Pismeni ispit iz "OTPORNOSTI MATERIJALA 1"

SVEUČILIŠTE U ZAGREBU
GRAĐEVINSKI FAKULTET
ZAVOD ZA TEHNIČKU MEHANIKU

06. srpnja 2018.

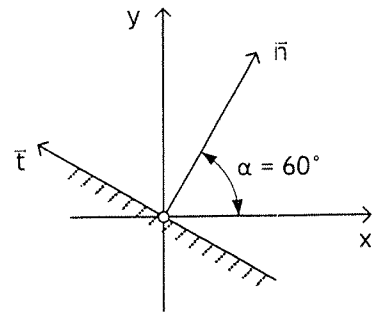
Ime i prezime:

1.] Zadane su komponente relativnih deformacija u ravnini prema slici:

$\varepsilon_x = 2 \cdot 10^{-4}$
$\varepsilon_y = 3,5 \cdot 10^{-4}$
$\varepsilon_{xy} = 1,8 \cdot 10^{-4}$

Odredite totalno naprezanje u ravnini sa zadanom normalom „n" te promjenu pravog kuta između osi „n" i „t".

$E = 2 \cdot 10^5$ MPa, $\nu = 0,3$

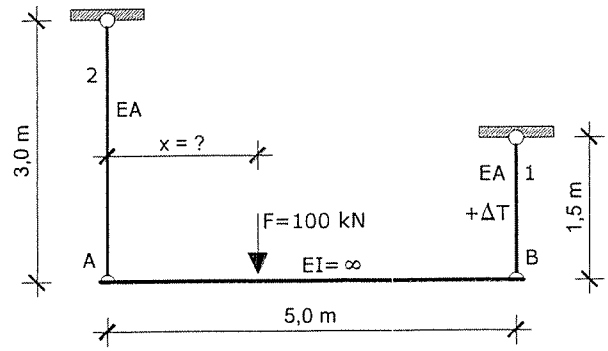


2.] Odredite:

A] Položaj sile F (vrijednost x) tako da su vertikalni pomaci točaka A i B jednaki.

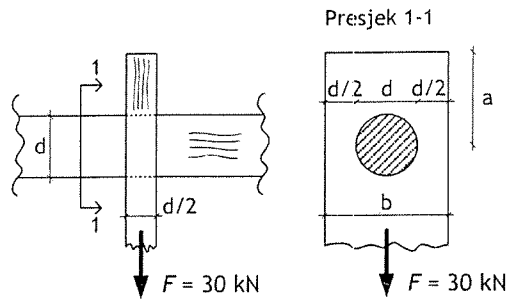
B] Položaj sile F tako da su naprezanja u štapovima 1 i 2 jednaka. Za određeni položaj odredi pomak hvatišta sile F.

$E = 2,1 \cdot 10^5$ MPa;
$\alpha_T = 1,25 \cdot 10^{-5}\ K^{-1}$;
$A = 900\ mm^2$;
$\Delta T = 40$ K.



3.] Dimenzionirajte spoj dvaju elemenata prikazan na slici (odredite „d", „a" i „b").

$\sigma^{\parallel}_{p,dop} = 5$ MPa
$\sigma^{\perp}_{p,dop} = 2$ MPa
$\sigma^{\parallel}_{v,dop} = 7$ MPa
$\tau^{\parallel}_{dop} = 1$ MPa
$\tau^{\perp}_{dop} = 2$ MPa



4.] Za nosač prikazan na slici treba odrediti:

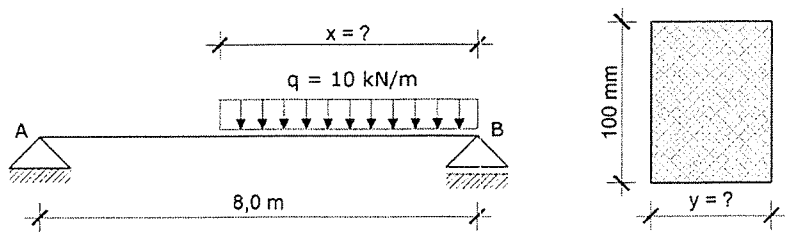
A] Duljinu kontinuiranog opterećenja (x na skici) tako da je veličina reakcije na ležaju A dvostruko manja nego reakcija na ležaju B.

$2R_A = R_B$

B] Za dobivenu duljinu opterećenja (x) potrebno je odrediti minimalnu širinu poprečnog presjeka (y na skici) ako je zadano:

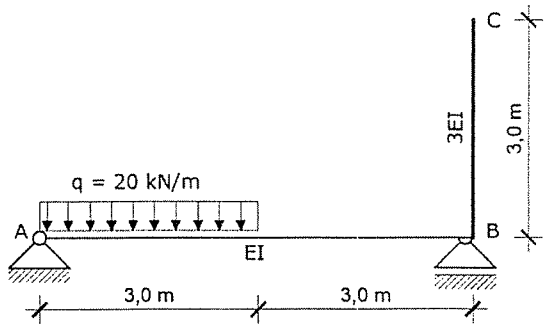
$\tau_{dop} = 10$ MPa

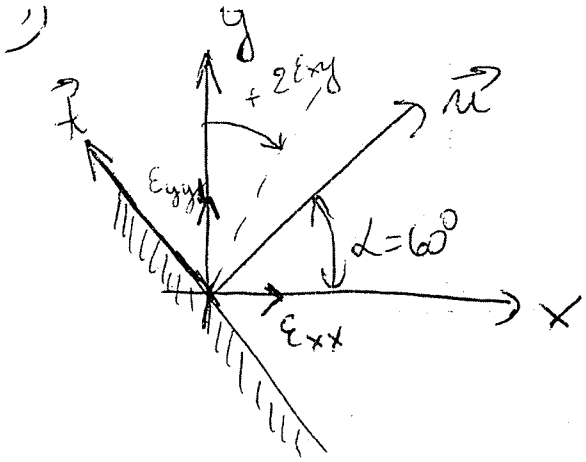
C] Za dobivene x i y odrediti i skicirati dijagrame maksimalnih normalnih i posmičnih naprezanja.



5.] Za zadani statički sustav treba **GRAFOANALITIČKIM POSTUPKOM** odrediti ukupni pomak i kut zaokreta točke C.

$E = 2 \cdot 10^5$ MPa;
$I = 8 \cdot 10^7$ mm^4.





$\varepsilon_{xx} = 2 \cdot 10^{-4}$

$\varepsilon_{yy} = 3{,}5 \cdot 10^{-4}$

$\varepsilon_{xy} = 1{,}8 \cdot 10^{-4}$

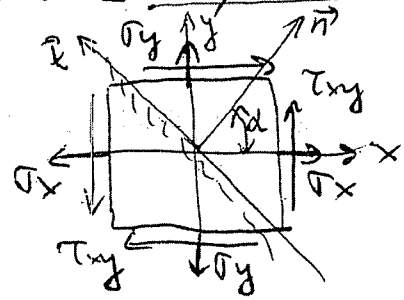
$E = 2 \cdot 10^5$ MPa

$\nu = 0{,}30$

$$\sigma_x = \frac{E}{1-\nu^2}(\varepsilon_{xx} + \nu\varepsilon_{yy}) = \frac{2 \cdot 10^5}{1-0{,}3^2}(2 + 0{,}3 \cdot 3{,}5) \cdot 10^{-4} = 67{,}03 \text{ MPa}$$

$$\sigma_y = \frac{E}{1-\nu^2}(\varepsilon_{yy} + \nu\varepsilon_{xx}) = \frac{2 \cdot 10^5}{1-0{,}3^2}(3{,}5 + 0{,}3 \cdot 2) \cdot 10^{-4} = 90{,}11 \text{ MPa}$$

$$\tau_{xy} = 2G \cdot \varepsilon_{xy} = \frac{E}{1+\nu}\varepsilon_{xy} = \frac{2 \cdot 10^5}{1+0{,}3} \cdot 1{,}8 \cdot 10^{-4} = 27{,}69 \text{ MPa}$$



$$\left(G = \frac{E}{2(1+\nu)} = \frac{2 \cdot 10^5}{2 \cdot (1+0{,}3)} = 0{,}7692 \cdot 10^5 \text{ MPa}\right)$$

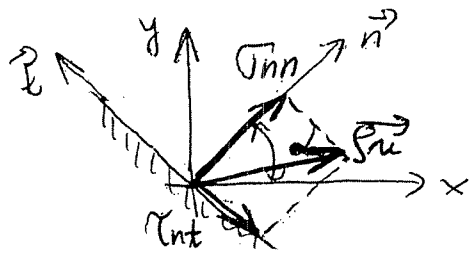
$$\sigma_{nn} = \sigma_x \cos^2\alpha + \sigma_y \sin^2\alpha + \tau_{xy} \sin 2\alpha = 67{,}03 \cos^2 60° + 90{,}11 \cdot \sin^2 60° +$$
$$+ 27{,}69 \cdot \sin(2 \cdot 60°) = 16{,}7575 + 67{,}5825 + 23{,}98 = 108{,}32 \text{ MPa}$$

$$\tau_{nt} = \frac{\sigma_y - \sigma_x}{2} \sin 2\alpha + \tau_{xy} \cos 2\alpha$$

$$\tau_{nt} = \frac{90{,}11 - 67{,}03}{2} \cdot \sin(2 \cdot 60°) + 27{,}69 \cdot \cos(2 \cdot 60°) = 11{,}54 \cdot 0{,}866 + 27{,}69 \cdot (-0$$

$$\tau_{nt} = 9{,}99 - 13{,}845 = -3{,}855 \text{ MPa}$$

$$\sigma_u = \sqrt{\sigma_{nn}^2 + \tau_{nt}^2} = \sqrt{(108{,}32)^2 + (-3{,}855)^2} = \sqrt{11733{,}22 + 14{,}86} = 108{,}388 \text{ MPa}$$



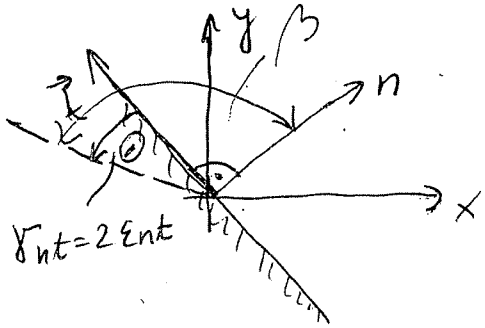
Promjena pravoga kuta između nit:

$\gamma_{nt} = 2\varepsilon_{nt}$; $\underline{\varepsilon_{nt} = \frac{\tau_{nt}}{2G} = -\frac{3,855}{2 \cdot 0,7692 \cdot 10^5} = -2,5 \cdot 10^{-5}}$

$\left[\varepsilon_{nt} = \frac{1+\nu}{E} \tau_{nt} = \frac{1+0,30}{2 \cdot 10^5} \cdot (-3,855) = -2,5 \cdot 10^{-5} \right]$

$\underline{\gamma_{nt} = 2 \cdot (-2,5 \cdot 10^{-5}) = -5 \cdot 10^{-5} = -0,00005 \text{ rad}}$

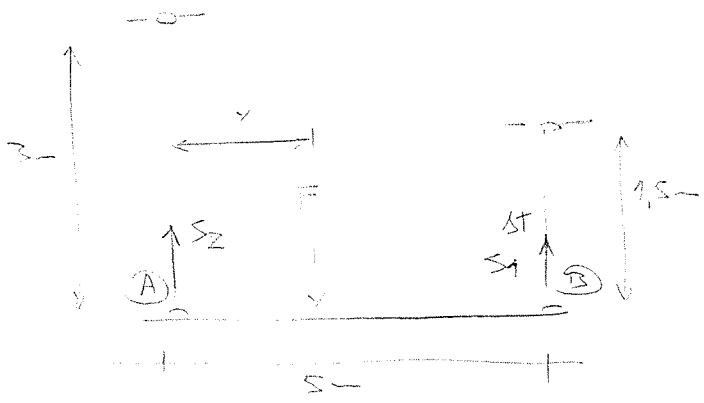
(povećanje pravog kuta između nit)



$\varphi^\circ = \frac{180^\circ}{\pi} \cdot 0,00005 = 0,00286^\circ = 0^\circ 0' 10''$

$\beta = 90^\circ + \varphi^\circ = 90,00286^\circ = 90^\circ 0' 10''$

2. ZADATAK



$\Sigma F_x = 0$

$S_1 + S_2 = F$

$\Sigma M_A = 0$

$35,158 \cdot 5 = 100 x$

$x = 1,75795$ m

A $\quad \delta_A = \delta_B$

$\delta_A = \frac{S_2 \cdot L_2}{EA}$

$\delta_B = \frac{S_1 L_1}{EA} + \alpha_T \Delta T \cdot L_1$

$\frac{S_2 L_2}{EA} = \frac{S_1 L_1}{EA} + \alpha_T \Delta T L_1$ (1)

$1,5573 \cdot 10^{-5} S_2 = 7,9365 \cdot 10^{-6} S_1 + 0,175$

$1,5873 \cdot 10^{-5} F - 1,5873 \cdot 10^{-5} S_1 = 7,9365 \cdot 10^{-6} S_1 + 0,175$

$2,38065 \cdot 10^{-5} S_1 = 0,837$

$S_1 = 35,158$ kN

$S_2 = 64,842$ kN

B $\quad \sigma_1 = \sigma_2 \rightarrow S_1 = S_2 = \frac{F}{2} = 50$ kN

$\delta_A = \frac{50 \cdot 10^3 \cdot 3000}{EA} = 0,79$ mm

$\delta_B = \frac{50 \cdot 10^3 \cdot 1500}{EA} + \alpha_T \Delta T L = 0,175 + 0,397 = 1,147$ mm

$\delta_F = \frac{\delta_A + \delta_B}{2} = 0,97$ mm

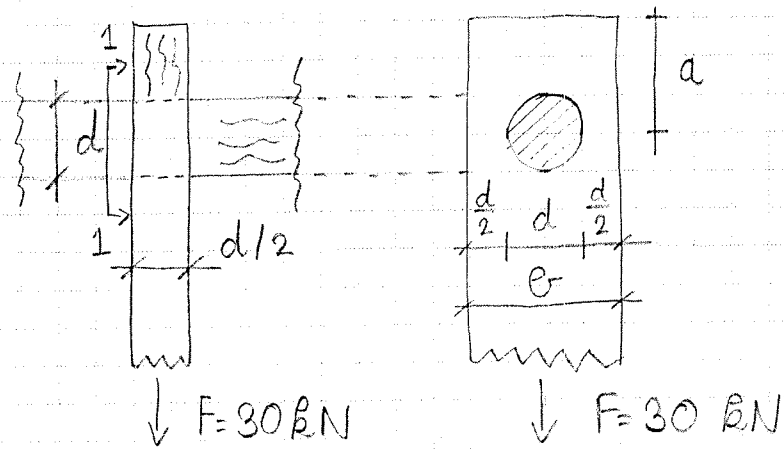
$\sigma''_{p,dop} = 5\ MPa$

$\sigma''_{v,dop} = 7\ MPa$

$\tau''_{dop} = 1\ MPa$

$\tau^{\perp}_{dop} = 2\ MPa$

$\sigma^{\perp}_{p,dop} = 2\ MPa$



$$\sigma^{\perp}_{p,max} = \frac{F}{d \cdot \frac{d}{2}} \leq \sigma^{\perp}_{p,dop} \Rightarrow \underline{d \geq 173{,}2\ mm}$$ Mjerodavno!

$\underline{d = 175 mm}$

$$\sigma''_{p,max} = \frac{F}{d \cdot \frac{d}{2}} \leq \sigma''_{p,dop} \Rightarrow \underline{d \geq 109{,}5\ mm}$$

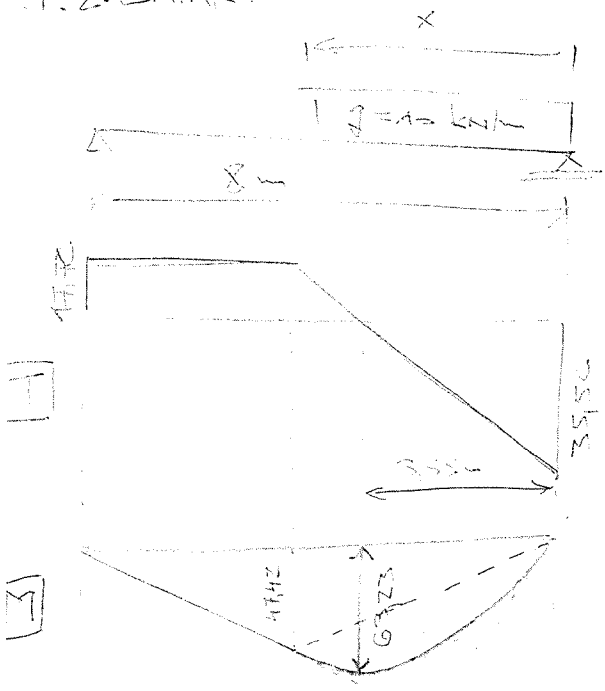
$$\sigma''_{v,max} = \frac{F}{d \cdot \frac{d}{2}} \leq \sigma''_{v,dop} \Rightarrow \underline{d \geq 92{,}6\ mm}$$

$$\tau''_{max} = \frac{F}{2 \cdot a \cdot \frac{d}{2}} \leq \tau''_{dop} \Rightarrow \underline{a \geq 171{,}4\ mm}$$ Ova mjerodavna se nadi na kraju, nakon što se odredi "d".

$$\tau^{\perp}_{max} = \frac{F}{2 \cdot \frac{d^2 \pi}{4_2}} \leq \tau^{\perp}_{dop} \Rightarrow \underline{d \geq 97{,}7\ mm}$$

$e = 2 \cdot d \Rightarrow \underline{e = 350\ mm}$

4. ZADATAK:



$\sum M_A = 0$

$\sum M_B = 0$

$Z_A \cdot 8 = \frac{10 \cdot 8^2}{2}$ (2)

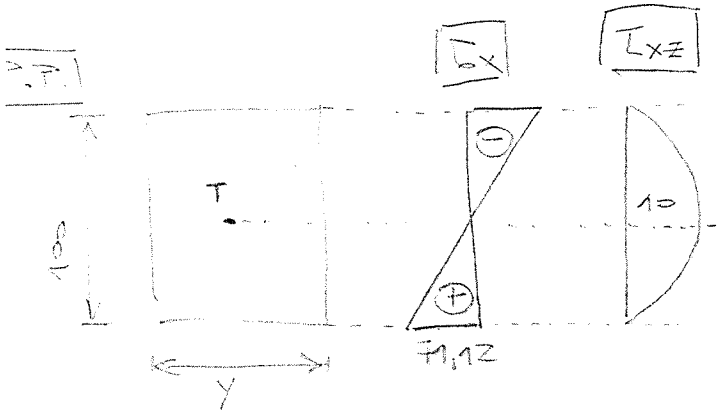
(1) → (2) $\frac{x \cdot 10x}{2} = \frac{10x^2}{2}$ $\rightarrow$ $x = 5,33$ m

$Z_A = 17,78$ kN

$Z_B = 35,56$ kN

$M_{MAX} = 63,23$ kNm

$T_{MAX} = 35,56$ kN



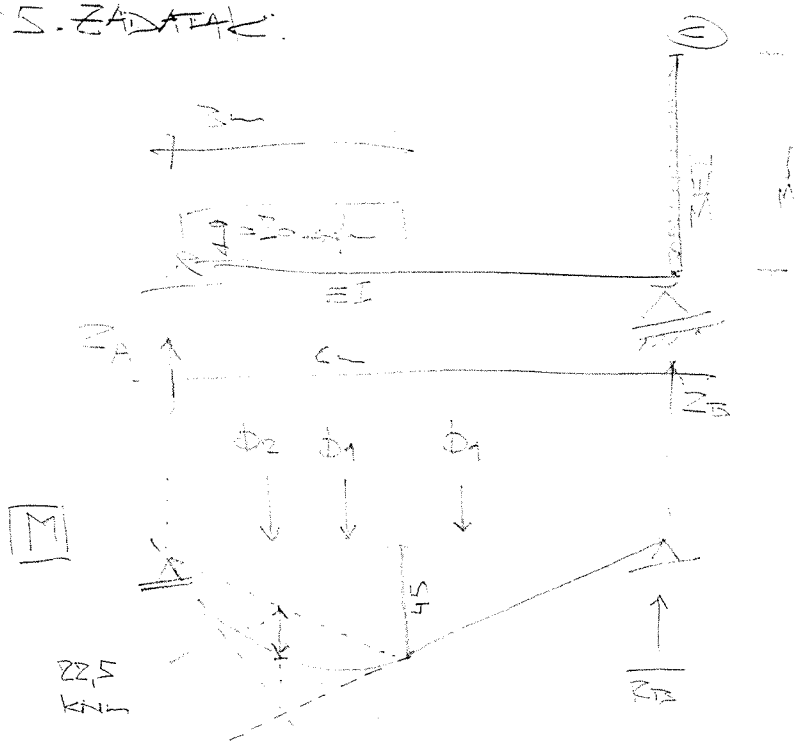
$\frac{T_z \cdot S_y}{I_y \cdot b} = 10 \rightarrow \frac{35,56 \cdot 50 \cdot 25 \cdot y \cdot 10^3}{y \cdot \frac{100^3}{12} \cdot y} = 10$ $\rightarrow$ $y = 53,34$ mm

$I_y = 4,445 \cdot 10^6$ mm^4

$\sigma_{xMAX} = \frac{63,23 \cdot 10^6}{4,445 \cdot 10^6} \cdot 50$

$\sigma_{xMAX} = 71,12$ MPa

5. ZADATAK



$\sum M_A = 0$

$20 \cdot 3 \cdot 1{,}5 = R_B \cdot 6$

$R_B = 15\ kN$

$R_A = 45\ kN$

$\frac{qL^2}{8} = 22{,}5$

$\Phi_1 = \frac{45 \cdot 3}{2} = 67{,}5\ kNm^2$

$\Phi_2 = 22{,}5 \cdot 3 \cdot \frac{2}{3} = 45\ kNm^2$

$\sum \overline{M}_A = 0$

$\overline{F}_B \cdot 6 - \Phi_1 \cdot 4 - \Phi_1 \cdot 2 - \Phi_2 \cdot 1{,}5 = 0$

$\overline{F}_B = 78{,}75\ kNm^2 \Rightarrow \overline{T}_3 = -\overline{R}_B$

$\varphi_B = \frac{\overline{T}_3}{EI} = \frac{-78{,}75 \cdot 10^3}{2 \cdot 10^5 \cdot 5 \cdot 10^7} = -0{,}00393219\ rad$ $\qquad \varphi_{13} = \varphi_C$

$\delta_C = \varphi_C \cdot 3000$

$\delta_C = 14{,}765\ mm$