

Novel, Efficient Iterative Procedure for the Structural Analysis - Generalisation of Modern Methods

key words: fast iterative method, Ritz method, sparse matrix storage, accuracy estimate, generalization of iterative methods

Summary: The idea of the project is to develop a fast iterative method for structural analysis which is, in the case of equal memory usage, faster and for larger memory usage, much faster than modern direct and iterative solvers. The procedure that we propose has additional importance, because the majority of modern iterative methods can be represented as a special case of our procedure. Therefore, it can be considered as a generalization of most modern iterative methods. In brief, it is actually an iterative procedure in which at each iteration step, discrete Ritz method is applied. At every step, coordinate vectors which form a subspace are generated, within which local energy minimum is sought, thus decreasing the total energy of the system, which therefore converges to the required minimum. The number of coordinate vectors (subspace dimension) is not limited, but aims to be small - much smaller than the number of unknowns. How to generate a good subspace - coordinate vectors is the central problem the project needs to address. Basically, with the method proposed it is possible to combine the good properties of several iterative methods simultaneously. Furthermore, every new vector generation procedure opens the possibility to fasten our method. Additionally, the property of conjugacy which underlies some iterative procedures and which is valid only in linear problems, in our case is not absolutely necessary. Therefore, the method can be successfully applied in nonlinear calculations in which the conjugacy is not even defined. This fact is really important, because in the case of nonlinear practical applications, iterative methods are used exclusively. Finally, applying our original method based on integer arithmetic, the exact solution of appropriate, practical, realistic (not just simple, benchmark) examples can be determined, so we can evaluate the convergence, stability and accuracy of any numerical procedure, including the method proposed.

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With continued intensive development and application of computing in all areas of science and technology, the need for efficient methods in solving large systems of algebraic equations is of crucial importance. In this research field, researchers aim to achieve as high speed of solution as possible and as small as possible storage requirement on the computer's memory. Principal ambition of the proposed project is to formulate and implement a novel fast iterative solver for structural analysis which is (in the case of the same demand on memory) faster than any modern direct and iterative solver and for the larger memory usage (due to the need for additional model information) much faster than any other solver to date. The proposed procedure comprises an additional importance in so far that most modern iterative solvers can be represented or interpreted as special cases of the proposed algorithm. Therefore, the proposed formulation can be considered as a generalisation of majority of iterative methods. This proposal represents a significant extension of an earlier concept for an iterative procedure, within which the discrete Ritz method is applied at each iteration step. At every step, suitable coordinate vectors are generated which form a subspace, within which the local energy minimum is sought, thus decreasing the total energy of the system and the procedure therefore eventually converges to the required minimum. The number of coordinate vectors (subspace dimension) is not limited, but aims to be small - much smaller than the number of unknowns. The

central problem the proposed project needs to address is how to generate coordinate vectors for a good quality subspace. With the proposed method it is possible to combine good properties of several iterative methods simultaneously. Furthermore, the formulation allows for any new coordinate vectors generation procedure (suggested by other independent researchers) to be easily implemented to make the proposed method potentially even faster. Moreover, the conjugacy property which underlies some iterative procedures and which is valid only for linear problems is not strictly necessary in the proposed procedure. Therefore, the method can be successfully applied to nonlinear problems as well, where the concept of conjugacy is not even defined. This issue is vitally important for nonlinear problems and practical applications, as in these cases iterative methods are used exclusively. The main project idea is to develop an algorithm for the fast and efficient generation of coordinate vectors which form a good subspace, so that the solution is obtained using a small number of fast iterations. The intention is to use as small number of coordinate vectors as possible (up to 6), which can be quickly generated within a single iteration. During this process, different structural model data can be additionally used: e.g. topological properties of the model and the corresponding finite element mesh or nodal residuals or some other suitable model property.

It is important to emphasize that within the proposed method it is possible to combine good properties of all known iterative methods simultaneously. Simply, one or more coordinate vectors represent each of the methods selected for the use in the computing process of creating a discrete Ritz subspace. Furthermore, by the application of integer arithmetics, the exact solution for a large scale test problem (representative of a practical structural engineering problem, not just a simple small scale benchmark) can be determined, so that an evaluation of the convergence, stability and accuracy of any numerical solution procedure, including the method proposed, can be conducted. Regarding the implementation, the aim is to utilise the FEAP 8.4 environment, the high quality open source research software package for the finite element analysis written in Fortran - which is very popular with many advanced mechanics research groups around the world (main developer prof. R. Taylor from the University of California at Berkeley) and implement a high quality sparse storage scheme together with the proposed iterative method. Thus, the analyses results will be directly comparable with the results obtained by all methods available within the FEAP package. In addition, it will also be possible to make comparisons with solutions obtained by using other popular programs for structural mechanics. In doing so, a number of very large size benchmark problems (with approximately fifty million or more unknowns) will be analysed. Finally, in order to assess the solution accuracy, the procedures which employ exact arithmetics, will be written in Mathematica and Sage, which both support symbolic algebra calculations.