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Optimization of stadium roof structure using force density method

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ABSTRACT: The paper presents the application of a form finding procedure to the optimization of spatial structures. In contrast to typically form finding procedure, focus is given to the structures which are primarily loaded by compressive forces. According to the tensile-compressive analogy, finding the form of an analogue tensile structure, bending moments in the structural elements can be eliminated. The geometry optimization is performed iteratively by the force density method through the kinematic constraints which provides the final shape of structure does not differ significantly from the initial. Finally, the method is applied to geometry optimization of the roof stadium Kantrida in Rijeka.

1 INTRODUCTION

1.1 *General design problem*

Structural design of long span structures is often very challenging and offers a wide range of possibilities for optimal structural system selection. To satisfy the aesthetics and functionality criteria it is necessary to pay attention to structural conception.

For tensile structures, form finding is well known procedure used to adapt the geometry of structure to the optimum structural shape. However, for the compressive structures, diversity of ideas and solutions is significantly lower. Tested and familiar regular forms are usually used ignoring the fact that design of compressive structures can be challenging just as tensile design.

Compressive structures achieve the load-bearing capacity for vertical actions primarily by the activation of compression forces. Finding the form of structures consisting primarily of compressive elements, while trying to stay in desired architectural limits, is not always an easy task. In order to facilitate the design process of compression structures, the shape of a mirrored tensile structure can therefore be adopted, based on the traditional tension-compression analogy. Hence, given the conditional initial shape, through slight, but important modifications, a more suitable structural form may be achieved. In particular, adding kinematic constraints to the original definition of the force density method, fixing the length of each structural element, the procedure may be applied iteratively until given conditions are satisfied (Fresl et al. 2013). Such iterative application of the force density method can be implemented to find structural forms based on computer analyses in the design of conventional structures. The final form, corresponding to a structure consisting primarily of compressive elements, must not differ much from the obtained mirrored tensile shape.

1.2 *Form finding based on tensile-compressive analogy*

The structural optimization is based on the tensile-compressive analogy originally developed for the solution of form finding problems applicable to tensile structures (Gidak 2013). For the application on compression structures, kinematic constraints must been added to the solver in order to fix the length of all elements in the given mesh and to repeat the iterative analysis until the corresponding conditioned form is achieved.

According to that, by definition, the element force density $q_{i,j}$ is proportional to its force value $S_{i,j}$, for a constant element length, the ratio of force values corresponding to two iteration steps is equal to the ratio of force densities:

$$q_{i,j}^{(k)} = q_{i,j}^{(k-1)} \frac{\overline{S}}{S_{i,j}^{(k-1)}} \tag{1}$$

Expression (1) is used in Maurin and Motro (2001) to describe the iterative procedure developed for the calculation of nets with evenly distributed tensile forces (nets of minimal length). However, such procedure does not allow any control of force values in boundary elements. Moreover, the attainment of different force values in various elements is not possible. With the aim of achieving nets with different force values, the force density in element (i, j), for iteration step k, can be defined as:

$$q_{i,j}^{(k)} = q_{i,j}^{(k-1)} \frac{\overline{S_{i,j}}}{S_{i,j}^{(k-1)}} = \frac{\overline{S_{i,j}}}{l_{i,j}^{(k-1)}} \tag{2}$$

The target element length can be achieved by defining the force density in element (i, j) ac-cording to:

$$q_{i,j}^{(k)} = \frac{S_{i,j}^{(k-1)}}{\overline{l_{i,j}}} \tag{3}$$

Expression (3) stems from the fact that, by definition, the force density is inversely proportional to the element length and with unchanged force value:

$$q_{i,j}^{(k)} / q_{i,j}^{(k-1)} = l_{i,j}^{(k-1)} / l_{i,j}^{(k)} \tag{4}$$

Terms (3) and (4) define the iterative application of the force density method, which has some advantages. In particular, the procedure converges to the wanted solution through a series of equilibrium configurations, while the first approximation does not need to be defined, because the system of equations, which are being solved, is linear.

In addition to the required element lengths, input parameters include external vertical concentrated forces in the free nodes of the structural mesh, as well as the coordinates of all free nodes and points of support. The free nodes are placed in the $z = 0$ plane, while support nodes are all in one plane. A solution of this problem, using the classical force density method, would result in the given plane of support nodes. However, due to the additional condition of fixed element lengths (and the application of external forces), the final form is spatial. Regarding known problems with force density method concerning compression forces, form finding algorithm will discard kinematic condition of those elements with force $S_{ij} \leq 10^{-5}$.

The process of geometry optimization is based on distribution of internal forces. According to that, the verification of structural analysis must be performed. The commercial structural analysis program SAP2000 was used. The model consists of frame (Timoshenko 3D beam) elements.

2 OPTIMIZATION OF SIMPLE ROOF STRUCTURE

2.1 *Geometry*

First, we will analyze a simple model of steel roof, which consists of compressive elements. The inner ring is placed at a height of 5 m above the points of support. The initial geometry is in the shape of irregular ellipsoid with large central opening and ring in horizontal plane (Figure 1). The load is 1.0 kN/m^2 uniformly distributed on the surface in vertical direction.

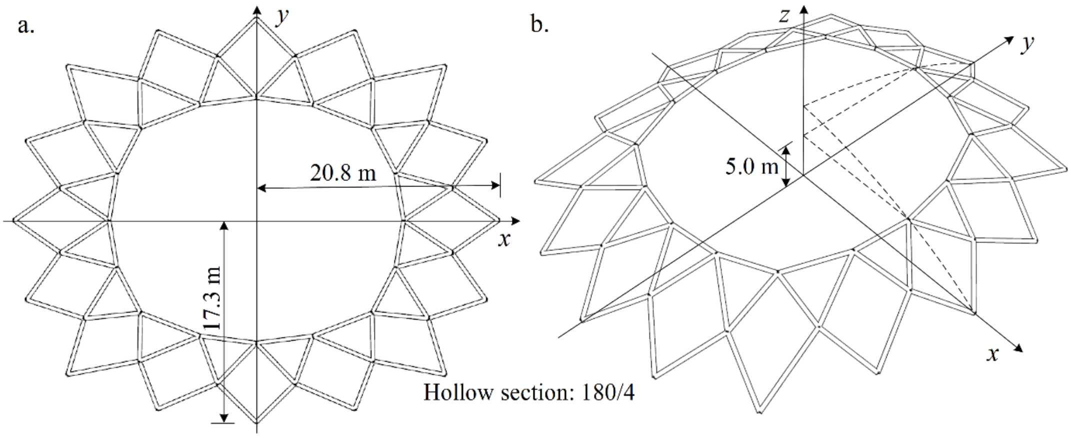

Figure 1. Original geometry before optimization (initial shape).

The initial geometry of the structure, due to the application of vertical loads, has considerable bending moments in the structural elements what leads to large cross-sections of structural elements and more complex connections. Also, the significant vertical displacement can be observed what may be unacceptable from the standpoint of serviceability.

2.2 *Form finding*

The objective is to find more efficient structural form, which does not deviate much from the initial. Considering a predominant compressive response of structure, the optimization has been made by finding the closest form including compressive elements of given lengths. The length is assumed to be constant because of the high axial stiffness of structural elements. Thus, that condition provides the final shape does not differ significantly from the initial. According to the tensile-compressive analogy, finding the form of an analogue tensile structure, bending moments in the structural elements can be eliminated.

An optimization process was carried out for the analogue mirrored tensile structure and the structural shape before and after optimization is compared (Fig. 2). Relatively small changes can be observed. The maximum deviations are primary in vertical direction. The central ring has moved downwards by 14 – 43 cm, while the nodes of second ring moved upwards approximately 22 - 26 cm. Central ring is no longer in the horizontal plane and it is curved according to the thrust line. It is important to notice that the symmetry of geometry is preserved. The structure didn't change the shape significantly and small modifications in nodes positions were sufficient to achieve the optimized geometry.

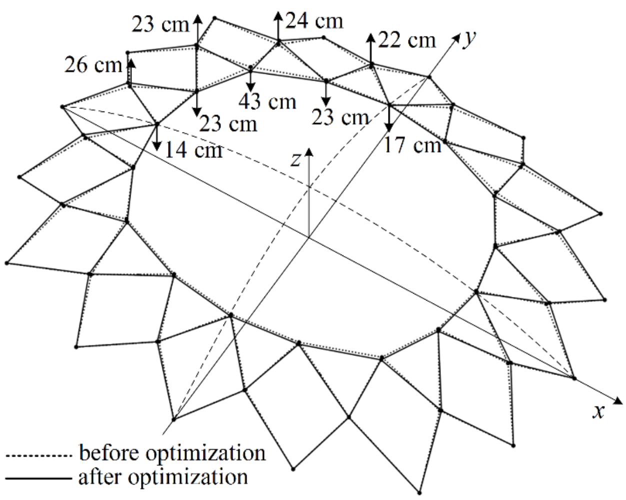

Figure 2. Differences between original geometry and geometry after form finding

2.3 *Structural analysis*

After optimization of geometry, the static analysis of the structure was carried out in Sap2000 and comparisons were made. The observed parameters were displacement and distribution of internal forces which are directly related to the limit states of structure.

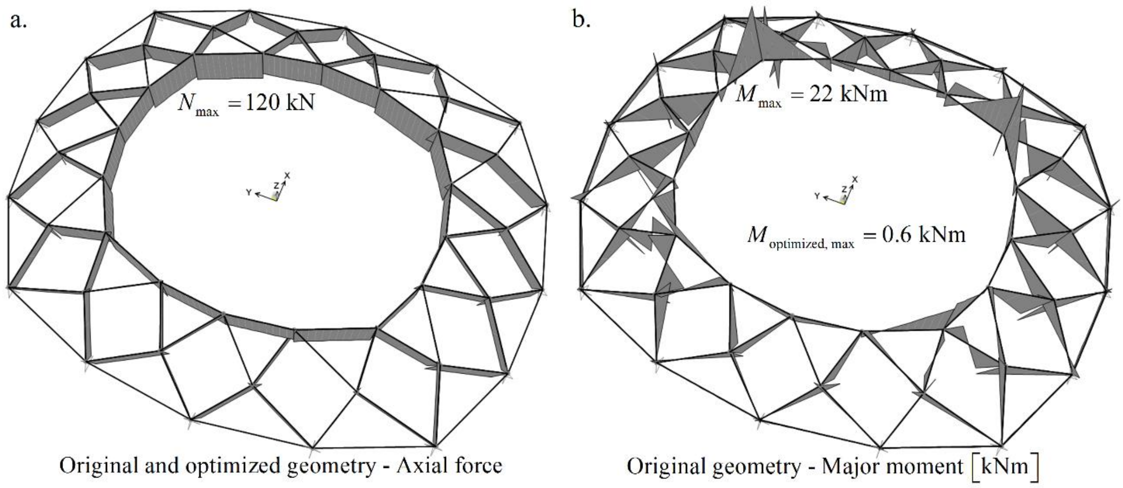

Figure 3. Comparison of axial force and bending moment before and after optimization.

By comparison of axial forces due to slight geometry modifications can be observed that values almost did not change (Fig. 3a). Other internal forces (bending moments) in all structural elements practically vanish (Fig. 3b). As it is mentioned above, result of tensile-compressive analogy is tensile structure (tensile polygon) without any bending moments. Distribution of internal forces is exclusively membrane. Moreover, evaluating the difference between the initial and final geometry, the affine image of the bending moment diagram corresponding to the original layout before the form finding can be obtained. However, in numerical model of real structure one needs to adapt the assumption of connection behavior. In this example the rigid connections are considered and that is the reason why bending moments in the structural analysis after optimization occur.

Furthermore, after optimization, the structure significantly increased load capacity and resists more efficiently to applied vertical load. Moreover, observing the nodal displacements for vertical load, reduction for more than 90% is achieved (Fig. 4). This is very favorable from the point of serviceability. Therefore, the construction is considerably more stiff meaning that it is less sensitive to wind vibrations.

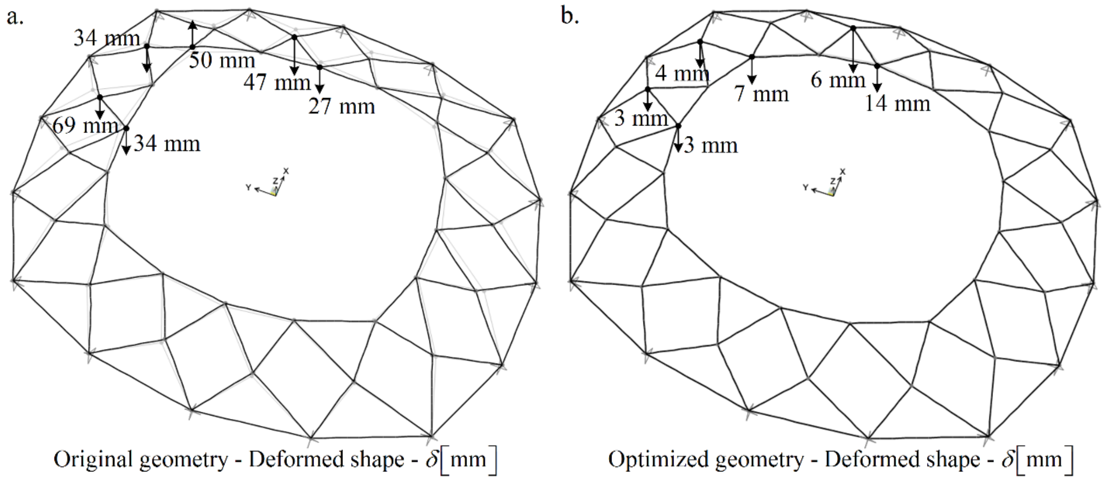

Figure 4. Comparison of node displacement before and after optimization.

3 ROOF OF THE STADUIM NK RIJEKA

3.1 *Geometry*

Described process of structural optimization based on tension-compression analogy is applied to the roof design of new stadium Kantrida in Rijeka, Croatia. Initial geometry of the roof was flattened ellipsoid, with large opening in the center. Span of the roof in the minor direction is 132 m, and in major 172 m. Height of the roof is approximately 14 m while dimensions of central opening are 75 m in minor direction to 116 m in major direction (Figure 5).

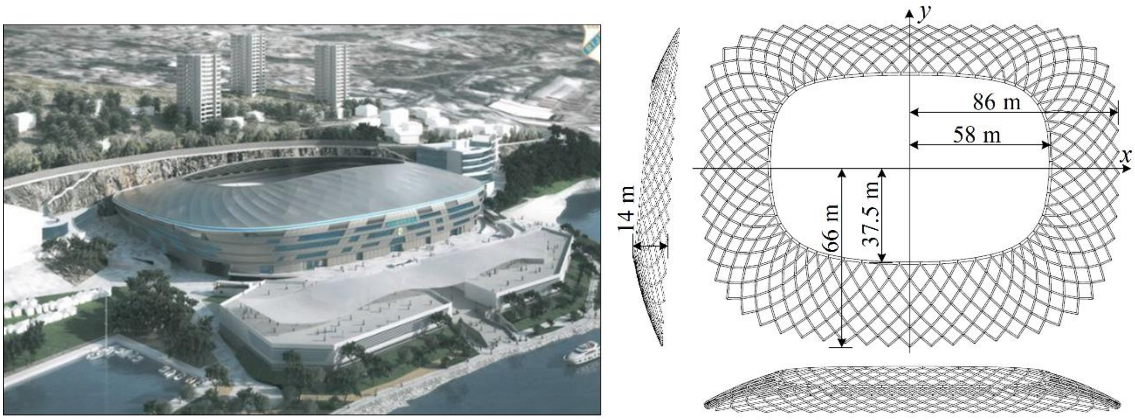

Figure 5. Illustration of the new stadium (final geometry) and initial geometry of the roof

Roof structure was originally designed as a single-layer reticulated steel dome with tension reinforced prestressed ring at the bottom. The grid pattern is shaped as rhomb approximately 6 m wide. All elements are steel tube profiles 610/12.5 mm except a central ring which is 1400/30 mm. The structure is loaded by self-weight and additional vertical loads of 1.0 kN/m^2.

Although, the initial form was favorable from the viewpoint of internal forces distribution, the irregular geometry of the roof, a large opening and significant roof span demanded unacceptably large dimensions of steel profiles in order to fulfill the ultimate and serviceability limit state. As a result, more detail form finding procedure was carried out.

3.2 *Form finding*

The intention of form finding is to find a more suitable structural form in the sense of structural efficiency, which does not excessively deviate from the initial architectural idea. As in the previous case, the length of all elements, which are part of the original mesh, are constrained in optimization process. It should be noted that elements in the final geometry might not have exclusively compression forces. In addition, this is often impossible to achieve. This depends primarily on the initial grid of elements and additional constraints (fixed length) in the process of form finding. The existence of compressive forces on the mirrored (tensile structure) geometry can be avoided with additional criteria. When force changes from tension to compression, the element length constraint has to be removed. As a result, the shortening of element occurs. This is generally of local character and should be monitored during the iterative procedure.

An optimization process was carried out for the analogue mirrored tensile structure and the structural shape before and after optimization is compared (Fig. 6a). Only characteristic points and their deviation from the initial geometry are displayed (Fig. 6b). Δ indicate a total deviation (in direction x, y and z) because the final deviation is not only vertical but has a significant horizontal component.

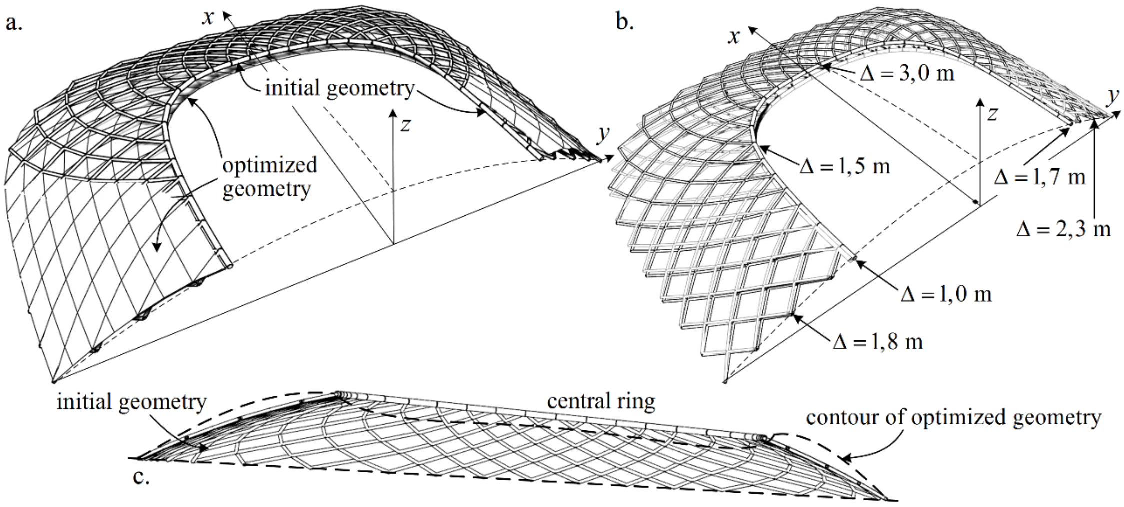

Figure 6. Geometry comparison before and after optimization.

The most important change can be noticed on the central ring, which is no longer in plane (Fig. 6c). Final shape is curved according to thrust line. At central position ring is slightly elevated and then lowered in plan's corner. A similar adjustment in shape can be seen on the surface of the roof. Despite the fact that optimized surface has similar dimensions as the initial, visually, the shape significantly changed. In the process of form finding this could be prevented by limiting movements of the central ring. This constraint would preserve the form of structure much better, but would have considerable less favorable internal force distribution. In addition, the result of form finding is primarily a form rather than amplitude. This means that a linear scaling (in gravity direction) does not change the favorable distribution of the internal forces. Thus it is possible, if necessary, to ensure the same height of the central ring.

3.3 *Structural analysis*

The structural analysis of the structure was carried out and conclusions are given for nodal displacement and distribution of internal forces. As well as in the previous example, the axial forces in grid elements before and after geometry optimization are almost the same (Figure 7).

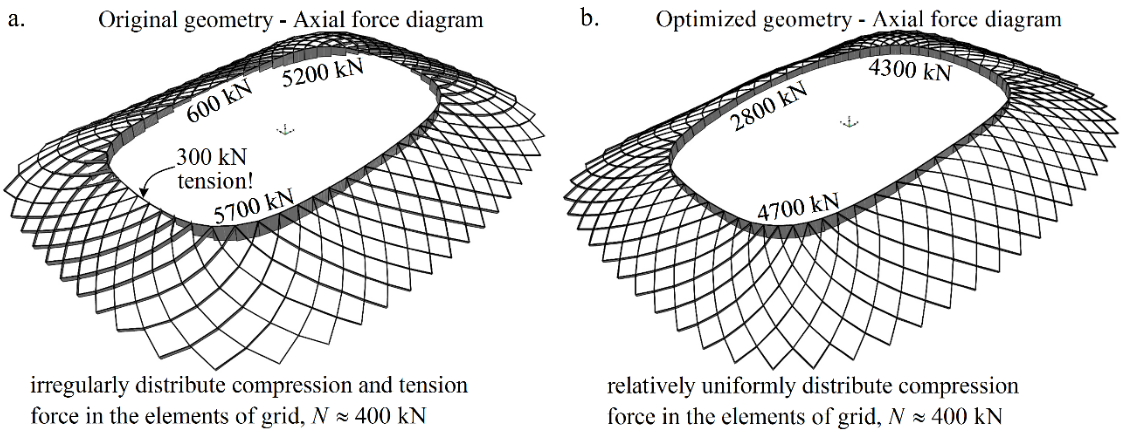

Figure 7. Comparison of axial force before and after optimization

The significant difference is only in the central ring and close to it where axial force is uniformly distributed after the optimization. This observation is essential because the irregularly distributed compression force (with occurrence of tensile force) in the top ring of reticulated shell emphasizes the problem of global stability.

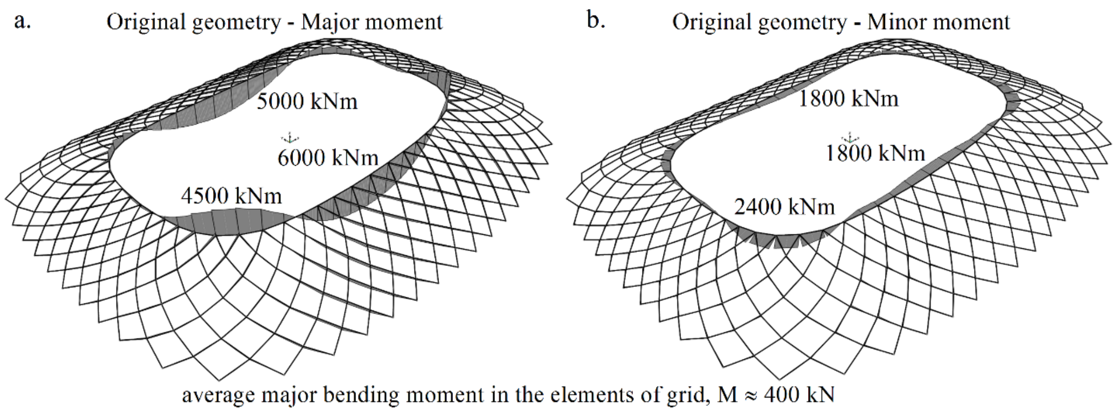

Figure 8. Bending moment before and after optimization

The bending moments in all structural elements practically vanish. Like in previous example the rigid connections are observed which causes small values of bending moments. The major and minor bending moments are shown on the initial geometry (Fig. 8). Due to the relatively high bending moments the ring is significantly deformable which means it cannot meet its primary objective to make a structure stiffer. Finally, the structure's load capacity to applied vertical load is considerably increased. Regarding ultimate serviceability state, significant reduction of nodal displacements is achieved (Fig. 9).

Described optimization process is largely determined by the geometry, grid of structure, boundary conditions and applied loads. It is possible that the resulting geometry after optimization does not satisfy structures esthetic and functional requirements. If so, different constraints in the form finding procedure can be applied and/or remove existing ones. Consequently, the solution is not unique but according to specified constraints. If necessary, it is possible to locally adjust the resulting shape and the global distribution of internal forces should not be significantly disturbed.

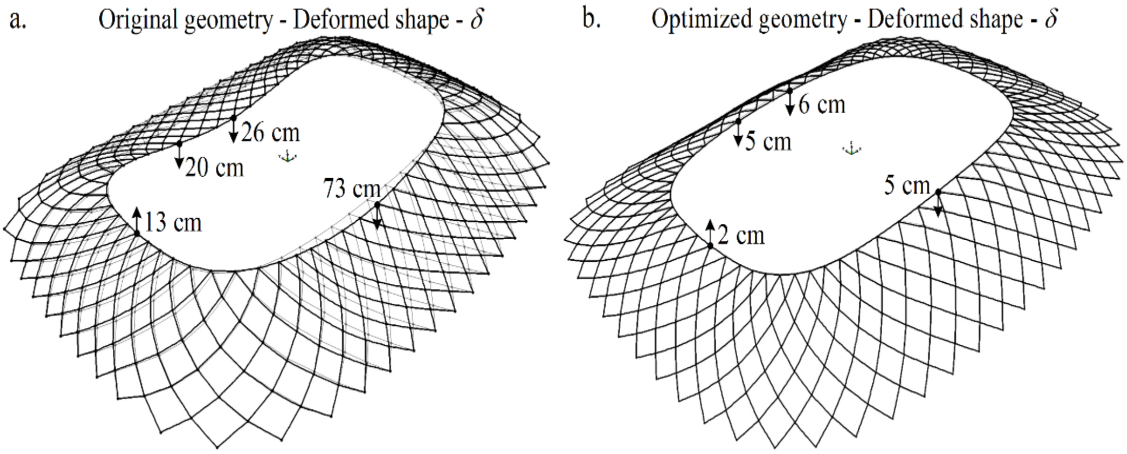

Figure 9. Comparison of node displacement before and after optimization.

4 CONCLUSION

This paper presents form finding procedure of spatial structures conceived to find a more suitable structural form, which is not so different from the initial architectural idea. The process is based on force density method complemented by kinematic constraints which is originally developed for tensile structures. Here is adapted in design of compressive structures like reticulated domes and shells.

This procedure gives an optimum distribution of internal forces and primary membrane state of stress in the structural elements reducing the bending moments to minimum. This results in elements of smaller dimensions and consequently a lighter structure. Finally, the structure significantly increases load capacity. In addition, substantial reduction of nodal displacements are achieved which is favorable from the point of serviceability limit state.

In conclusion, the described form finding procedure for compression structures is flexible in terms of applied constraints. Consequently, the solution is not unique offering plenty of options to the designer.

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REFERENCES

Fresl, K., Gidak, P. & Vrančić, R. 2013. Generalized minimal nets in form finding of prestressed cable nets. *Građevinar* 65: 13-16.

Gidak, P. 2014. *Stability assessment of the form finding procedures applied to prestressed cable nets. Doctoral thesis*. Zagreb: Faculty of Civil Engineering. University of Zagreb.

Maurin, B. & Motro, R. 2001. Investigation of minimal forms with conjugate gradient method. *International Journal of Solids and Structures*. 38: 2387-2399.