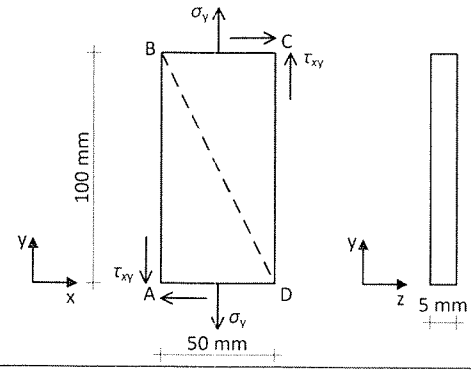
OTPORNOST MATERIJALA 1
PISMENI ISPIT 24. lipnja 2016. godine

Ime i prezime: _____

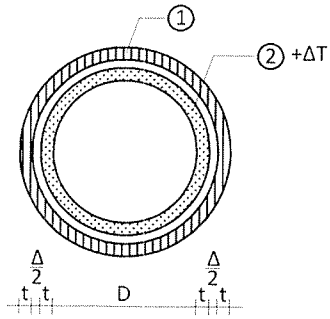
Za element prikazan na slici treba odrediti promjenu duljine dijagonale BD te apsolutnu promjenu volumena elementa.

1. Zadano:
σ_y = 120 MPa
τ_{xy} = 20 MPa
$E = 2{,}0 \cdot 10^5$ MPa
ν = 0,30



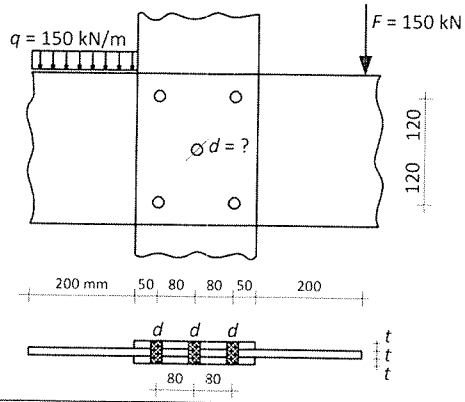
Odredite promjenu temperature prstena 2 koja uzrokuje naprezanje od 80 MPa u prstenu 1.

Zadano:
2. $E_1 = 2{,}0 \cdot 10^5$ MPa, $E_2 = 1{,}8 \cdot 10^5$ MPa
$\alpha_{t2} = 1{,}0 \cdot 10^{-5}$ 1/K
t = 5 mm
Δ = 0,2 mm
D = 600 mm



Tri lima jednake debljine (t = 16 mm) spojena su sa 5 zakovica promjera d. Na spoj djeluju sila F i kontinuirano opterećenje q. Odredite promjer zakovica.

3. Zadano:
$\sigma_{ob,\,dop}$ = 260 MPa
τ_{dop} = 100 MPa



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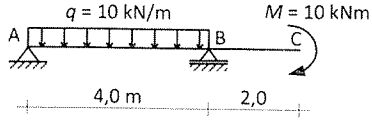
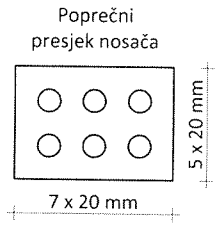
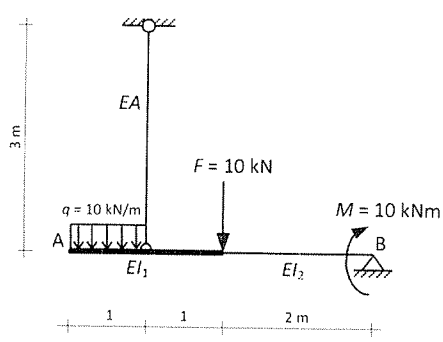
4. Za zadani nosač odredite najveće normalno i posmično naprezanje kritičnim presjecima.

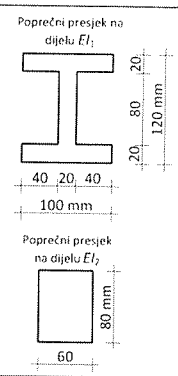




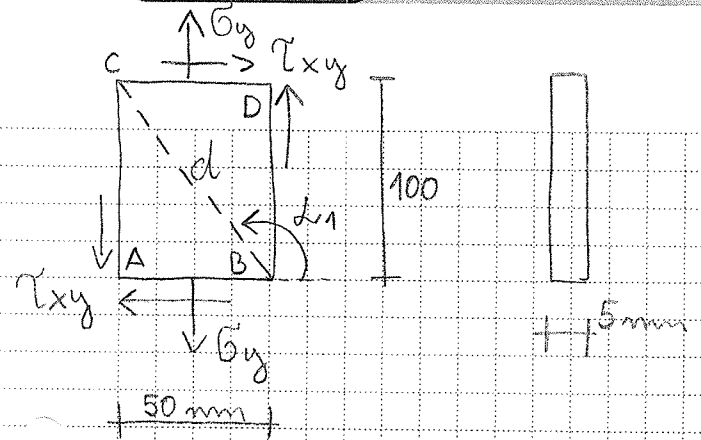
Za sustav prikazan na slici, grafoanalitičkim postupkom odredite ukupni pomak točke A i ukupni kut nagiba presjeka u točki B.

5. Zadano:
$E = 2{,}0 \cdot 10^5$ MPa
A = 100 mm^2





1. ZADATAK



$\sigma_y = 120\ MPa$

$\tau_{xy} = 20\ MPa$

$E = 2 \cdot 10^5\ MPa$

$\nu = 0,3$

$tg\alpha = \frac{50}{100}$, $\alpha = 26,56°$, $\alpha_1 = \alpha + 90° = 116,56°$

$\varepsilon_d = \varepsilon_{xx} \cos^2\alpha_1 + \varepsilon_{yy} \sin^2\alpha_1 + \varepsilon_{xy} \sin 2\alpha_1$

$\varepsilon_{xx} = -\nu \frac{\sigma_{yy}}{E}$ $\quad \varepsilon_{yy} = \frac{\sigma_{yy}}{E}$ $\quad \varepsilon_{xy} = \frac{\tau_{xy}(1+\nu)}{E}$

$\varepsilon_{xx} = -1,8 \cdot 10^{-4}$ $\quad \varepsilon_{yy} = 6 \cdot 10^{-4}$ $\quad \varepsilon_{xy} = 1,3 \cdot 10^{-4}$

$\cos^2\alpha_1 = 0,2$ $\quad \sin^2\alpha_1 = 0,8$ $\quad \sin 2\alpha_1 = -0,8$

$\varepsilon_d = -1,8 \cdot 10^{-4} \cdot 0,2 + 6 \cdot 10^{-4} \cdot 0,8 - 1,3 \cdot 10^{-4} \cdot 0,8 = 3,4 \cdot 10^{-4}$

$\varepsilon_d = \frac{\Delta d}{d}$, $\Delta d = 3,4 \cdot 10^{-4} \cdot \sqrt{100^2 + 50^2} = 380,13 \cdot 10^{-4}\ mm$

$\varepsilon_v = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} = -\frac{\nu \sigma_{yy}}{E} + \frac{\sigma_{yy}}{E} - \frac{\nu \sigma_{yy}}{E} =$

$\varepsilon_v = -1,8 \cdot 10^{-4} + 6 \cdot 10^{-4} - 1,8 \cdot 10^{-4} = 2,4 \cdot 10^{-4}$

$\Delta V = \varepsilon_v \cdot V = 2,4 \cdot 10^{-4} \cdot 100 \cdot 50 \cdot 5 = 6\ mm^3$

$\varepsilon_{zz} = -1,8 \cdot 10^{-4}$

ZADATAK 3

$\sigma_1 = 80\ MPa$

$E_1 = 2 \cdot 10^5\ MPa$

$E_2 = 1{,}8 \cdot 10^5\ MPa$

$t = 5\ mm$

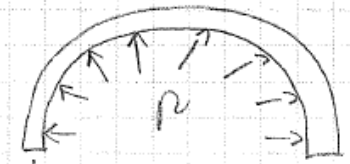
$\alpha_{t2} = 1 \cdot 10^{-5}\ 1/K$

$\Delta = 0{,}2\ mm$

$D = 600\ mm$

$\Delta T_2 = ?$

Prsten 1



$D_1 = D + 2t + \Delta$

$D_1 = 610{,}2\ mm$

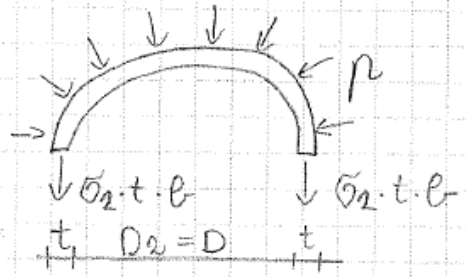
$2\sigma_1 \cdot t \cdot b - p \cdot D_1 \cdot b = 0$

$\sigma_1 = \frac{p \cdot D_1}{2t} \Rightarrow p = \frac{\sigma_1 \cdot 2t}{D_1} = \frac{80 \cdot 2 \cdot 5}{610{,}2}$

$p = 1{,}31\ MPa$

$\varepsilon_1 = \frac{\sigma_1}{E_1} = \frac{p \cdot D_1}{2t \cdot E_1}$

Prsten 2



$$2\sigma_2 \cdot t \cdot b + p \cdot (D + 2t) \cdot b = 0$$

$$\sigma_2 = -\frac{p \cdot (D + 2t)}{2t}$$

$$\varepsilon_2 = \frac{\sigma_2}{E_2} + \alpha_t \Delta T = -\frac{p \cdot (D + 2t)}{2t \cdot E_2} + \alpha_t \Delta T$$

Uvjet deformacije

$\varepsilon_2 = \frac{\Delta}{D_2} + \varepsilon_1 \Rightarrow$

$$-\frac{p \cdot (D + 2t)}{2t \cdot E_2} + \alpha_t \Delta T = \frac{\Delta}{D_2} + \frac{p \cdot D_1}{2t \cdot E_1}$$

$$\Delta T = \frac{\frac{p}{2t} \cdot \left(\frac{D_1}{E_1} + \frac{D + 2t}{E_2}\right) + \frac{\Delta}{D_2}}{\alpha_t} = \frac{\frac{1{,}31}{10} \cdot \left(\frac{610{,}2}{2 \cdot 10^5} + \frac{610}{1{,}8 \cdot 10^5}\right) + \frac{0{,}2}{600}}{1 \cdot 10^{-5}}$$

$$\Delta T = \frac{0{,}131 \cdot (30{,}51 \cdot 10^{-4} + 33{,}89 \cdot 10^{-4}) + 3{,}33 \cdot 10^{-4}}{1 \cdot 10^{-5}} = \boxed{117{,}66\ \mathrm{K}}$$

ZADATAK 3

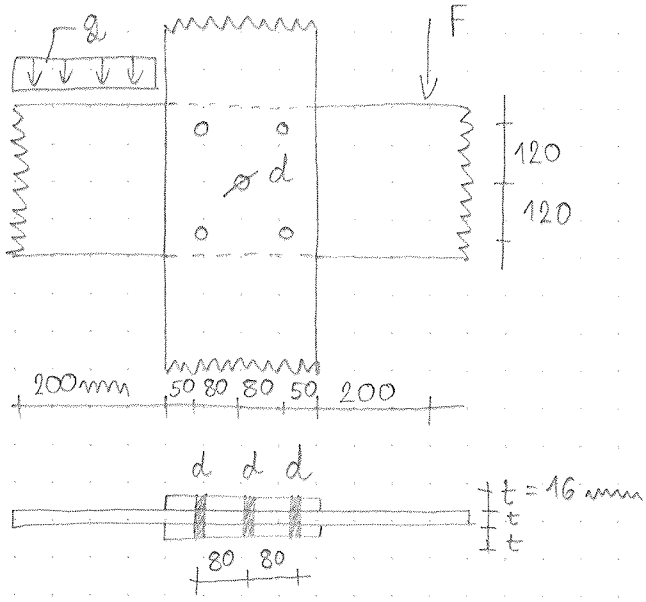
$\tau_{dop} = 100\,MPa$

$\sigma_{ok,dop} = 260\,MPa$

$F = 150\,kN$

$q = 150\,kN/m$

$t = 16\,mm$



$F_{ukupno} = F + q \cdot 0{,}2 = 180\,kN$

$S_V = \frac{F_{ukupno}}{5} = 36\,kN$

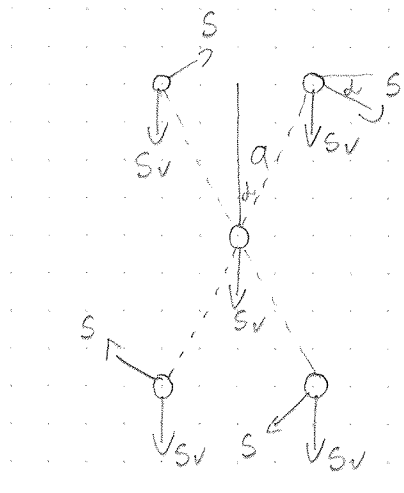
$M = -F \cdot 0{,}33 + q \cdot 0{,}2 \cdot 0{,}23$

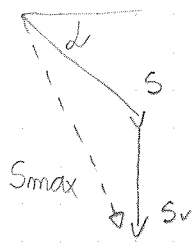
$M = -42{,}6\,kNm$

$a = \sqrt{120^2 + 80^2} = 144{,}22\,mm$

$M = 4 \cdot S \cdot a$

$S = 73{,}85\,kN$





$tg\,d = \frac{80}{120}$

$d = 33{,}69°$

$S_{max} = \sqrt{(S\cos d)^2 + (S\sin d + S_V)^2}$

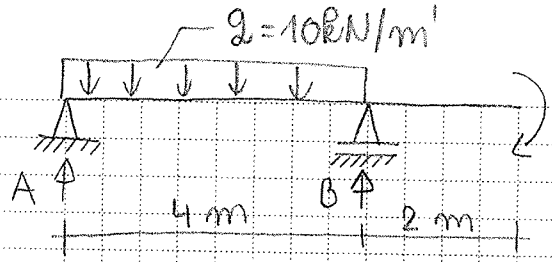
$S_{max} = 98{,}48\,kN$

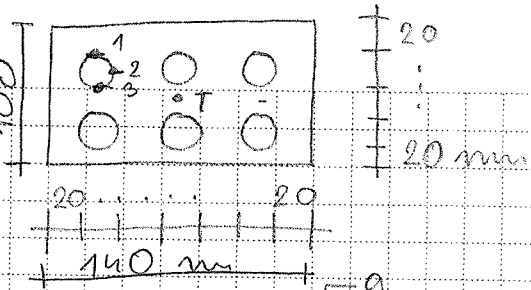
$\tau_{max} = \frac{S_{max} \cdot 4}{2\,d^2\,\pi} \leq \tau_{dop}\ ;\ d \geq \sqrt{\frac{4\,S_{max}}{2\pi\,\tau_{dop}}} \geq 25{,}04\,mm$

MJERODAVNO!

$\sigma_{ok,max} = \frac{S_{max}}{d \cdot t} \leq \sigma_{ok,dop}\ ;\ d \geq \frac{S_{max}}{\sigma_{ok,dop} \cdot t} \geq 23{,}67\,mm$

4. ZADATAK



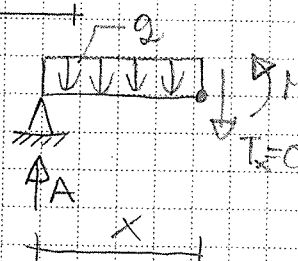


$\Sigma M_A = 0$

$-10 + B \cdot 4 - 10 \cdot 4 \cdot 2 = 0$; $\quad B = 22{,}5\ kN$

$\Sigma M_B = 0$

$-A \cdot 4 + 10 \cdot 4 \cdot 2 - 10 = 0$; $\quad A = +17{,}5\ kN$



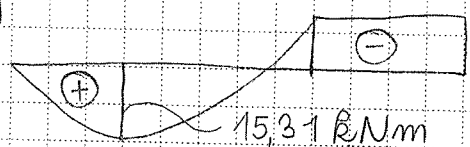
$\Sigma M_x = 0$

$A - q \cdot x = 0$

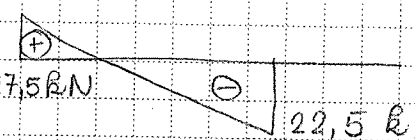
$x = 1{,}75\ m$

$-A \cdot 1{,}75 + 10 \cdot 1{,}75 \cdot \frac{1{,}75}{2} + M_x = 0$; $\quad M_x = 15{,}31\ kNm$

(M)



(T)



$J_y = \frac{140 \cdot 100^3}{12} - 6 \cdot \left(\frac{\pi \cdot 20^4}{64} + \frac{20^2 \cdot \pi}{4} \cdot 20^2 \right) = 11{,}67 \cdot 10^6 -$

$- 6 \cdot 0{,}13 \cdot 10^6 = 10{,}89 \cdot 10^6\ mm^4$

$\sigma_{max\ min} = \frac{M_{max}}{J_y} \cdot z_{max} = \frac{15{,}31 \cdot 10^6}{10{,}89 \cdot 10^6} \cdot 50 = \pm 70{,}30\ MPa$

$S_{y1} = 140 \cdot 20 \cdot 40 = 112\,000\ mm^3$

$S_{y2} = 140 \cdot 30 \cdot 35 - 3 \cdot \frac{20^2 \cdot \pi}{4} \cdot \frac{1}{2} \cdot (10 + 10 + 0{,}424 \cdot 10)$

$S_{y2} = 135\,577{,}17\ mm^3$

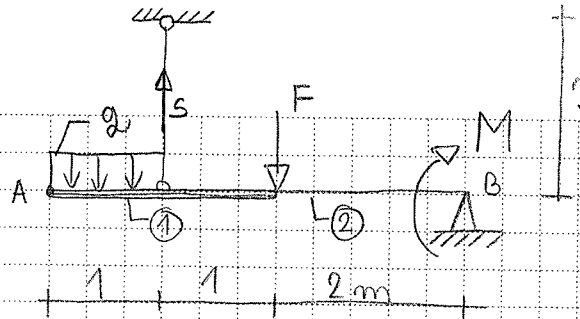
$S_{y3} = 140 \cdot 40 \cdot 30 - 3 \cdot \frac{20^2 \cdot \pi}{4} \cdot 20 = 149\,150{,}44\ mm^3$

$S_{yT} = 140 \cdot 50 \cdot 25 - 3 \cdot \frac{20^2 \cdot \pi}{4} \cdot 20 = 156\,150{,}44\ mm^3$

$\tau_i = \frac{22{,}5 \cdot 10^3 \cdot S_{yi}}{10{,}89 \cdot 10^6 \cdot e_i}$ $\quad (e_1 = 140\ mm,\ e_2 = 80\ mm,\ e_3 = 140\ mm,\ e_T = 140\ mm)$

$\tau_1 = 1{,}65\ MPa$; $\quad \tau_2 = 3{,}5\ MPa$; $\quad \tau_3 = 2{,}2\ MPa$; $\quad \tau_T = 2{,}3\ MPa$

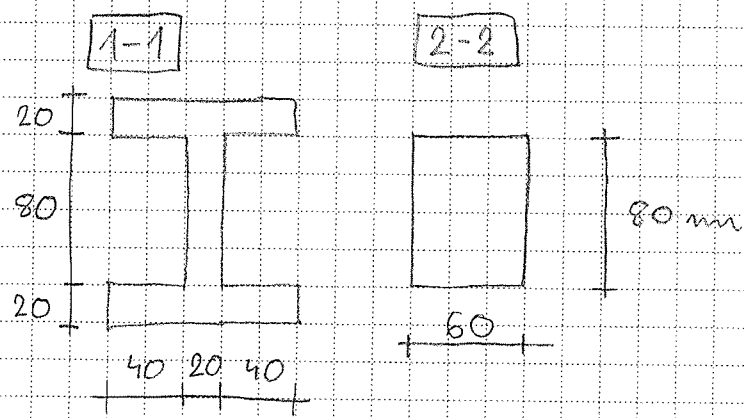
5. ZADATAK



$q = 10\ kN/m'$ $E = 2 \cdot 10^5\ MPa$

$F = 10\ kN$ $A = 100\ mm^2$

$M = 10\ kNm$ $w_A, \varphi_B = ?$



$\Sigma M_B = 0$

$10 \cdot 3{,}5 + 10 \cdot 2 - 10 - S \cdot 3 = 0$

$S = 15\ kN$

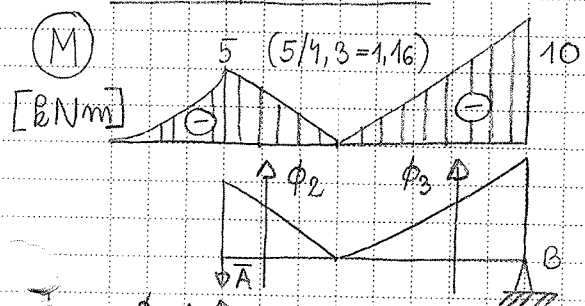
$\Delta h = \frac{15 \cdot 10^3 \cdot 3 \cdot 10^3}{2 \cdot 10^5 \cdot 100}$

$\Delta h = 2{,}25\ mm$

$J_{y1} = \frac{100 \cdot 120^3}{12} - \left(\frac{40 \cdot 80^3}{12}\right) \cdot 2$

$J_{y1} = 10{,}99 \cdot 10^6\ mm^4$

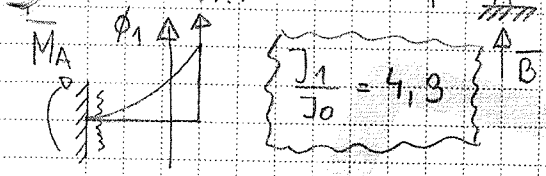
$J_{y2} = \frac{60 \cdot 80^3}{12} = 2{,}56 \cdot 10^6\ mm^4 = J_0$



$\varphi_1 = \frac{1}{3} \cdot 1 \cdot 1{,}16 = 0{,}39\ kN\,m^2$

$\varphi_2 = 0{,}5 \cdot 1 \cdot 1{,}16 = 0{,}58\ kNm^2$

$\varphi_3 = 0{,}5 \cdot 10 \cdot 2 = 10\ kNm^2$



$\Sigma \bar{M}_B = 0$

$\bar{A} \cdot 3 - \varphi_2 \cdot 2{,}67 - \varphi_3 \cdot 0{,}67 = 0$, $\bar{A} = 2{,}75\ kN\,m^2$

$\Sigma \bar{M}_{AL} = 0$; $A \cdot 1 + \varphi_1 \cdot 0{,}75 - \bar{M}_A = 0$; $\bar{M}_A = 3{,}04\ kNm^3$; $w_A' = 5{,}94\ mm$

$\Sigma \bar{M}_{štap} = 0$; $\varphi_2 \cdot 0{,}33 + \varphi_3 \cdot 2{,}33 + \bar{B} \cdot 3 = 0$; $\bar{B} = -7{,}83\ kNm^2$

$\frac{\Delta h}{3} = \frac{\delta_A}{4}$, $\delta_A = 3\ mm$ $\Rightarrow$ $w_{Aub} = w_A' + \delta_A = 8{,}94\ mm$

$\varphi_B'' = \frac{\Delta R}{3000} = -7{,}5 \cdot 10^{-4}$ $\Rightarrow$ $\varphi_{Bub} = \varphi_B' + \varphi_B'' = 0{,}014$

$\varphi_B' = 0{,}015\ rad$