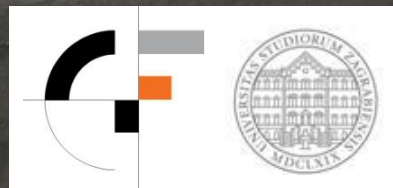


**Optimization of stadium roof structure  
using force density method**

**Mario Uroš  
Petra Gidak  
Damir Lazarević**

University of Zagreb, Faculty of Civil Engineering  
Zagreb, Croatia



Faculty of Civil Engineering, University of Zagreb, Croatia  
Department of technical mechanics  
Chair for statics, dynamics and stability of structures  
Supported by Croatian Science Foundation



## Introduction

- **OBJECTIVE:**
  - design a long span roofs according to the criteria of aesthetics, functionality, **structural stability**
  - structural shape is most important factor
- **tensile structures**
  - form finding is well known procedure for optimum structural shape (tension polygon)
- **compressive structures**
  - diversity of ideas and solutions is significantly lower (tested and regular shapes)
- **IDEA is to implement procedure of design tensile structures to compressive structures**
  - traditional tension-compression analogy (shape of a mirrored tensile structure)
  - from **initial shape**, through **minor** modifications, get a more suitable structural form
  - **minor (or not?)** modifications are insured by **kinematic constraints**
- **numerical procedure**
  - force density method, dynamic relaxation method...
  - adding kinematic constraints to the original definition (length, slope, force...).

## Numerical procedure - Force density method

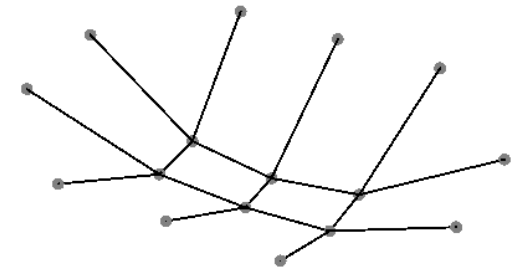
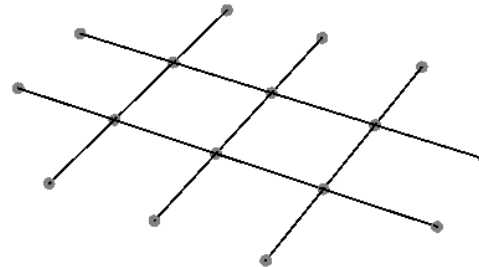
- structural optimization is based on the tensile-compressive analogy originally developed for the solution of form finding problems applicable to tensile structures (code by Gidak and Fresl)
  - implementation of kinematic constraints
- verification by dynamic relaxation method
- static and dynamic analyses by commercial software (SAP2000)

$$q_{i,j}^{(k)} = q_{i,j}^{(k-1)} \frac{\bar{S}}{S_{i,j}^{(k-1)}}$$

$$q_{i,j}^{(k)} = q_{i,j}^{(k-1)} \frac{\bar{S}_{i,j}}{S_{i,j}^{(k-1)}} = \frac{\bar{S}_{i,j}}{l_{i,j}^{(k-1)}}$$

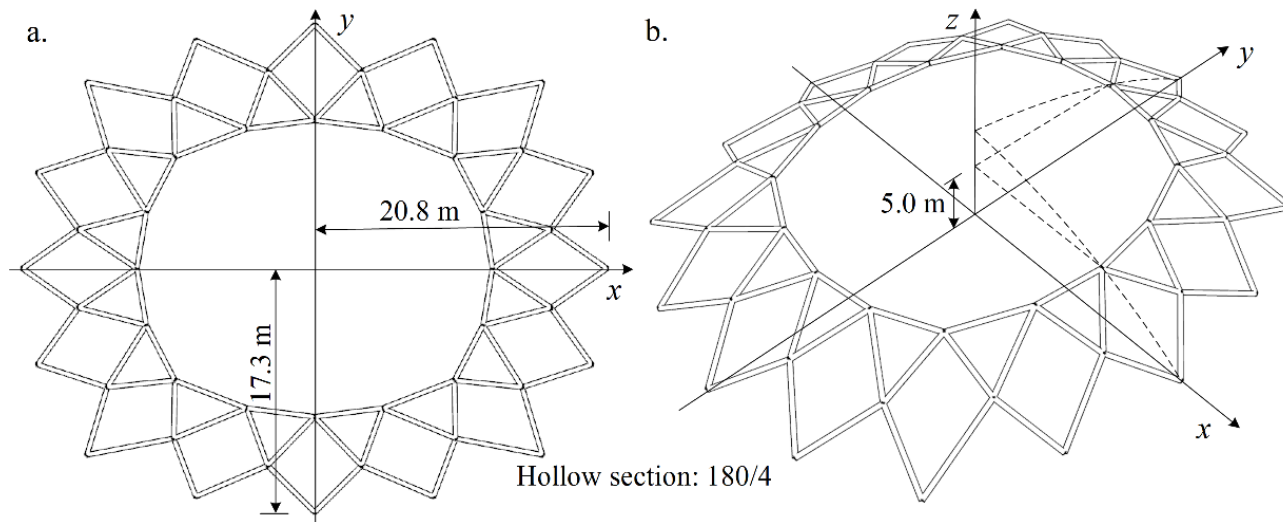
$$q_{i,j}^{(k)} / q_{i,j}^{(k-1)} = l_{i,j}^{(k-1)} / l_{i,j}^{(k)}$$

$$q_{i,j}^{(k)} = \frac{S_{i,j}^{(k-1)}}{l_{i,j}}$$



## Optimization of simple roof structure

- vertical load -  $1.0 \text{ kN/m}^2$  uniformly distributed
- initial geometry is in the shape of irregular ellipsoid with large central opening

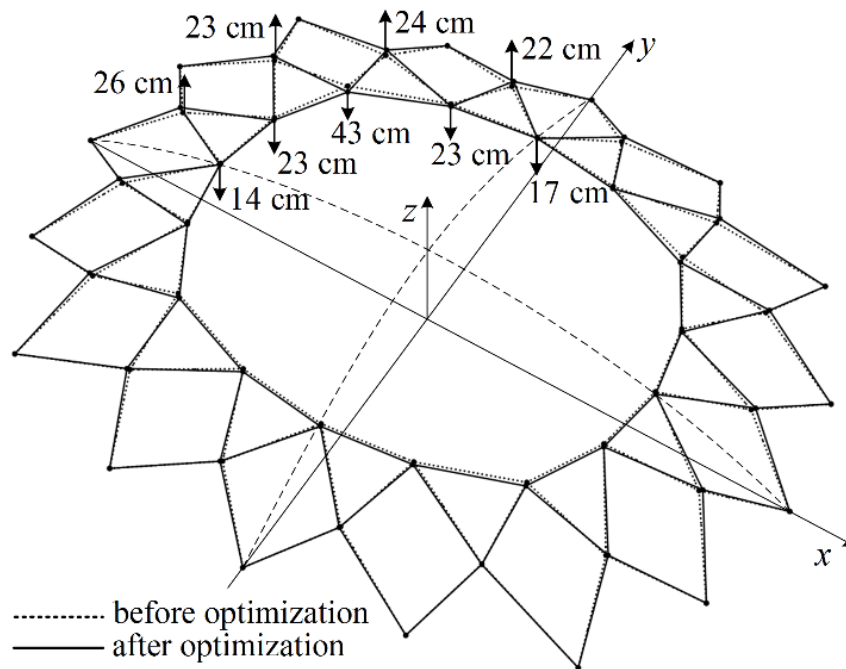


- initial geometry of the structure has considerable bending moments for vertical loads
  - large cross-sections of structural elements and complex connections
  - the significant vertical displacement
- **OBJECTIVE: find more efficient structural form, which does not deviate much from the initial**



## Optimization of simple roof structure

- optimization has been made by form finding of mirrored tensile structure
- element lengths are fixed - the final shape does not differ significantly from the initial



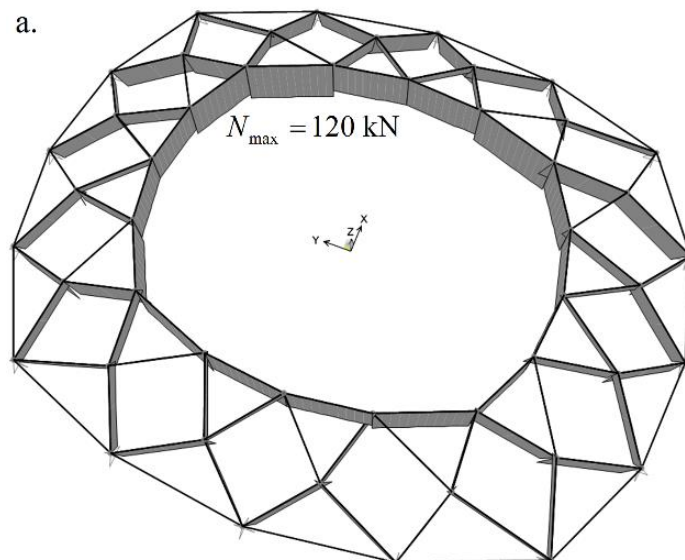
## RESULTS

- central ring has moved downwards by 14 – 43 cm
- nodes of second ring moved upwards 22 - 26 cm
- central ring is no longer in the horizontal plane and it is curved according to the thrust line

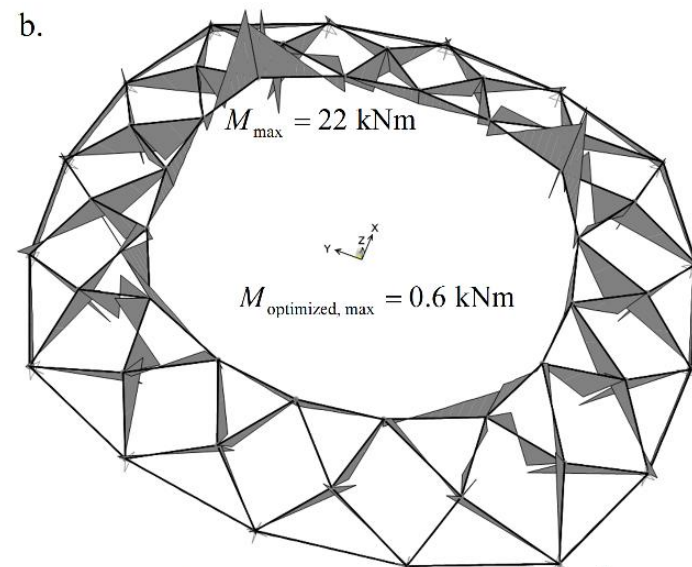
**NOTICE:** The symmetry of geometry is preserved. The structure didn't change the shape significantly and small modifications in nodes positions were sufficient to achieve the optimized geometry

## Optimization of simple roof structure

- static analysis of the structure was carried out in Sap2000
- axial forces due to slight geometry modifications almost did not change
- bending moments practically vanish - distribution of internal forces is exclusively membrane
  - affine image of the bending moment diagram



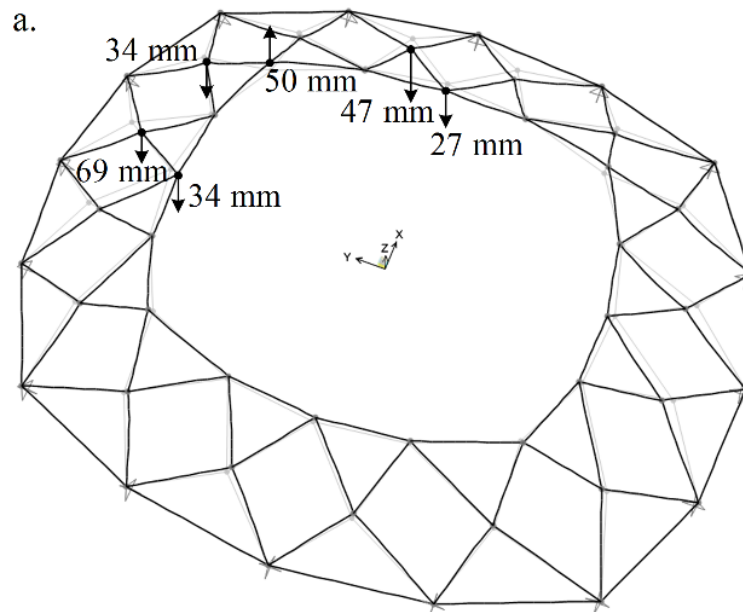
Original and optimized geometry - Axial force



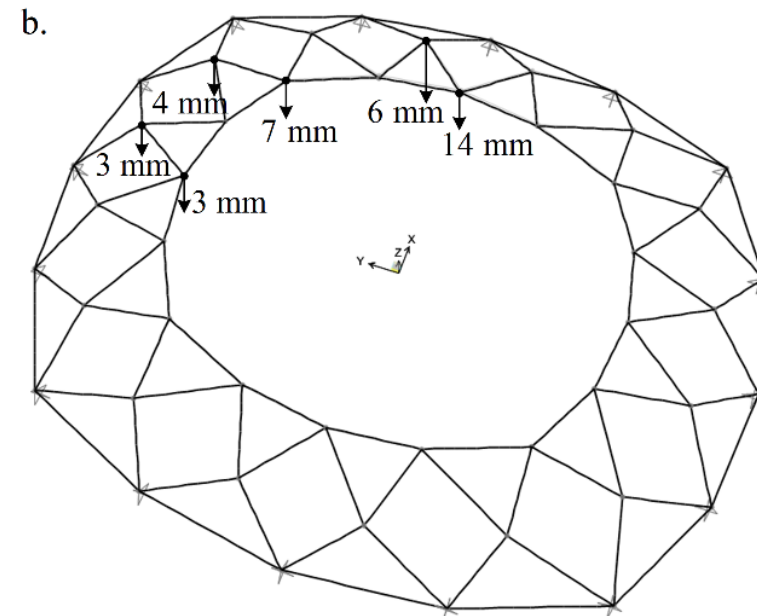
Original geometry - Major moment [kNm]

## Optimization of simple roof structure

- deflection and stiffness of structure – effects on structural stability



Original geometry - Deformed shape -  $\delta$  [mm]

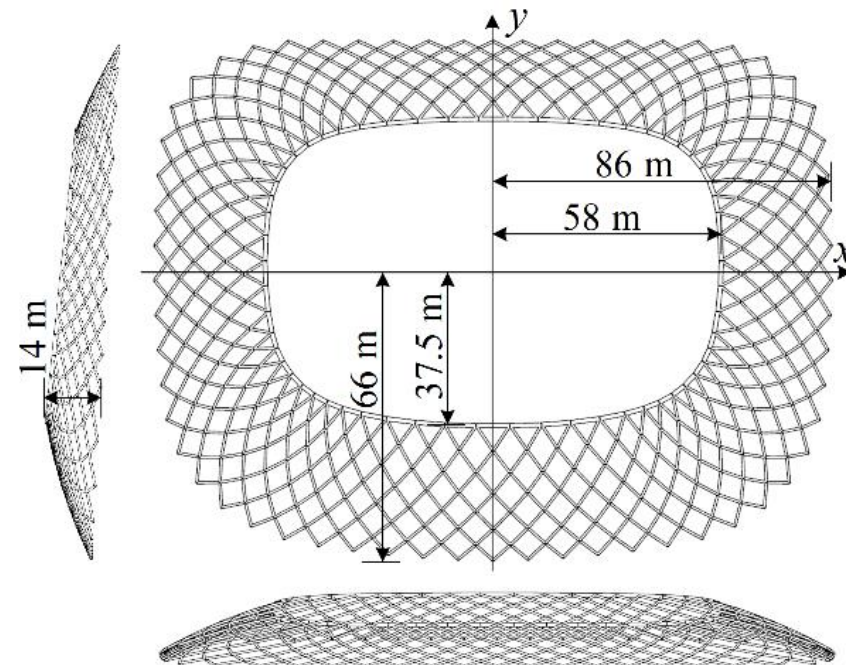


Optimized geometry - Deformed shape -  $\delta$  [mm]

- structure significantly increased load capacity and resists more efficiently to applied vertical load - reduction for more than 90% is achieved
- construction is considerably more stiff - less sensitive to wind vibrations

## Roof of the stadium Rijeka

- initial geometry was flattened ellipsoid with large opening in the center (steel tube profiles)
- single-layer reticulated steel dome (grid pattern is rhomb approximately 6 m wide)
- self-weight and additional vertical loads  $1.0 \text{ kN/m}^2$  (wind not included)
- irregular geometry, large opening and span - large dimensions of steel tubes and displacements





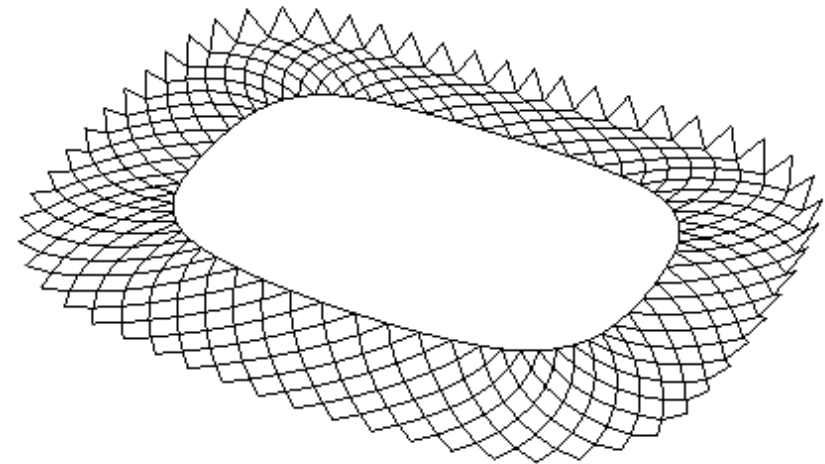
## Roof of the stadium Rijeka

### OPTIMIZATION PROCESS

- form finding of mirrored tensile structure
- force density method - iteration process
- **length of all elements are constrained**

### CONSEQUENCE of constraint

- elements in the final geometry might not have exclusively compression forces !
- compressive forces on the mirrored geometry can be avoided with additional criteria



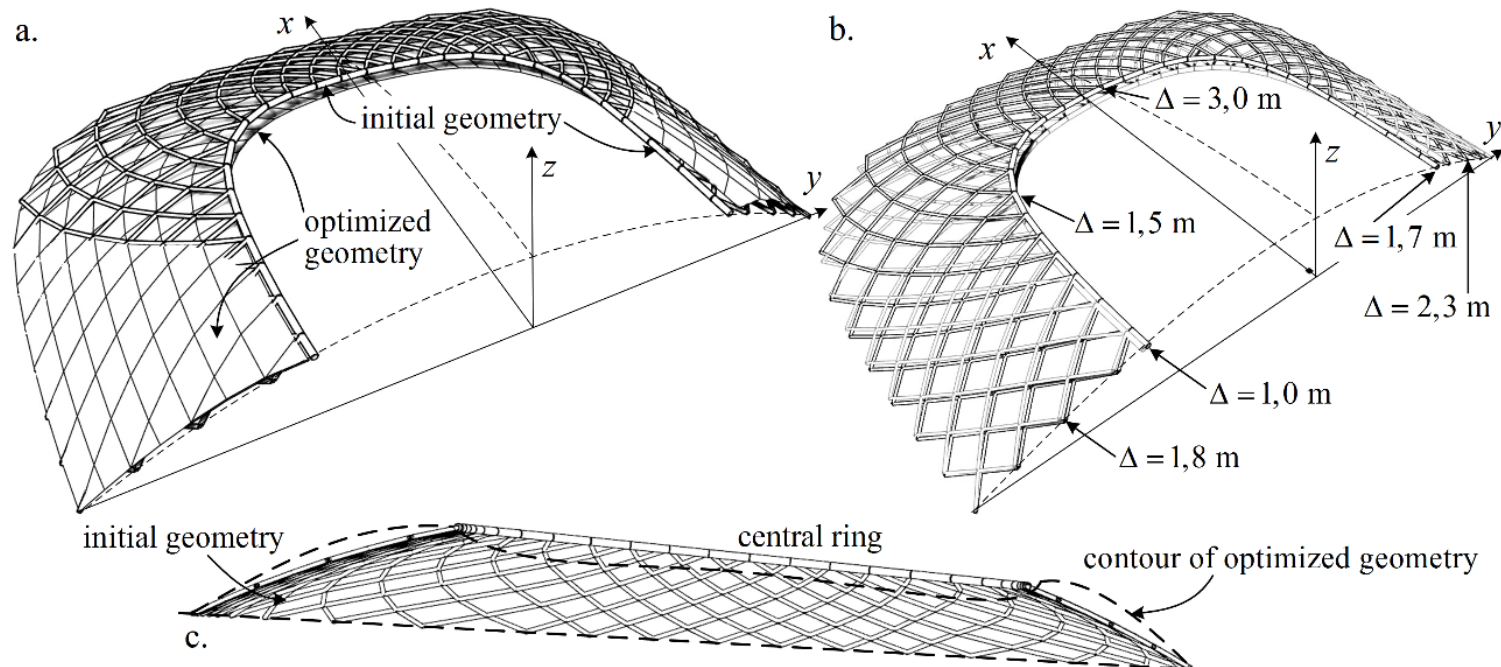
### RESULT

- shortening of compression elements occurs



## Roof of the stadium Rijeka

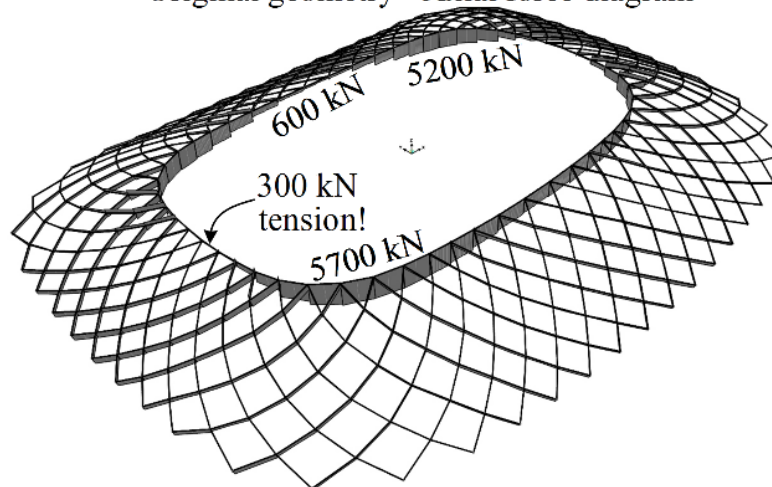
- central ring is no longer in plane (final shape is curved according to thrust line)
- optimized surface has similar shape as the initial, but the ring shape significantly changed
  - possibility of additional constraints of central ring ↔ less optimum internal force distribution
- adapting global geometry - **result of form finding is a form - no amplitude** (linear scaling can be applied)



## Roof of the stadium Rijeka

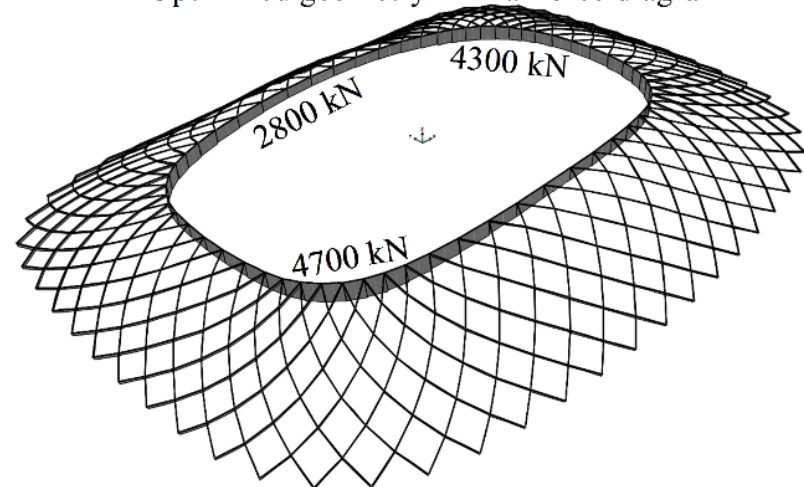
- static analysis of the structure was carried out in Sap2000
- irregularly distribution of axial force in the center ring (with occurrence of tensile)
- **problem of global stability of single layer reticulated shell**
- axial forces in grid elements before and after geometry optimization are almost the same

a. Original geometry - Axial force diagram



irregularly distribute compression and tension force in the elements of grid,  $N \approx 400$  kN

b. Optimized geometry - Axial force diagram

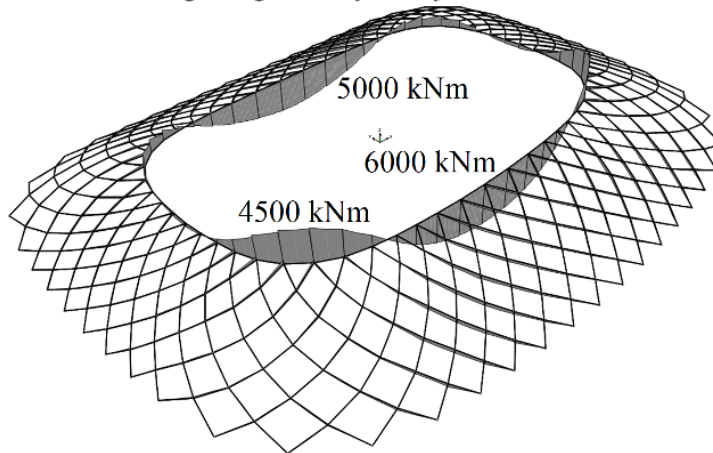


relatively uniformly distribute compression force in the elements of grid,  $N \approx 400$  kN

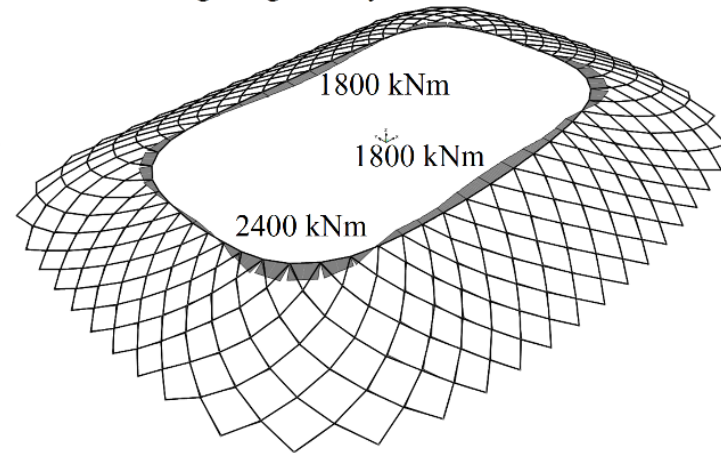
M. Uroš, P. Gidak & D. Lazarević

- bending moments in all structural elements practically vanish

a. Original geometry - Major moment

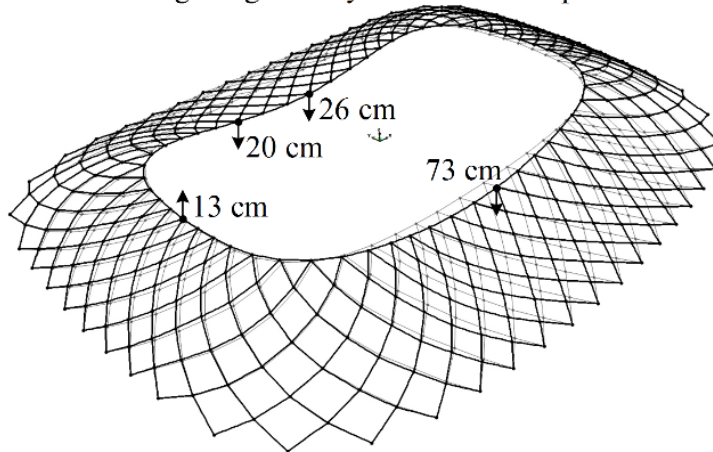


b. Original geometry - Minor moment

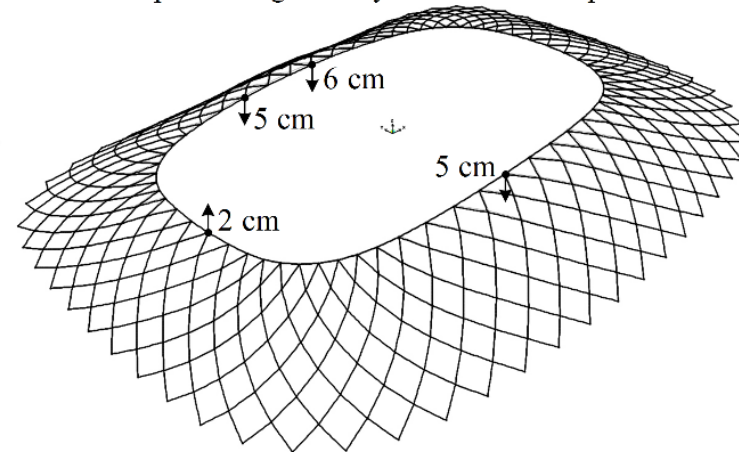


- deflection and stiffness of structure – positive effects on structural stability

a. Original geometry - Deformed shape -  $\delta$



b. Optimized geometry - Deformed shape -  $\delta$



## Conclusion

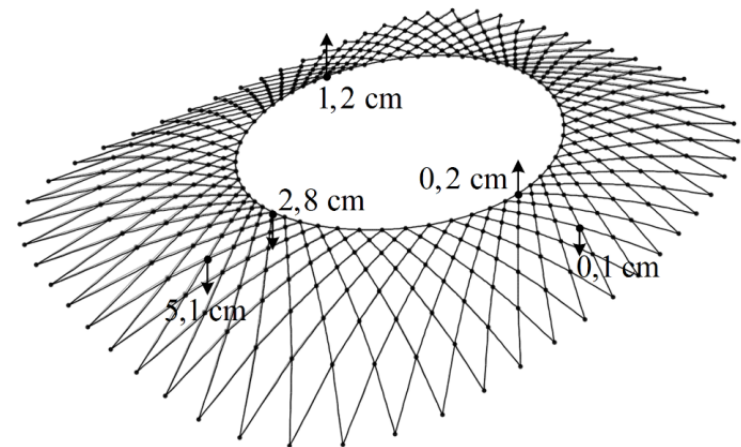
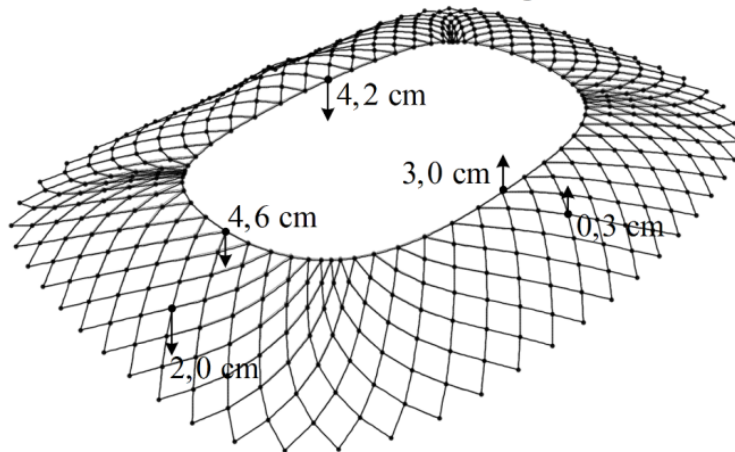
- form finding procedure of tensile structures can be adopted to design compressive structures
- process is based on force density method complemented by kinematic constraints
- procedure gives an optimum (!?) distribution of internal forces and primary membrane state of stress in the structural elements reducing the bending moments to minimum
- structure significantly increases load capacity
- stiffness of structure increases - reduction of nodal displacements (SLS)  
(smaller dimensions of elements and a lighter structure)
  
- form finding procedure for compression structures is flexible in terms of applied constraints
- solution is not unique offering plenty of options to the designer
  
- optimization process is largely determined by the global geometry, grid of structure, boundary conditions, constraints and applied loads



## Variants of stadium roofs

Some other solutions:

- elements of the same length (6.40 m), ring have uniform axial force 2000 kN
- target force in inner ring 2100 kN and in all other elements 150 kN



# Thank You

Research presented in this paper has been financially supported by  
Croatian Science Foundation under the project IP-2014-09-2899.



Supported by Croatian science foundation



Faculty of Civil Engineering, University of Zagreb, Croatia