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Some examples of static equivalency in space using descriptive geometry and Grassmann algebra

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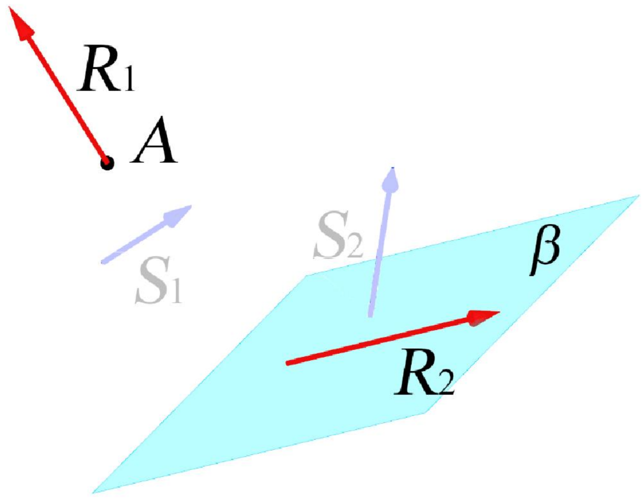
CREATIVITY IN STRUCTURAL DESIGN

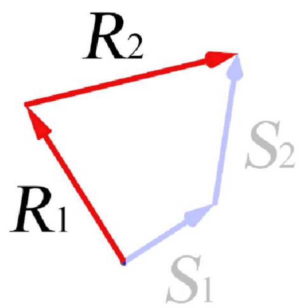
The problem – static equivalency

- In many static problems, it is convenient to replace existing force system with another, usually simpler, statically equivalent force system.

Form diagram

Force diagram





Content:

1. The idea
2. Why we use descriptive geometry, Grassmann algebra and Plücker coordinates
3. Examples
4. Future work

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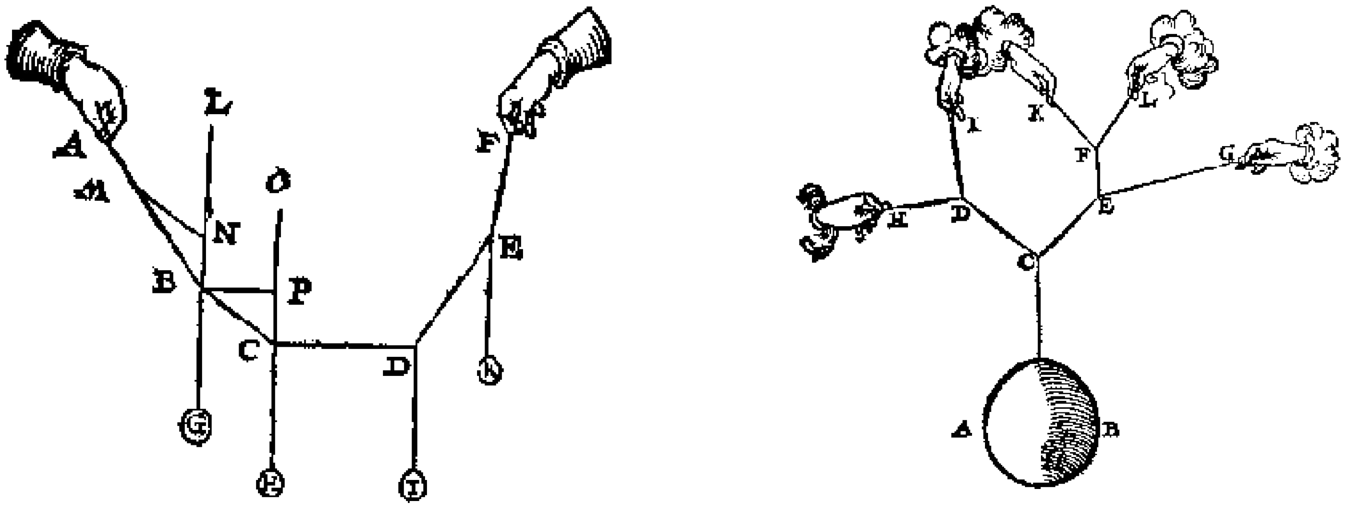
1. The idea for geometric constructions



- Ropes stretched by applied forces, Stevin [1608]

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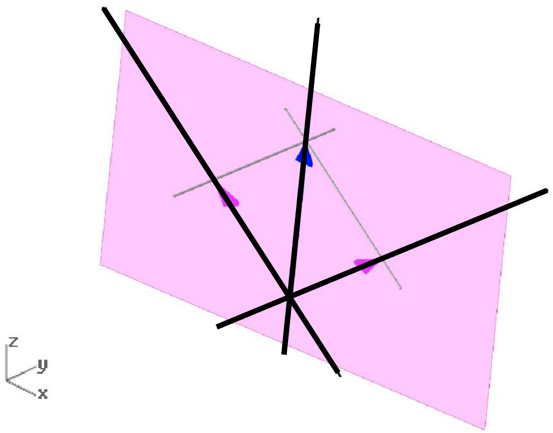
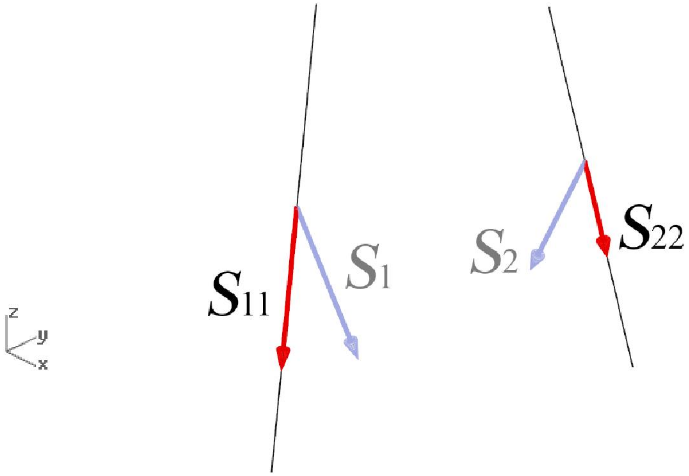
The idea - The geometric construction is based on two principles:

1. principle:



2. principle:



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2. Why we use descriptive geometry, Grassmann algebra and Plücker coordinates

- **Descriptive geometry** emphasizes “visual thinking” – essential for graphic statics.
- **Grassman algebra** translates operations of descriptive geometry into algebraic expressions and thereafter into a program code.
- **Grassmann coordinates** uniformly manage points, lines and planes in space.
- **Plücker coordinates** are a special case of Grassmann coordinates.

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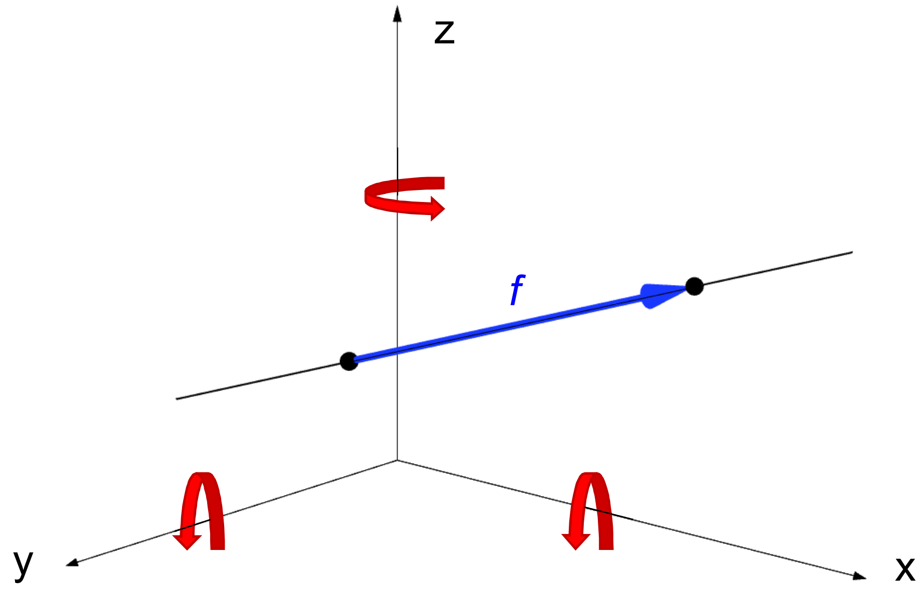
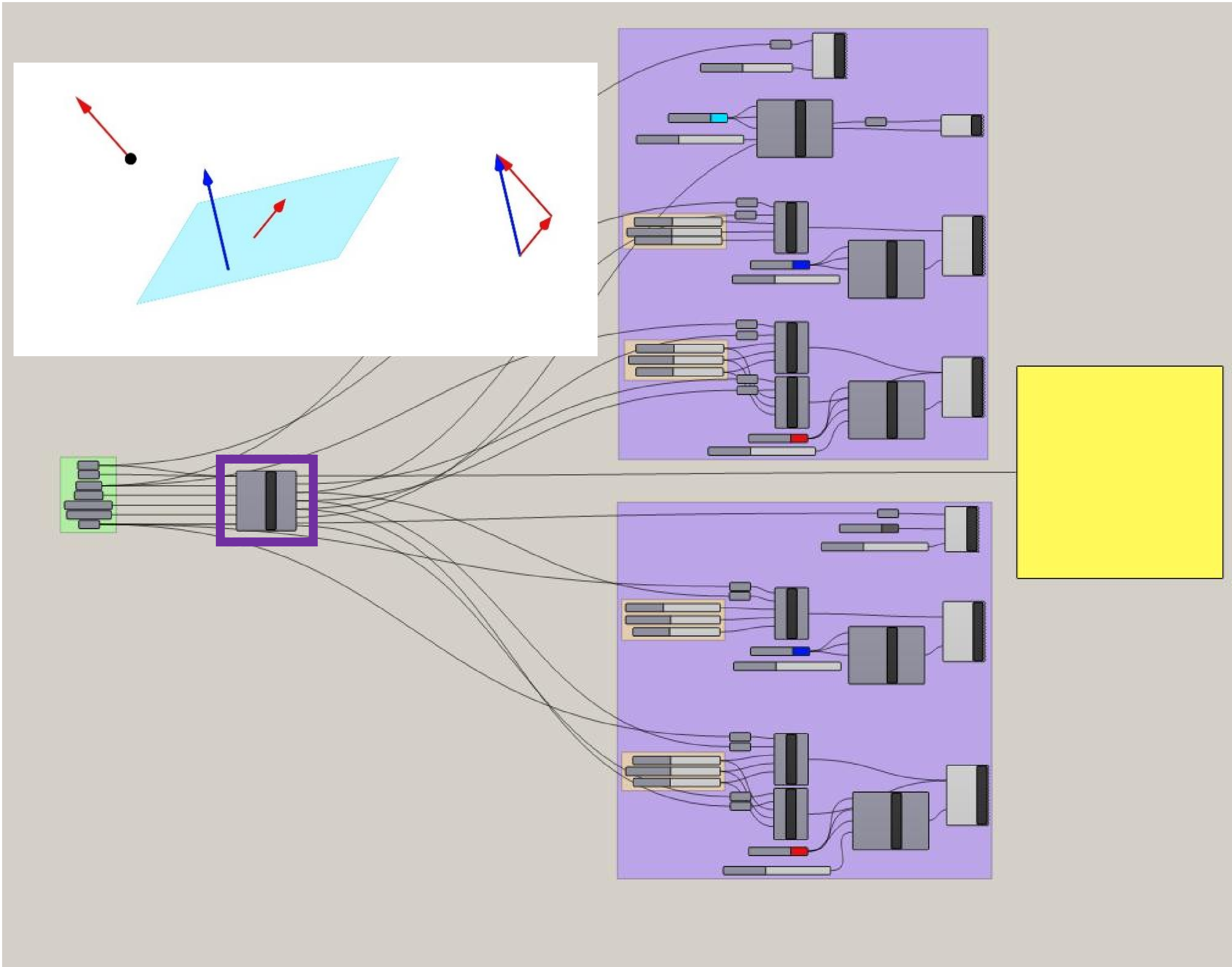
Lines and forces expressed with Plücker coordinates



Line as a span of two points: $l = [P_1\ P_2] = (\underbrace{l_{01}, l_{02}, l_{03}}_{l_0}, \underbrace{l_{23}, l_{31}, l_{12}}_{\bar{l}}) \rightarrow l_{ij} = x_i\, y_j - x_j\, y_i$

Force expressed with six coordinates: $F = [P_1\ P_2] = (\underbrace{f_{01}, f_{02}, f_{03}}_{f}, \underbrace{f_{23}, f_{31}, f_{12}}_{m}) \rightarrow f \cdot m = 0$

Rhinoceros / Grasshopper / GhPython



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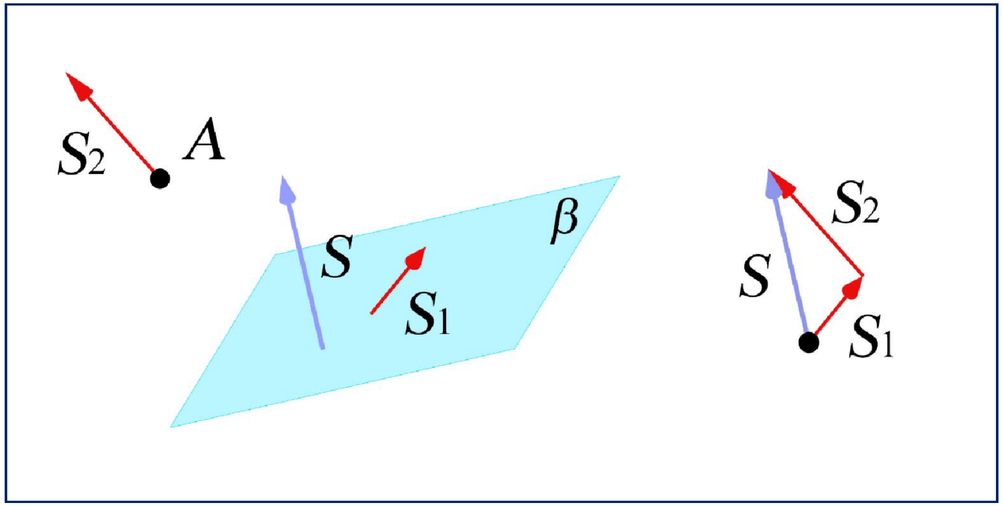
3. Example 1 :

- Replacing a single force with a force acting at the given point A and a force lying in the given plane β

Form diagram   Force diagram



Form diagram   Force diagram

$A = (a_0, a_1, a_2, a_3)$

$\beta = (\beta_0, \beta_1, \beta_2, \beta_3)$

$S = (s_{01}, s_{02}, s_{03}, s_{23}, s_{31}, s_{12}) = (s_0, \bar{s})$

$B = O + s_0$

$\sigma = A \wedge s$

$r = \sigma \vee \beta = (r_0, r) \rightarrow r' = O + r_0$

$P = \beta \vee s$

$p = A \wedge P = (p_0, p) \rightarrow p' = B + p_0$

$C = r' \vee p'$

⬇

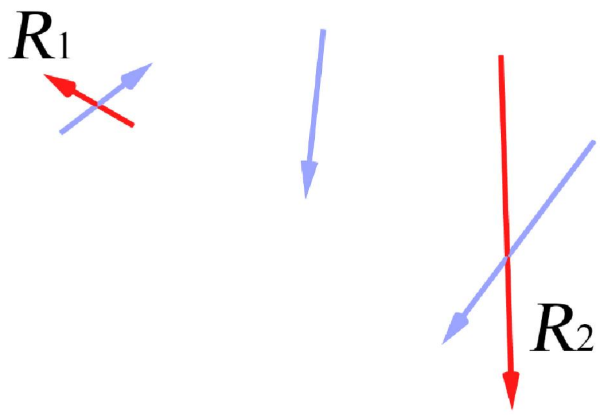
$S_1 = O \wedge C$

$S_2 = C \wedge B$

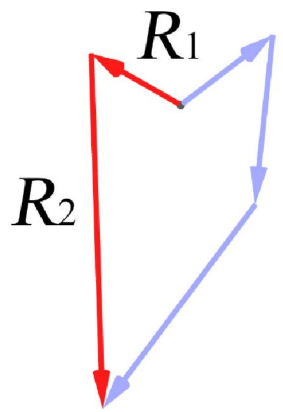
3. Example 2 :

- Replacing three skew forces with two forces acting along conjugate lines

Form diagram



Force diagram



3. Example 3 :

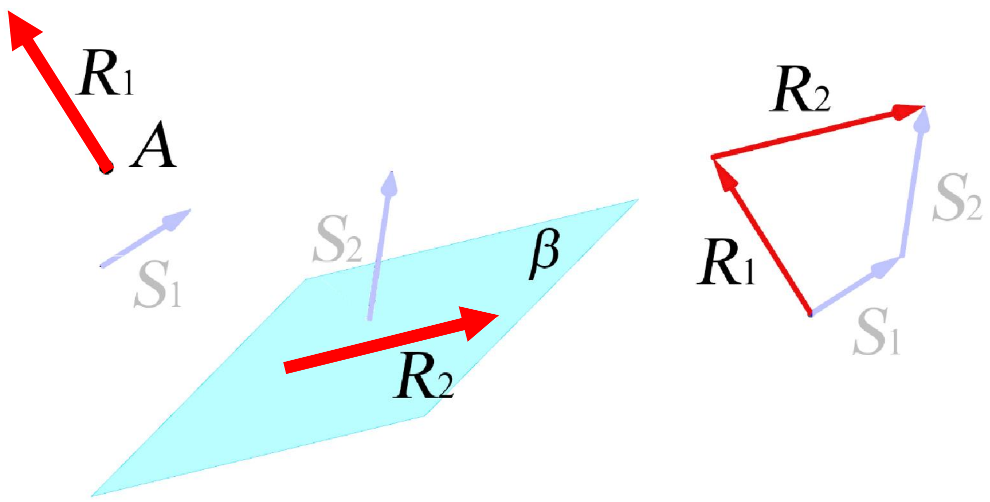
- Replacing two forces with a force acting at given point A and a force lying in given plane β

Two special cases:

1) $A \rightarrow$ □
2) $\beta \rightarrow$ □

Form diagram

Force diagram



3. Example 3 :

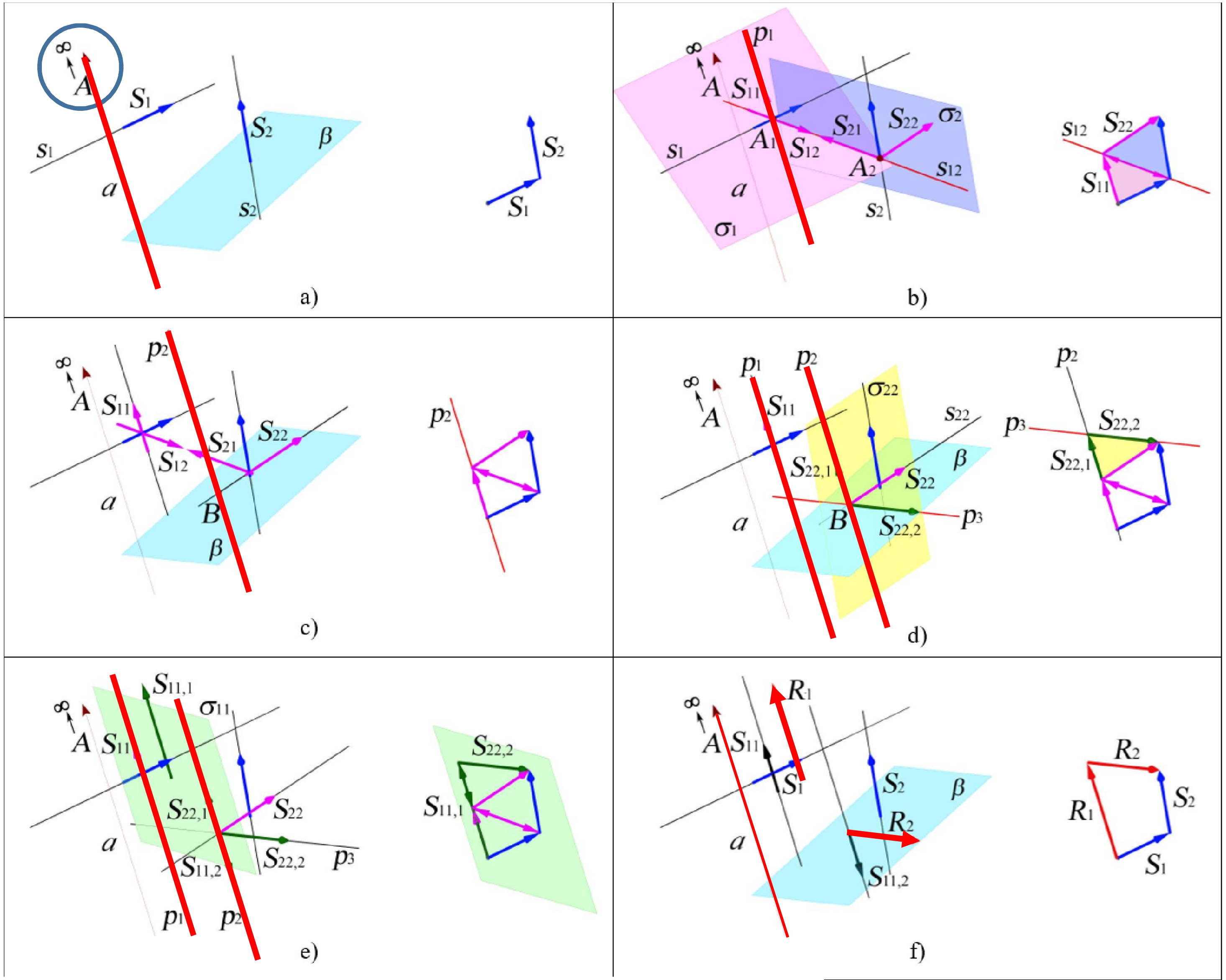
First special case –

A is point at infinity

$A = (0, a_1, a_2, a_3)$

$a = (a_1, a_2, a_3, 0, 0, 0)$

$a \parallel p_1 \parallel p_2$

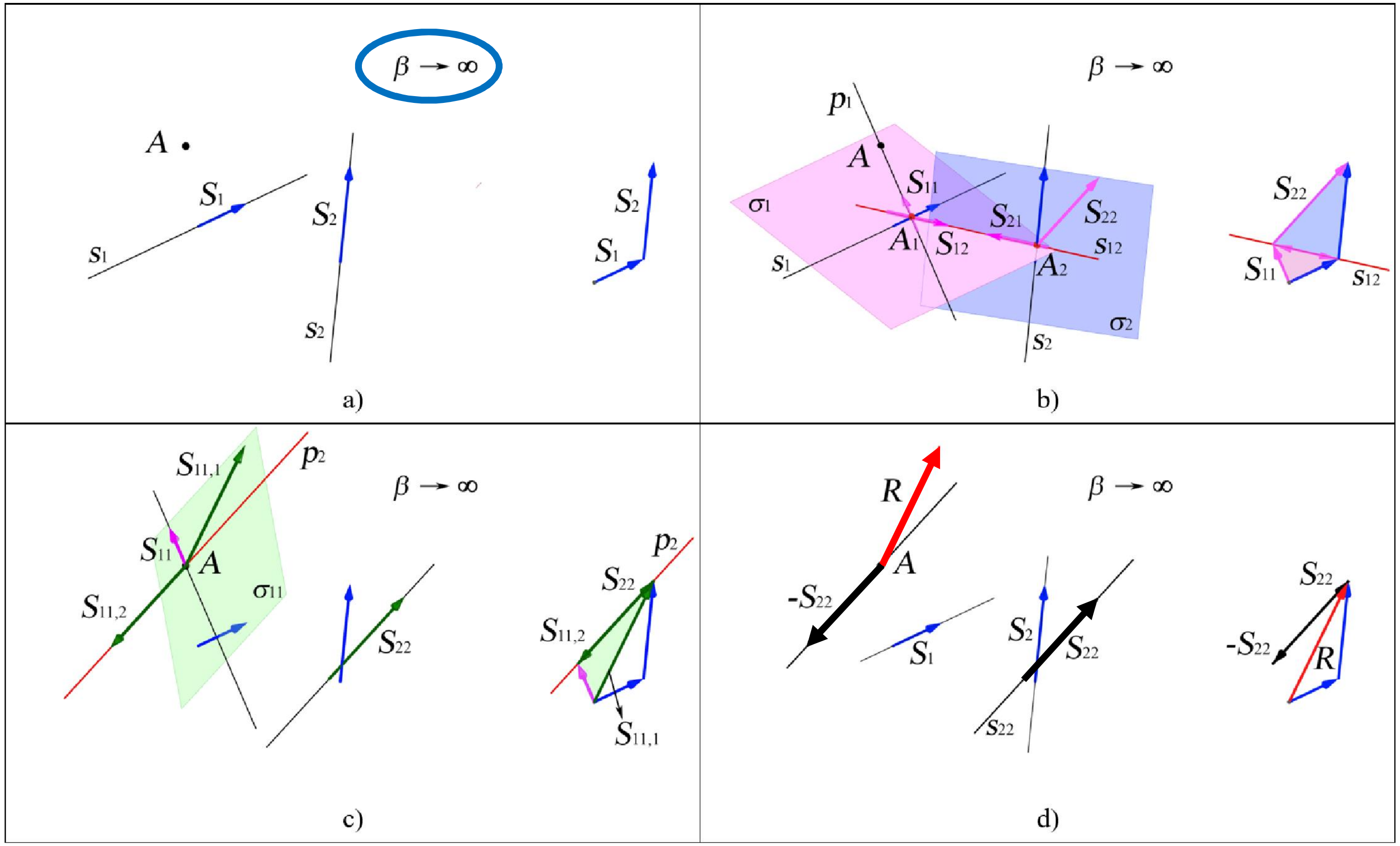


3. Example 3 :

Second special case –

β is plane at infinity

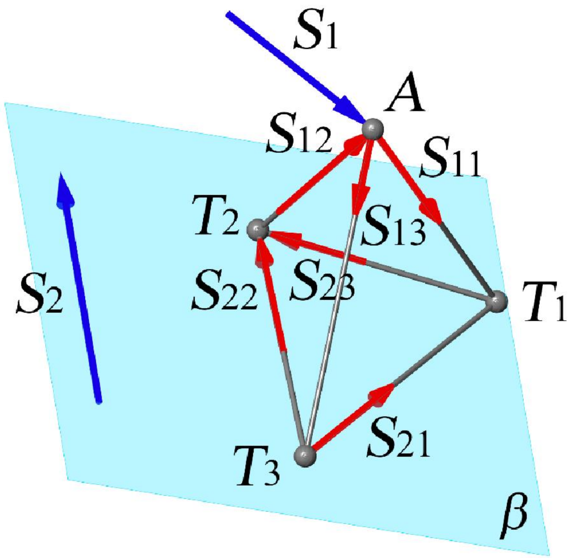
$\beta = (\beta_0, 0, 0, 0)$



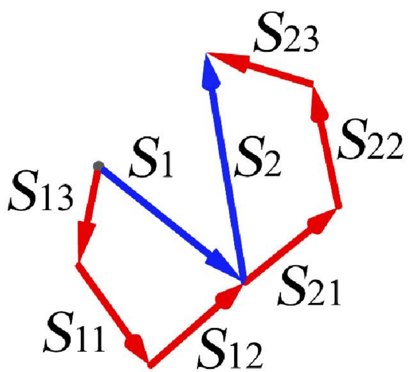
3. Example 4 :

- Replacing two forces with six forces acting along edges of a tetrahedron

Form diagram



Force diagram



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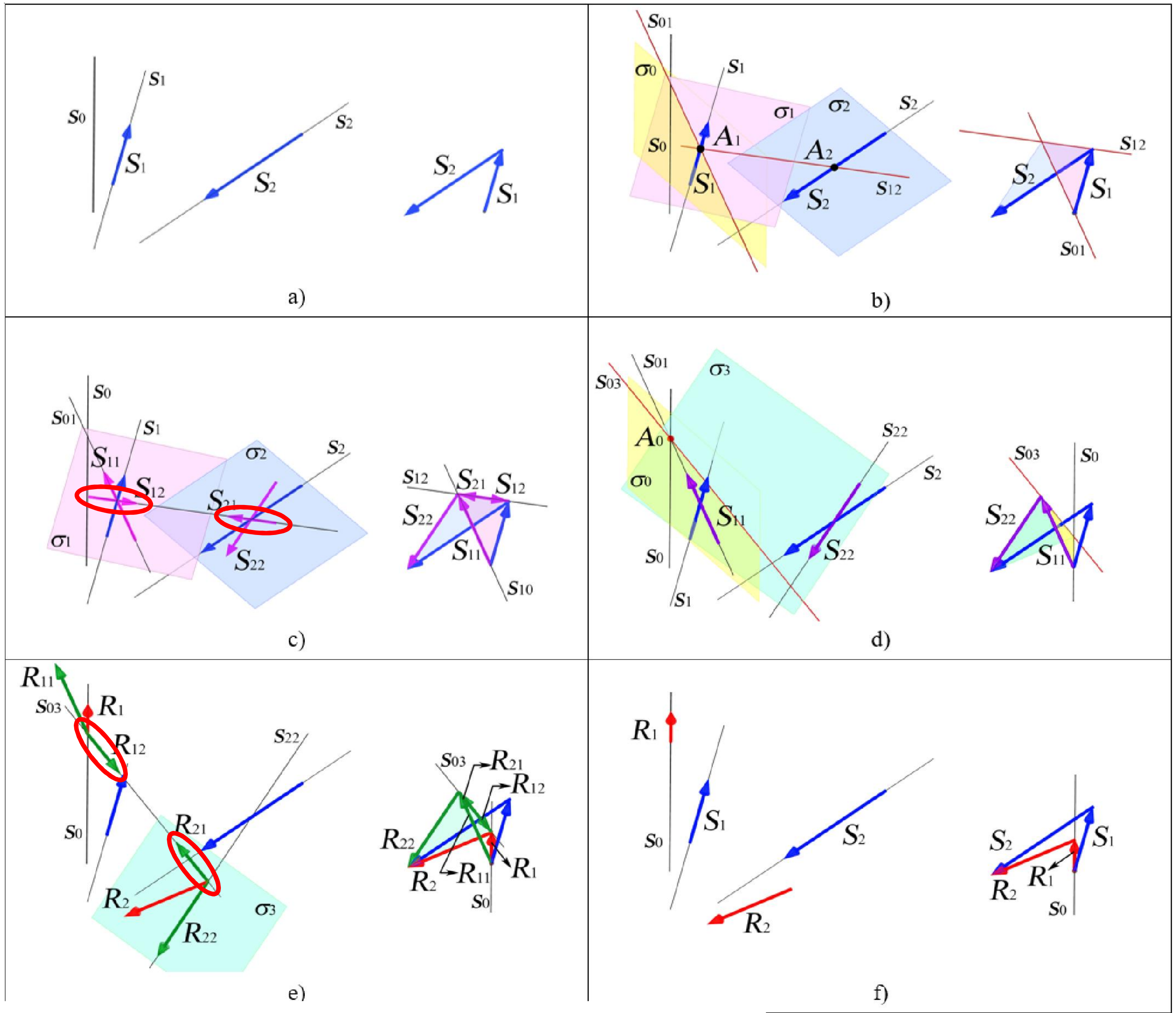
3. Example 5 :

- Replacing two forces with two forces of which one lies on a given line



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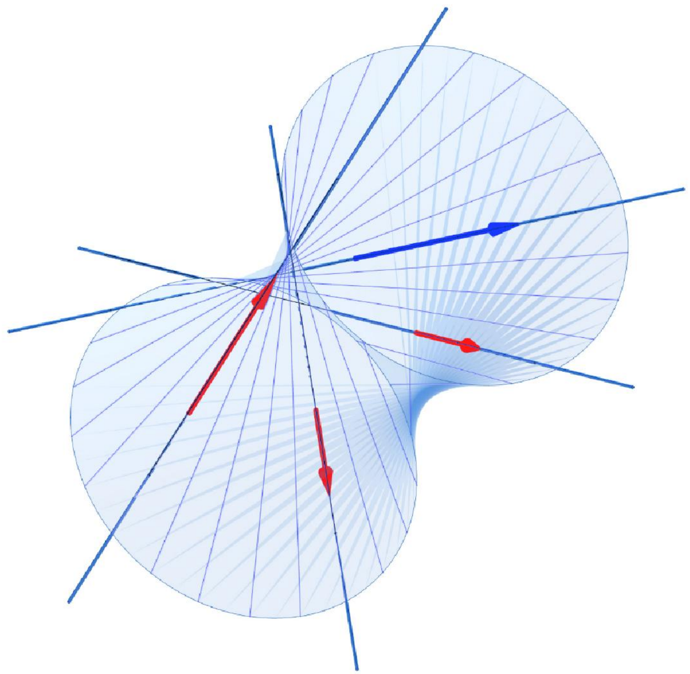
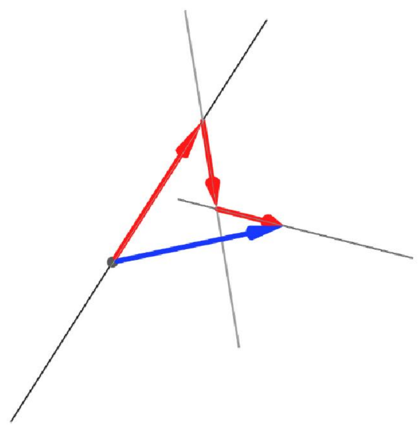
3. Example 6 :

- Replacing a single force with three forces acting along generators of the same system of a regulus

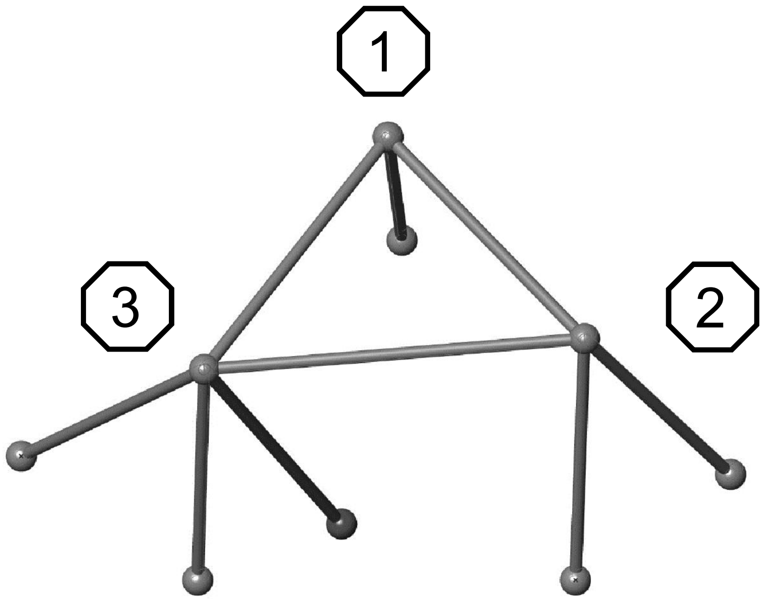
Form diagram

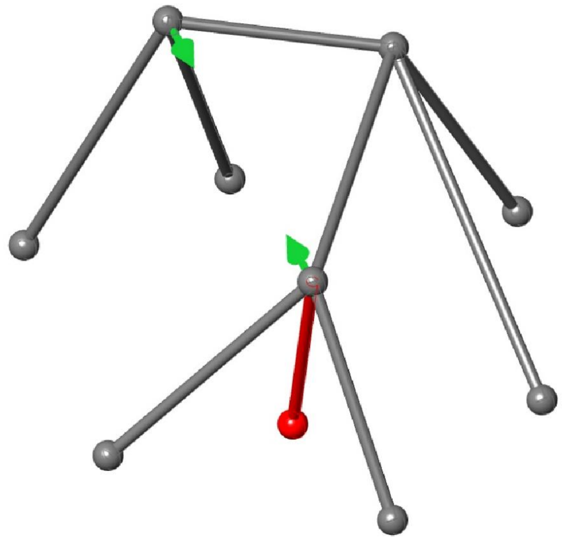


Force diagram



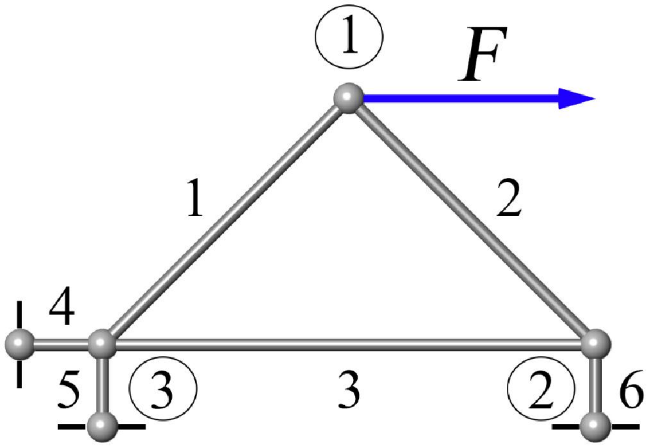
4. Future work:

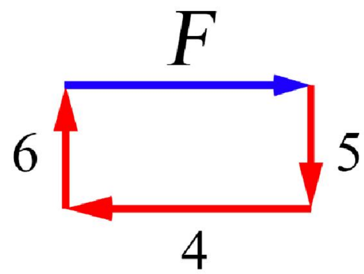




4. Future work:

Final result





Thank you for your attention!

Acknowledgements

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Questions