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# Shape optimization of compression structures

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## Abstract

Finding the form of structures consisting primarily of compressive elements is not always an easy task. In order to facilitate the design process of compression structures, the shape of a mirrored tensile structure can therefore be adopted, based on the traditional tension-compression analogy. Adding kinematic constraints to the original definition of the force density method, fixing the length of each structural element, the procedure may be applied iteratively until given conditions are satisfied Fresl *et al* [1]. Beside fixed element length, different kinematic constrain can be used in the sense of wanted element forces. Moreover, for some elements one can define wanted length and for others target tensile force. This paper will investigate different forms of compressive structure as a result of form finding with tension - compression analogy using diverse kinematic constraints applied to the same layout of tension elements.

**Keywords**: form finding, tension – compression analogy, kinematic constrain, compression structure

## 1. Form finding of compression structures

Structural design of long span structures is often very challenging and offers a wide range of possibilities for optimal structural system selection. To satisfy the aesthetics and functionality criteria it is necessary to pay attention to structural conception.
For tensile structures, form finding is used aimed to find the geometry of structure which is optimum in its shape. However, for the compressive structures, diversity of ideas and solutions is significantly lower. Tested and familiar regular forms are usually used ignoring the fact that designing compressive structures can be exciding just as tensile design.

Compressive structures achieve the load-bearing capacity for vertical actions primarily by the activation of compression forces. In order to facilitate the design process of compression structures, the shape of a mirrored tensile structure can therefore be adopted, based on the traditional tension-compression analogy. Adding kinematic constraints to the original definition of the force density method, fixing the length of each structural element, the procedure may be applied iteratively until given conditions are satisfied (Fresl *et al* [1]). Such iterative application of the force density method can be implemented to find structural forms based on computer analyses in the design of conventional structures. The final form, corresponding to a structure consisting primarily of compressive elements, must not differ much from the obtained mirrored tensile shape.

## 2. Force density method with kinematic constraints

Finding different geometries of compression structures with all elements in compression is based on the tensile-compressive analogy originally developed for the solution of form finding problems applicable to tensile structures (Gidak [2]). For the application on compression structures, kinematic constraints must be added to the solver in order to fix the length of elements or to set target forces in elements of the given mesh and to repeat the iterative analysis until the corresponding conditioned form is achieved. Regarding known problems with force density method concerning compression forces, form finding algorithm will discard kinematic condition of those elements with force $S_{ij} \leq 10^{-5}$. In addition to the required element lengths and/or target element forces, input parameters include coordinates of all free nodes and points of support. The free nodes are placed in the z = 0 plane, while support nodes are all in one plane. A solution of this problem, using the classical force density method, would result in the given plane of support nodes. However, due to the additional kinematic conditions (and the application of external forces whose values we will explain below), the final form is spatial.

By definition, the element force density $q_{i,j}$ is proportional to its force value $S_{i,j}$, for a constant element length, the ratio of force values corresponding to two iteration steps is equal to the ratio of force densities:

$$q_{i,j}^{(k)} = q_{i,j}^{(k-1)} \frac{\overline{S}}{S_{i,j}^{(k-1)}} \tag{1}$$

Maurin and Motro [3] describe the iterative procedure developed for the calculation of nets with evenly distributed tensile forces (nets of minimal length). However, such procedure does not allow any control of force values in boundary elements. Moreover, the attainment of different force values in various elements is not possible. With the aim of achieving nets with different force values, the force density in element (i, j), for iteration step k, can be defined as:

$$q_{i,j}^{(k)} = q_{i,j}^{(k-1)} \frac{\overline{S_{i,j}}}{S_{i,j}^{(k-1)}} = \frac{\overline{S_{i,j}}}{l_{i,j}^{(k-1)}} \tag{2}$$

In expression (2) $\overline{S_{i,j}}$ is the target force in element (*i*, *j*) which can be different for every element.

By definition, the force density is inversely proportional to the element length and with unchanged force value:

$$q_{i,j}^{(k)} / q_{i,j}^{(k-1)} = l_{i,j}^{(k-1)} / l_{i,j}^{(k)} \tag{3}$$

The target element length can be achieved by defining the force density in element (*i*, *j*) according to (where $\overline{l_{i,j}}$ is wanted element (*i*, *j*) length):

$$q_{i,j}^{(k)} = \frac{S_{i,j}^{(k-1)}}{\overline{l_{i,j}}} \tag{3}$$

In classical form finding procedure nodes are not loaded because the ratio of the prestress and other loads (such as self-weight of cables and other permanent loads) is significant in favor of prestress. But in compression structures weight of elements cannot be ignored, in fact the value of self-weight is by far higher than any other permanent loads if we consider lightweight cover. Accordingly, in every step solver is calculating concentrated vertical load of free nodes by summing half of all elements length which share the same free node. If the cover cannot be defined as lightweight, concentrated vertical load must be calculated from the area between elements. Authors will deal with this, not so trivial problem, in the future.

The process of geometry optimization is based on distribution of internal forces. According to that, the verification of structural analysis must be performed. The commercial structural analysis program SAP2000 was used. The model consists of frame (Timoshenko 3D beam) elements.

## 3. Definition of case study

Initial geometry, in plane dimension and element connectivity was inherited from the structure described in Uroš *et al* [4] where structural optimization based on tension-compression analogy is applied to the roof design of new stadium Kantrida in Rijeka, Croatia. Initial geometry of the roof was flattened ellipsoid, with large opening in the center (Figure 1, single-layer reticulated steel dome with tension reinforced prestressed ring at the bottom). The grid pattern is shaped as rhomb approximately 6m wide. Model contains 480 nodes (of which 60 are support nodes with prevented displacement in all three directions) and 900 elements whit intersections in nodes.

After structural optimization on the structure shown below (the length of all elements, which are part of the original mesh, are constrained in optimization process), extension of the study was done in the sense of exploring possible geometries as a result of different kinematic constraints

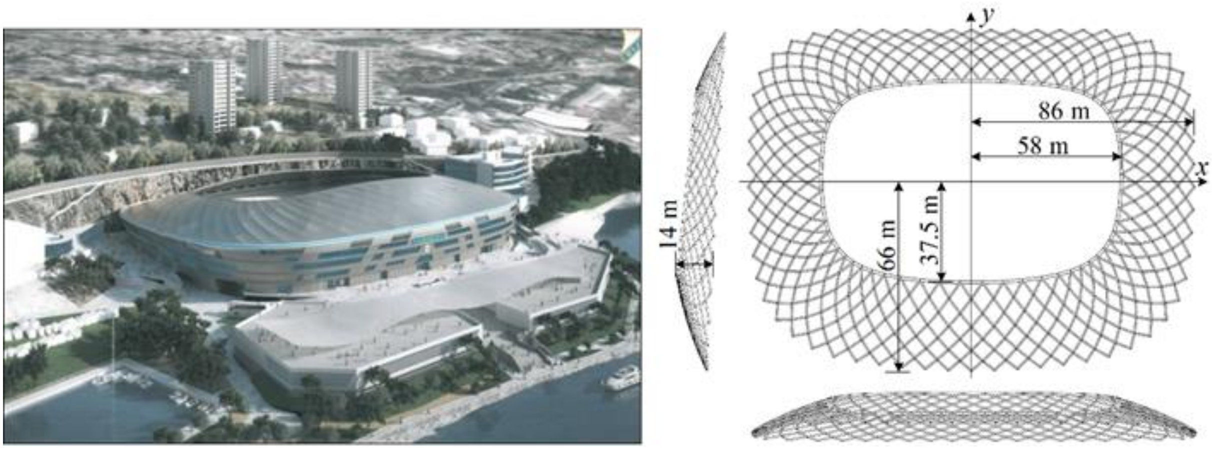


Figure 1: Illustration of the new stadium (final geometry) and initial geometry of the roof, Uroš *et al* [4]

For the first set of kinematic constrains (model A) target force in inner ring was set to 2100kN and in all other elements 150kN. After 2519 iterations equilibrium shape of model A was achieved, shown in Figure 2.

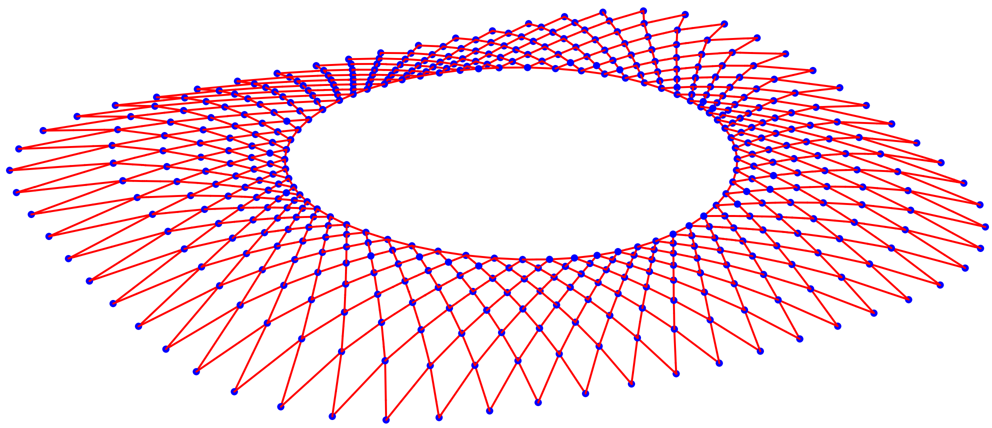

Figure 2: Perspectival view of equilibrium shape of model A

In model B different type of constraints were used. Final geometry had to consist of elements of the same length (6.40m) while for elements of inner ring axial force had to be 2000kN. For this constraints equilibrium shape (after 1542 iterations) is presented in Figure 3.

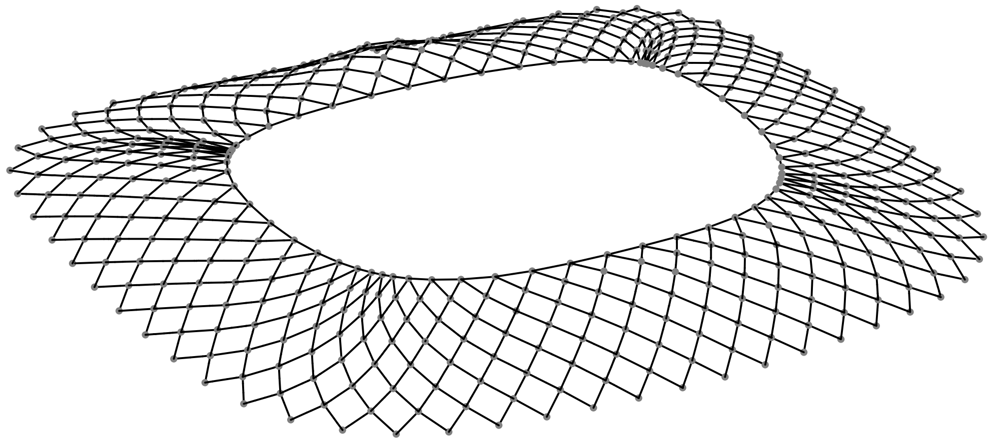

Figure 3: Perspectival view of equilibrium shave of model B

Both models are smaller in height by 2m compering then with initial geometry from Figure 1. Disposition of elements in model A is favorable from the point of construction because in model B accumulation of elements occur. Compering the area closed with inner ring, it can be notice that they are fairly similar.

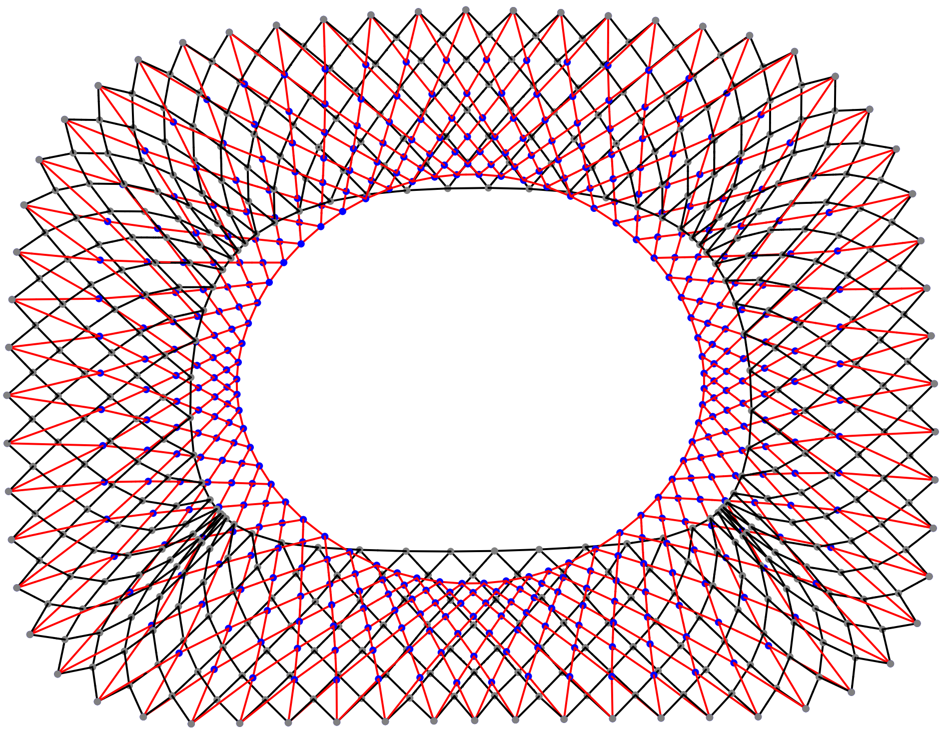

Figure 4: Plan view of model A (red lines) and model B (black lines)

## 4. Structural analysis

After optimization of geometry, the static analysis of the structure was carried out in SAP2000 and comparisons were made between initial geometry inherited from roof of new stadium Kantrida and

model A and B. The observed parameters were displacement and distribution of internal forces which are directly related to the limit states of structure.
In all three models the same cross section was used (for the inner ring steel tube 813/25mm and for other elements tube 457/12,5mm) and external vertical concentrated load is calculated from the length of elements.

Outcome of tensile-compressive analogy is tensile structure (tensile polygon) without any bending moments which is analogue to mirrored compression structure. Distribution of internal forces is exclusively membrane. However, in numerical model of real structure one needs to adapt the assumption of connection behaviour. In this example the rigid connections are considered and that is the reason why bending moments in the structural analysis after optimization occur.

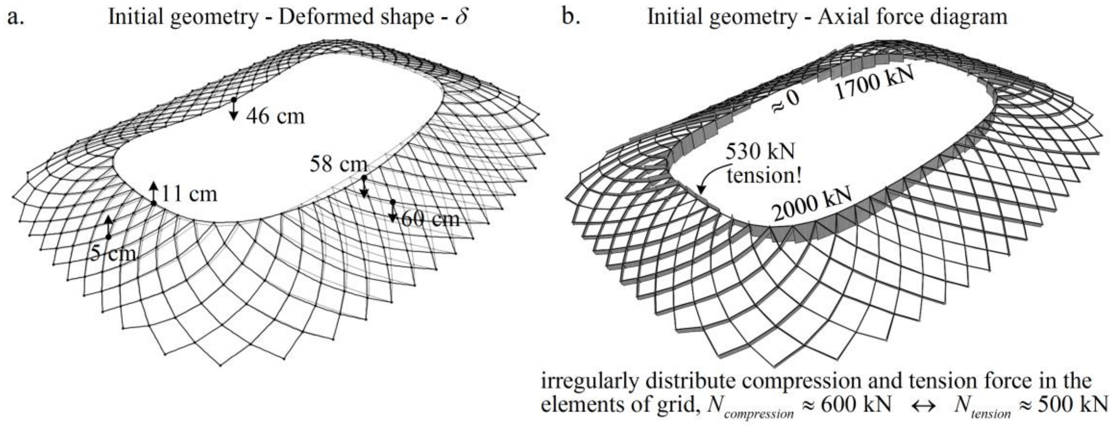


Figure 5: a) node displacement of initial geometry; b) axial force in the ring of initial geometry

Compering nodal displacement from initial geometry (Figure 5a) with one shown in Figures 6a and 7a, it is obvious that with models A and B significant reduction of nodal displacement is achieved, especially with model A.

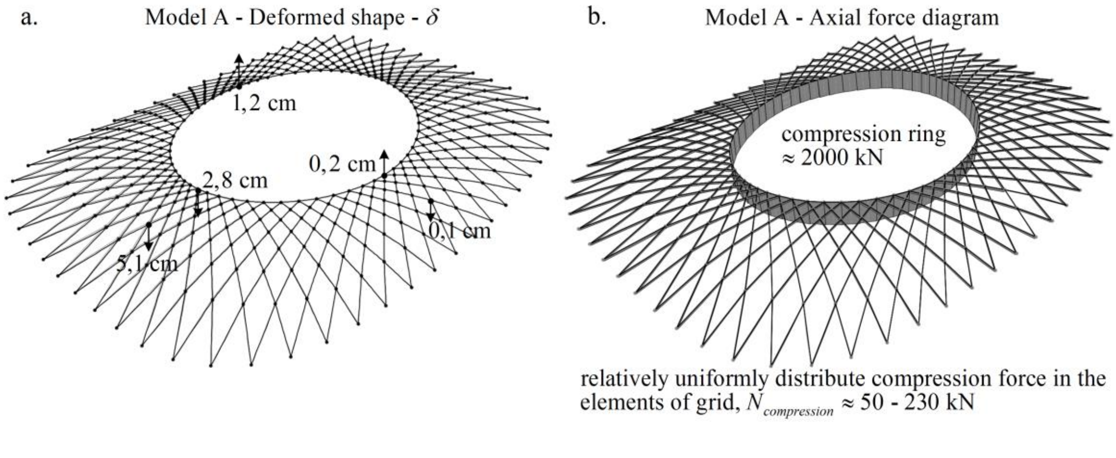


Figure 6: a) node displacement of model A; b) axial force in the ring of model A

Major improvement was made in distribution of axial force in inner ring; axial force is uniformly distributed after the optimization (Figure 6b and Figure 7b) where in initial geometry axial force varies significantly and even turns into tension (Figure 5b). Obtained axial forces in inner ring of model A and B slightly varies from target forces in optimisation process. This is the repercussion of the facts that in compression structures element have significant stiffness and are not connected with hinges.

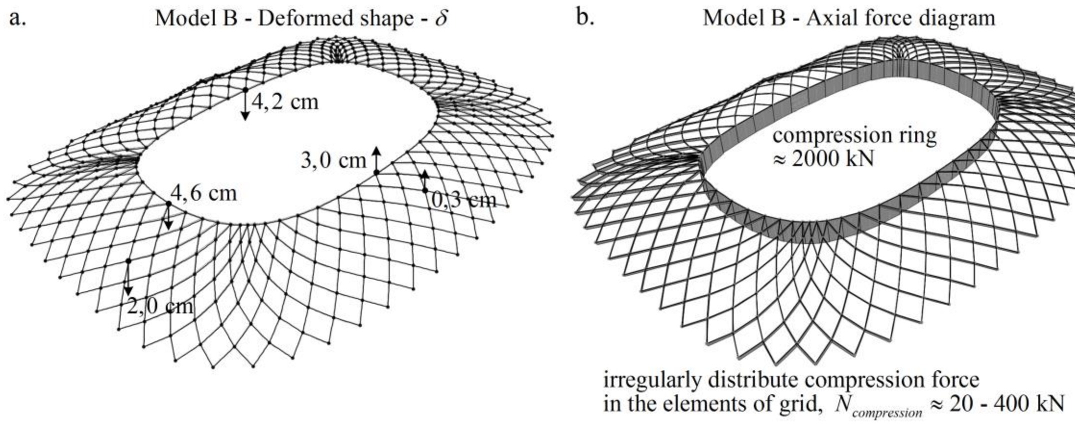


Figure 7: a) node displacement of model B; b) axial force in the ring of model B

Bending moments in model A in all structural elements practically vanish (unlike moments in initial geometry shown in Figure 8), to be more precise maximum major moment is 50kNm and minor 30kNm. But in model B interesting minor moments occur in elements on the "corners" of inner ring, with value of ±254kNm. In this "corners" accumulation of elements (with significant stiffness) outside the ring occurs and acts as a support of the curved beam (i.e. inner ring).

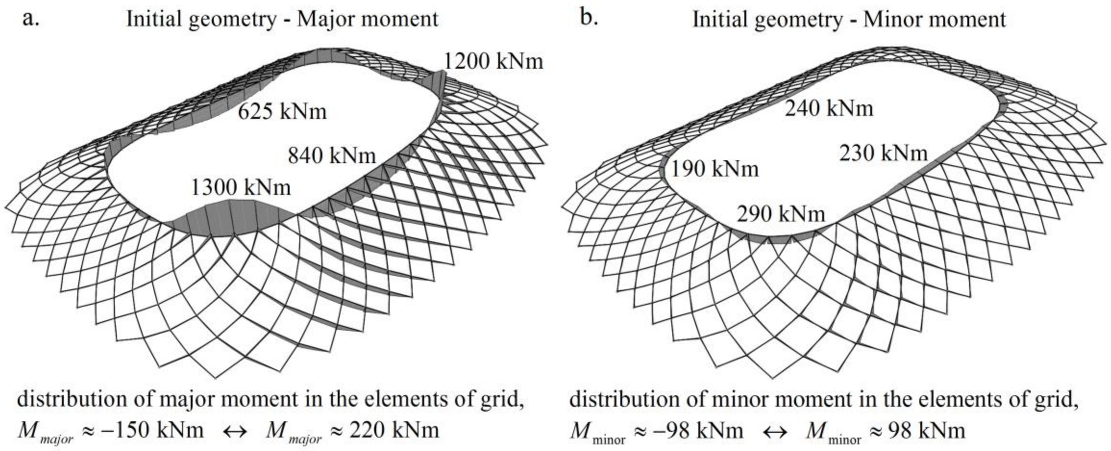


Figure 8: a) major bending moment of initial geometry; b) minor bending moment of initial geometry

## 5. Conclusion

In this paper form finding procedure is used to find equilibrium shapes of compressed structure using tension – compression analogy. The process is based on force density method complemented by kinematic constraints which is originally developed for tensile structures. Different kinematic constraints were used; in model A, target element forces were defined and in model B, all elements (except elements of inner ring) have the same target length while for elements of inner ring, target force was chosen.

Concentrated vertical load was calculated in every step of iterative procedure as sum of half of lengths of elements that share the same node. Because lightweight cover was chosen, this principle can be accepted. But for future analysis where weight of the cover is greater, solver must be upgraded and able to calculated concentrated vertical load from the area between elements.

Cinematic constraints for model A caused equilibrium geometry with inner ring close to circle and elements outside inner ring to form line-like curves. Because of the small curvature of equilibrium geometry in model A and the fact that nodes in compression structure are not hinges while elements have significant stiffness, small differences occur when static analysis of the structure was carried out in SAP2000.

This procedure gives an optimum distribution of internal forces and primary membrane state of stress in the structural elements reducing the bending moments to minimum. Also, substantial reduction of nodal displacements was achieved which is favorable from the point of serviceability limit state.

In conclusion, the described form finding procedure for compression structures is flexible in terms of applied constraints.

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