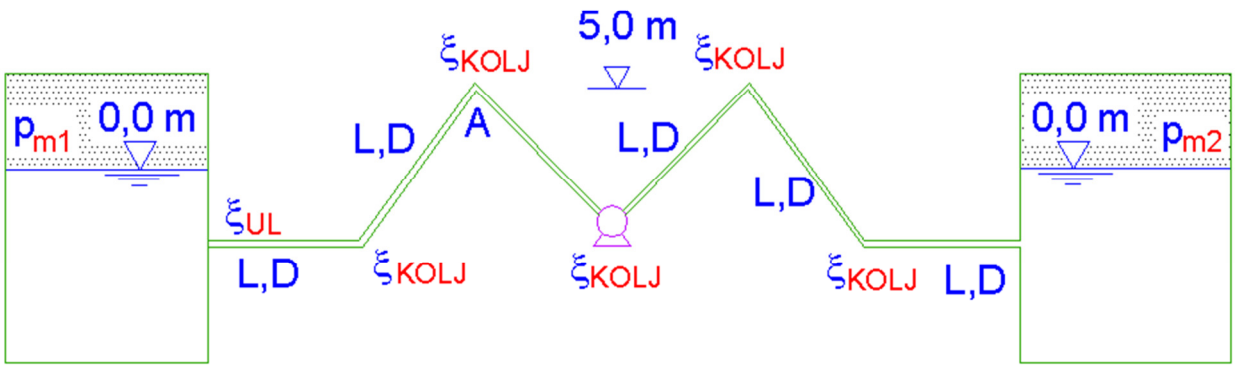
$$10+\frac{p_m}{\rho g}+H_P=\frac{v_1^2}{2g}\left(\xi_{UL}+\xi_{RAČ}+\lambda\cdot\frac{L}{D}\right)+\frac{v_2^2}{2g}\left(\lambda\cdot\frac{2L}{D}+1\right)$$

$$10-2+H_P=0{,}516\left(0{,}5+0{,}3+15\right)+0{,}025\left(30+1\right)$$

$$H_P=0{,}93\,m$$

37. Za cjevovod sa slike potrebno je provjeriti da li se ugradnjom pumpe s tlačnom visinom H_P = 10 m, ostvaruje takav režim strujanja da se u točki *A* ne premaši minimalno dozvoljeni podtlak od $p_A/(\rho g)$ = -8,5 metara vodnog stupca.

Zadano je: L = 10 m; p_{m1} = 19,62 kPa; p_{m2} = - 9,81 kPa; D = 100 mm;
ξ_{UL} = 0,5; ξ_{KOLJ} = 0,3; λ = 0,02; ρ = 1000 kg/m^3



$$\frac{p_{m1}}{\rho g}+H_P=\frac{p_{m2}}{\rho g}+\frac{v^2}{2g}\left(\xi_{UL}+5\cdot\xi_{KOLJ}+\lambda\cdot\frac{6\cdot L}{D}+1\right)$$

$$10+2=-1+\frac{v^2}{2g}\left(0{,}5+5\cdot 0{,}3+6\cdot 2+1\right)$$

$$\frac{v^2}{2g}=0{,}867\ m$$

$$\frac{p_{m1}}{\rho g}=5+\frac{p_A}{\rho g}+\frac{v^2}{2g}+\frac{v^2}{2g}\left(\xi_{UL}+2\xi_{KOLJ}+\lambda\cdot\frac{2\cdot L}{D}\right)$$

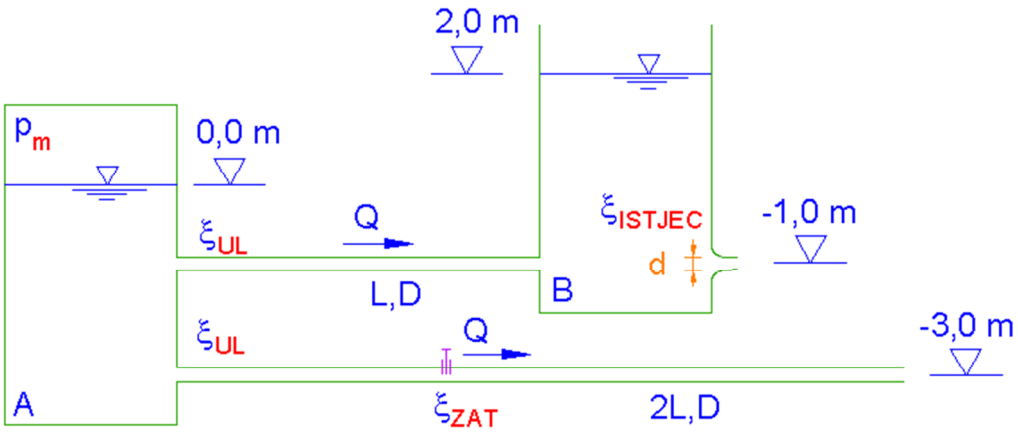
$$2=5+\frac{p_A}{\rho g}+0{,}867+0{,}867\left(0{,}5+2\cdot 0{,}3+2\cdot 2\right)$$

$$\frac{p_A}{\rho g}=-8{,}29\ m$$

$-8{,}29\ m\ >\ -8{,}5\ m \quad \rightarrow \quad$ *UVJET ZADOVOLJAVA!*

38. Za cjevovod sa slike potrebno je odrediti koeficijent lokalnog gubitka na zatvaraču $\xi_{ZAT,}$ kako bi se kroz donju izlaznu cijev iz komore A ostvario isti protok kao i kroz gornju cijev koja povezuje komore *A* i *B*. Razine vodnih lica u komorama A i B su stacionarne.

Zadano je: $d = 100$ mm; $D = 250$ mm; $L = 50$ m; $\varepsilon = 0{,}25$ mm; $\xi_{UL} = 0{,}5$; $\xi_{ISTJEC} = 0{,}3$; $C_C = 1$



$$2 = -1 + \frac{v_d^{\,2}}{2g}(\xi_{ISTJEC} + 1) \quad \rightarrow \quad v_d = 6{,}73\, m/s \quad \rightarrow \quad Q_d = 0{,}053\, m^3/s$$

$$Q_d = Q \quad (stacionarno\ strujanje)$$

$$v_D = \frac{4Q}{D^2\pi} = 1{,}08\, m/s \quad \rightarrow \quad \frac{v_D^{\,2}}{2g} = 0{,}06\, m/s$$

$$\frac{p_m}{\rho g} = 2 + \frac{v_D^2}{2g}\left(\xi_{UL} + \lambda \cdot \frac{L}{D} + 1\right)$$

$$\frac{p_m}{\rho g} = 2 + 0{,}06\,(0{,}5 + 4 + 1) = 2{,}33\, m$$

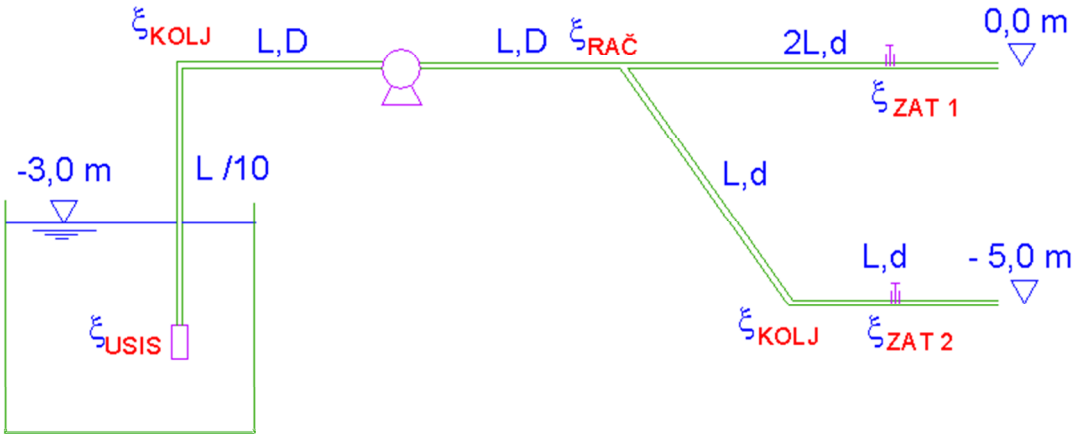
$$\frac{p_m}{\rho g} = -3 + \frac{v_D^2}{2g}\left(\xi_{UL} + \lambda \cdot \frac{2L}{D} + \xi_{ZAT} + 1\right)$$

$$2{,}33 = -3 + 0{,}06\,(0{,}5 + 8 + \xi_{ZAT} + 1)$$

$$\xi_{ZAT} = 79{,}33$$

39. Za cjevovod sa slike potrebno je odrediti koeficijente lokalnih gubitaka na zatvaračima $\xi_{ZAT\,1}$ i $\xi_{ZAT\,2}$ kojim se omogućuje jednaki protok u gornjoj i donjoj grani izlaznog cjevovoda. Nacrtati energetsku i piezometarsku liniju.

Zadano je: $H_P = 10$ m; $N_P = 2000$ W; $\eta = 0{,}7$; $\rho = 1000$ kg/m^3;
λ(za sve cijevi) $= 0{,}02$; $L = 50$ m; $D = 0{,}2$ m; $d = 0{,}1$ m;
$\xi_{USIS} = 0{,}1$; $\xi_{KOLJ} = 0{,}3$; $\xi_{RAČ} = f(v_D) = 0{,}25$



$$N_P = \frac{\rho g Q H_P}{\eta} \quad \rightarrow \quad Q = \frac{\eta \cdot N_P}{\rho \cdot g \cdot H_P} = \frac{0{,}7 \cdot 2000}{1000 \cdot 9{,}81 \cdot 10} = 0{,}0143\ m^3/s$$

$$Q_1 = Q_2 = \frac{Q}{2} = 0{,}00715\ m^3/s$$

$$v_D = \frac{4Q}{D^2\pi} = \frac{4 \cdot 0{,}0143}{0{,}2^2\pi} = 0{,}46\ m/s \quad \rightarrow \quad \frac{v_D^2}{2g} = 0{,}01\ m$$

$$v_d = \frac{4(Q/2)}{d^2\pi} = \frac{4 \cdot 0{,}00715}{0{,}1^2\pi} = 0{,}91\ m/s \quad \rightarrow \quad \frac{v_d^2}{2g} = 0{,}04\ m$$

$$-3 + H_P = \frac{v_D^2}{2g}\left(\xi_{USIS} + \xi_{KOLJ} + \xi_{RAČ} + \lambda \cdot \frac{2.1L}{D}\right) + \frac{v_d^2}{2g}\left(\xi_{ZAT\,1} + \lambda \cdot \frac{2L}{d} + 1\right)$$

$$-3 + 10 = 0{,}01(0{,}1 + 0{,}3 + 0{,}25 + 10{,}5) + 0{,}04(\xi_{ZAT\,1} + 20 + 1)$$

$$\xi_{ZAT\,1} = 151{,}21$$

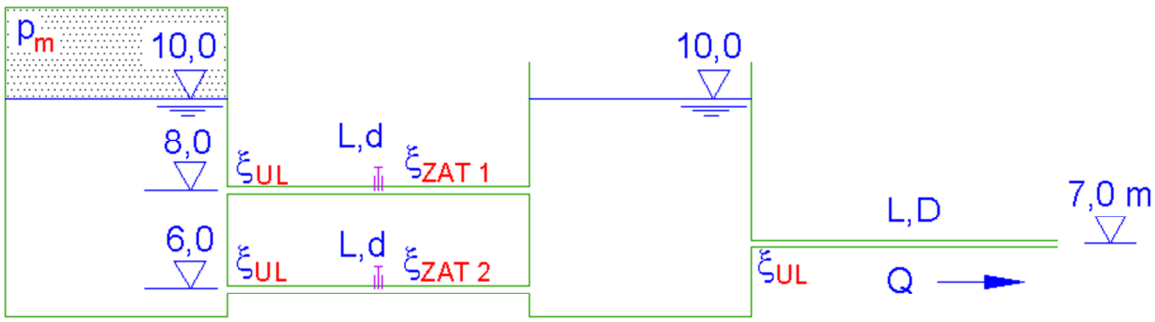
$$-3 + H_P = -5 + \frac{v_D^2}{2g}\left(\xi_{USIS} + \xi_{KOLJ} + \xi_{RAČ} + \lambda \cdot \frac{2{,}1L}{D}\right) + \frac{v_d^2}{2g}\left(\xi_{KOLJ} + \xi_{ZAT\,2} + \lambda \cdot \frac{2L}{d} + 1\right)$$

$$-3 + 10 = -5 + 0{,}01(0{,}1 + 0{,}3 + 0{,}25 + 10{,}5) + 0{,}04(0{,}3 + \xi_{ZAT\,2} + 20 + 1)$$

$$\xi_{ZAT\,2} = 275{,}9$$

40. Potrebno je odrediti koeficijente lokalnih gubitaka na zatvaračima $\xi_{ZAT\,1}$ i $\xi_{ZAT\,2}$ za slučaj stacionarnog tečenja iz lijeve u desnu komoru s ukupnim protokom Q i istjecanje iz desne komore, također s protokom Q. Potrebno je nacrtati piezometarsku i energetsku liniju.

Zadano je: $d = 200$ mm; $D = 250$ mm; $\varepsilon = 0{,}2$ mm; $L = 100$ m; $\xi_{UL} = 0{,}5$;
$\rho = 1000$ kg/m^3; $p_m = 29{,}43$ kPa



$$\frac{\varepsilon}{d} = \frac{0{,}2}{200} = 0{,}001 \quad \rightarrow \quad \lambda_d = 0{,}02$$

$$\frac{\varepsilon}{D} = \frac{0{,}2}{250} = 0{,}0008 \quad \rightarrow \quad \lambda_D = 0{,}019$$

$$10 = 7 + \frac{v_D^2}{2g}\left(\xi_{UL} + \lambda_D \cdot \frac{L}{D} + 1\right)$$

$$10 = 7 + \frac{v_D^2}{2g}(0{,}5 + 7{,}6 + 1)$$

$$\frac{v_D^2}{2g} = 0{,}33\,m \quad \rightarrow \quad v_D = 2{,}54\,m/s \quad \rightarrow \quad Q = v_D \frac{D^2\pi}{4} = 0{,}125\,m^3/s$$

$$Q/2 = 0{,}0625\,m^3/s \quad \rightarrow \quad v_d = \frac{4(Q/2)}{d^2\pi} = 1{,}99\,m/s \quad \rightarrow \quad \frac{v_d^2}{2g} = 0{,}2\,m$$

Napomena: kroz obje cijevi promjera d, protječe protok od $Q/2$ zbog istih karakteristika cijevi i istih rubnih uvjeta

$$\frac{p_m}{\rho g} = \frac{v_d^2}{2g}\left(\xi_{UL} + \lambda_d \cdot \frac{L}{d} + \xi_{ZAT} + 1\right)$$

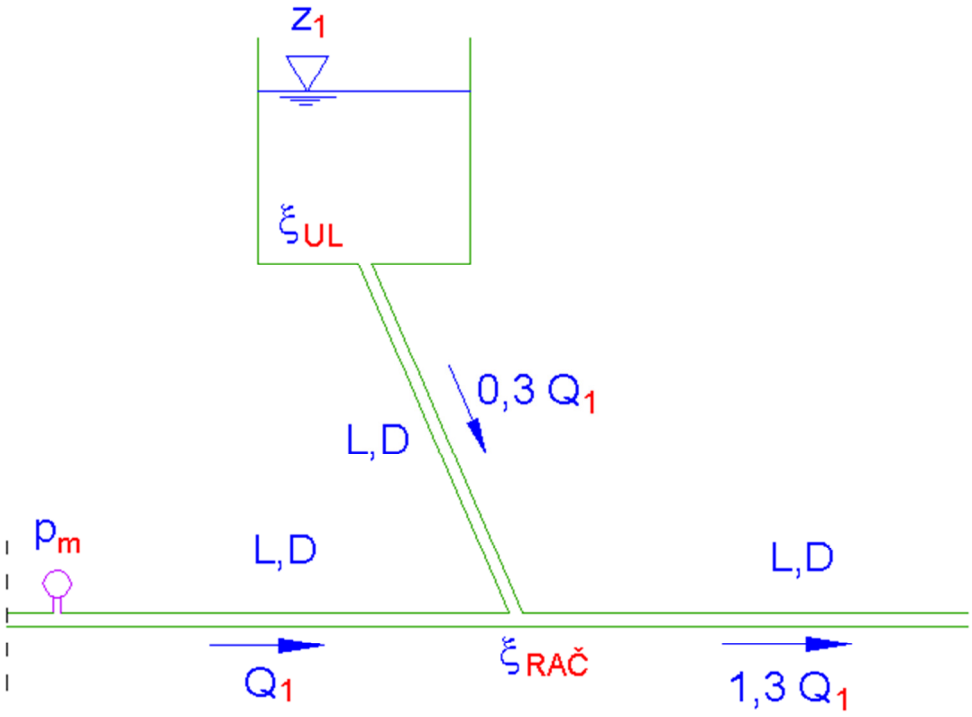
Napomena: ista B.J. vrijedi za obje cijevi pa, stoga, vrijedi i $\xi_{ZAT\,1} = \xi_{ZAT\,2} = \xi_{ZAT}$

$$3 = 0{,}2(0{,}5 + 10 + \xi_{ZAT} + 1)$$

$$\xi_{ZAT} = 3{,}5$$

41. Potrebno je odrediti razinu vodnog lica z_1 u rezervoaru sa slike te vrijednost tlaka p_m na manometru s kojim se omogućuje stacionarni režim tečenja realne tekućine s protocima u omjerima definiranim na slici. Nacrtati P.L. i E.L.

Zadano je: $\rho = 1000$ kg/m^3; $Q_1 = 120$ l/s; $L = 250$ m; $D = 0{,}2$ m;
$\lambda = 0{,}02$ (za sve cijevi); $\xi_{UL} = 0{,}5$; $\xi_{RAČ} = 0{,}3 = f(v_{nizvodno})$



$$A = \frac{D^2\pi}{4} = 0{,}0314\ m^2$$

$$\frac{p_m}{\rho g} + \frac{Q_1^2}{2gA^2} = \frac{(1{,}3Q_1)^2}{2gA^2} + \frac{Q_1^2}{2gA^2}\lambda\frac{L}{D} + \frac{(1{,}3Q_1)^2}{2gA^2}\left(\xi_{RAČ} + \lambda\frac{L}{D}\right)$$

$$\frac{p_m}{\rho g} + 0{,}7444 = 1{,}258 + 0{,}7444\cdot 25 + 1{,}258\left(0{,}3 + 25\right)$$

$$\frac{p_m}{\rho g} = 50{,}95\ m \qquad \rightarrow \qquad p_m = 499{,}83\ kPa$$

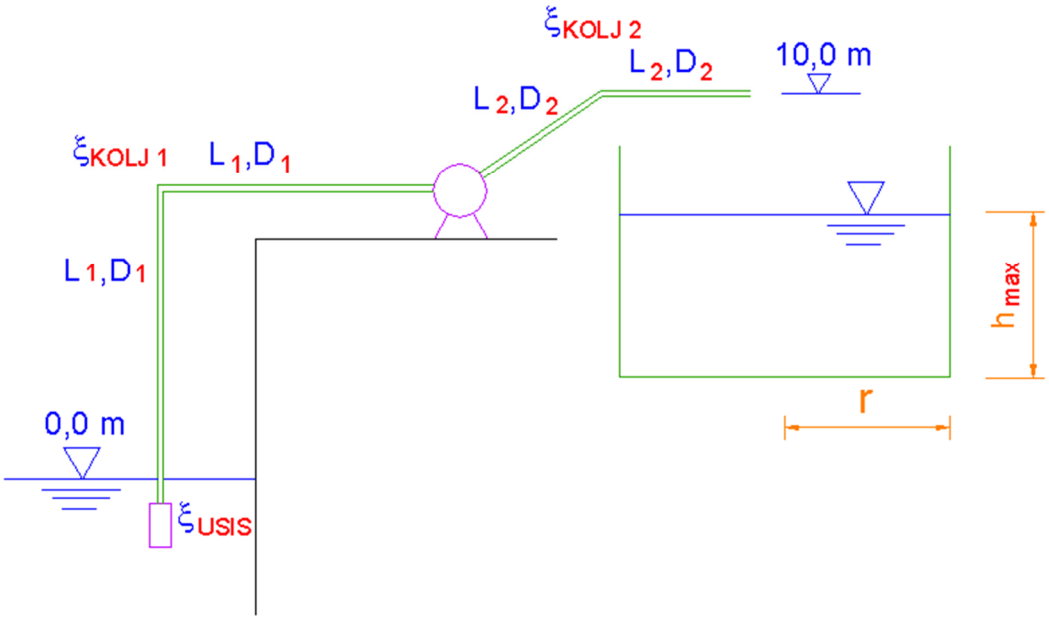
$$z_1 = \frac{(1{,}3Q_1)^2}{2gA^2} + \frac{(0{,}3Q_1)^2}{2gA^2}\left(\xi_{UL} + \lambda\frac{L}{D}\right) + \frac{(1{,}3Q_1)^2}{2gA^2}\left(\xi_{RAČ} + \lambda\frac{L}{D}\right)$$

$$z_1 = 1{,}258 + 0{,}067\left(0{,}5 + 25\right) + 1{,}258\left(0{,}3 + 25\right)$$

$$z_1 = 34{,}79\ m$$

42. Koje je vrijeme potrebno da se bazen polumjera r napuni do maksimalne visine h_{max}, ukoliko je dotok osiguran preko cjevovoda s pumpom?

Zadano je: $H_P = 12$ m; $D_1 = 0{,}15$ m; $D_2 = 0{,}2$ m; $\eta = 0{,}7$;
$\lambda = 0{,}02$; $L_1 = 100$ m; $L_2 = 50$ m; $h_{max} = 2$ m;
$r = 2$ m; $\xi_{KOLJ\,1} = 0{,}3$; $\xi_{KOLJ\,2} = 0{,}2$; $\xi_{USIS} = 0{,}1$



$$V = r^2\pi \cdot h_{\max} = 2^2\pi \cdot 2 = 25{,}13\,m^3$$

$$v_{D1}\frac{D_1^{\,2}\pi}{4} = v_{D2}\frac{D_2^{\,2}\pi}{4} \quad \rightarrow \quad v_{D1} = v_{D2}\frac{D_2^{\,2}}{D_1^{\,2}} = v_{D2}\frac{0{,}2^2}{0{,}15^2} = 1{,}778 v_{D2}$$

$$H_P = 10 + \frac{v_{D1}^2}{2g}\left[\xi_{USIS} + \xi_{KOLJ\,1} + \lambda\cdot\frac{2L_1}{D_1}\right] + \frac{v_{D2}^2}{2g}\left[\xi_{KOLJ\,2} + \lambda\cdot\frac{2L_2}{D_2} + 1\right]$$

$$12 = 10 + \frac{3{,}16\cdot v_{D2}^2}{2g}\left[0{,}1 + 0{,}3 + 26{,}667\right] + \frac{v_{D2}^2}{2g}\left[0{,}2 + 10 + 1\right]$$

$$\frac{v_{D2}^2}{2g} = 0{,}021\,m \quad \rightarrow \quad v_{D2} = 0{,}64\,m/s$$

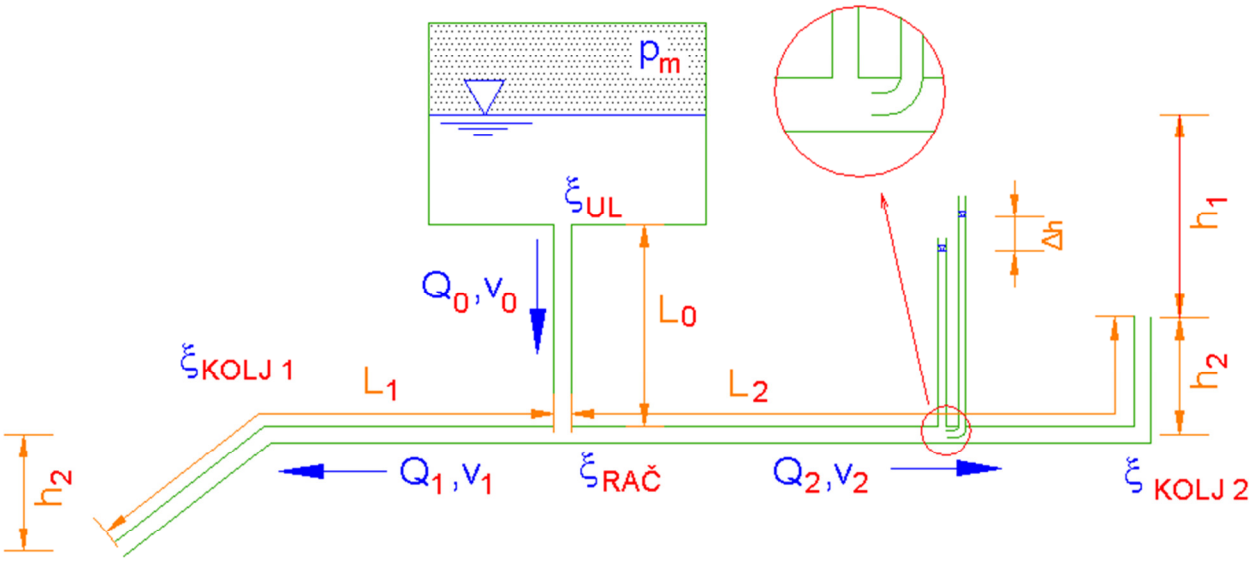
$$v_{D1} = 1{,}778\cdot 0{,}64 = 1{,}14\,m/s \quad \rightarrow \quad \frac{v_{D1}^2}{2g} = 0{,}066\,m$$

$$Q = v_{D1}\frac{D_1^{\,2}\pi}{4} = 1{,}14\cdot\frac{0{,}15^2\pi}{4} = 0{,}02\,m^3/s$$

$$t = \frac{V}{Q} = \frac{25{,}13}{0{,}02} = 1257\,s = 20\,min, 57\,s$$

43. Strujanje u cjevovodu na slici je stacionarno. U Prandtl - Pitotovoj mjernoj cijevi koja je postavljena na granu 2 cjevovoda, izmjerena je razlika razina Δh kao na slici. Potrebno je odrediti protoke Q_0, Q_1 *i* Q_2. Također je potrebno odrediti duljinu cjevovoda L_1. Nacrtati energetsku i piezometarsku liniju.

Zadano je: $\rho = 1000$ kg/m^3; $p_m = -19{,}62$ kPa; $h_1 = 5$ m;
$h_2 = 2$ m; $\Delta h = 0{,}09$ m; $L_0 = 5$ m;
$L_2 = 200$ m; D(za sve cijevi) = 300 mm; λ(za sve cijevi) = 0,02;
$\xi_{UL} = 0{,}5$; $\xi_{RAČ} = f(v_0) = 0{,}5$; $\xi_{KOLJ\,1} = 0{,}3$; $\xi_{KOLJ\,2} = 0{,}4$



$$\Delta h = 0{,}09\,m = \frac{v_2^2}{2g} \qquad \rightarrow \qquad v_2 = 1{,}33\,m/s$$

$$h_1 + \frac{p_m}{\rho g} = \frac{v_0^2}{2g}\left(\xi_{UL} + \lambda \cdot \frac{L_0}{D} + \xi_{RAČ}\right) + \frac{v_2^2}{2g}\left(\lambda \cdot \frac{L_2}{D} + \xi_{KOLJ\,2} + 1\right)$$

$$5 - 2 = \frac{v_0^2}{2g}(0{,}5 + 0{,}33 + 0{,}5) + 0{,}09(13{,}33 + 0{,}4 + 1)$$

$$\frac{v_0^2}{2g} = 1{,}26\,m \qquad \rightarrow \qquad v_0 = 4{,}97\,m/s$$

$$Q_0 = Q_1 + Q_2 \qquad \rightarrow \qquad v_0 = v_1 + v_2$$

$$v_1 = v_0 - v_2 = 4{,}97 - 1{,}33 = 3{,}64\,m/s \qquad \rightarrow \qquad \frac{v_1^2}{2g} = 0{,}68\,m$$

$$5 + 2 \cdot 2 - 2 = 1{,}26(0{,}5 + 0{,}33 + 0{,}5) + 0{,}68\left(0{,}02 \cdot \frac{L_1}{0{,}3} + 0{,}3 + 1\right)$$

$$L_1 = 97{,}9\,m$$