1. KOLOKVIJ iz "OTPORNOSTI MATERIJALA 1" GRUPA B

SVEUČILIŠTE U ZAGREBU
GRAĐEVINSKI FAKULTET
ZAVOD ZA TEHNIČKU MEHANIKU

14. 11. 2016.

Ime i prezime:_____

1. Na čeličnoj ploči poznate su komponente naprezanja:

$\sigma_x = 100MPa$
$\sigma_y = 80MPa$
$\tau_{xy} = 90MPa$

Treba odrediti ukupnu promjenu duljine dijagonale DB i promjenu volumena ploče.

$E = 2 \cdot 10^5\ MPa$
$\nu = 0.3$

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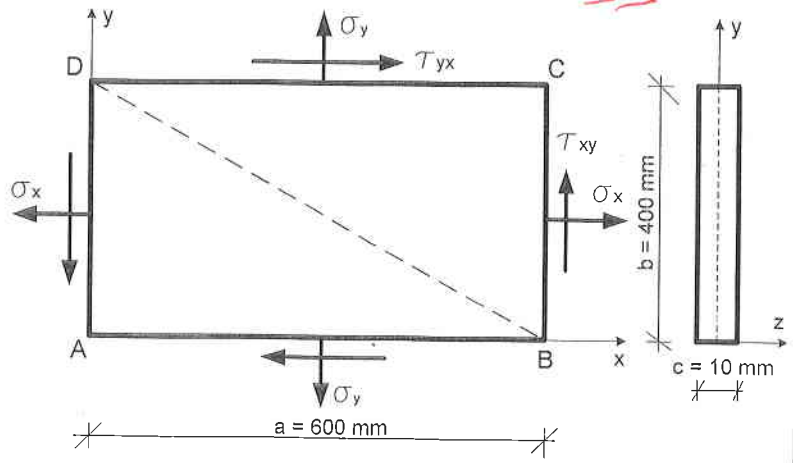

2. Četiri štapa spojena su u točki A. Treba odrediti naprezanja u štapovima i pomak točke A zbog promjene temperature štapova 1 za $\Delta T = +40K$.

$A_1 = 100\ mm^2$
$A_2 = 70\ mm^2$
$E_1 = 1.6 \cdot 10^5\ MPa$
$E_2 = 2 \cdot 10^5\ MPa$
$\alpha_T = 1.2 \cdot 10^{-5}\ 1/K$

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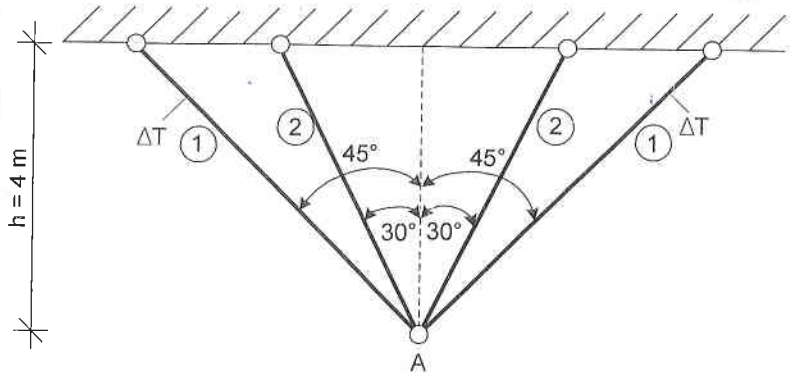

3. Prsten i valjak se dodiruju. Treba odrediti stanje naprezanja u prstenu i valjku ako izvana na prsten djeluje pritisak $p = 6\ MPa$, a temperatura ~~prstena~~ VALJKA se poveća za $\Delta T = +60K$.

$D = 400\ mm$
$h = 10\ mm$
$E_1 = 1.2 \cdot 10^5\ MPa$
$E_2 = 2 \cdot 10^5\ MPa$
$\alpha_T = 10^{-5}\ 1/K$
$\nu = 0.25$

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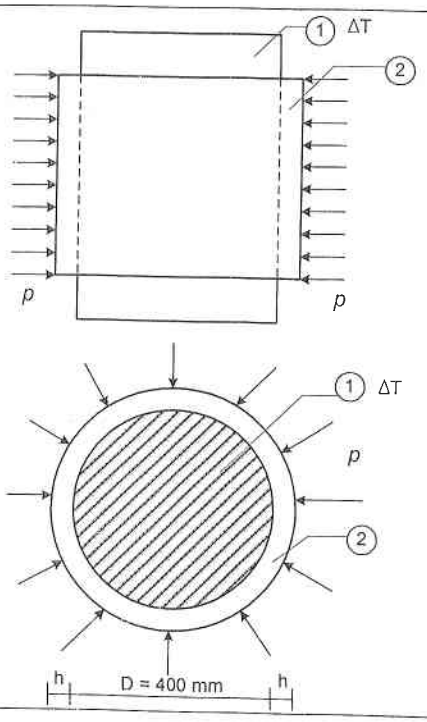


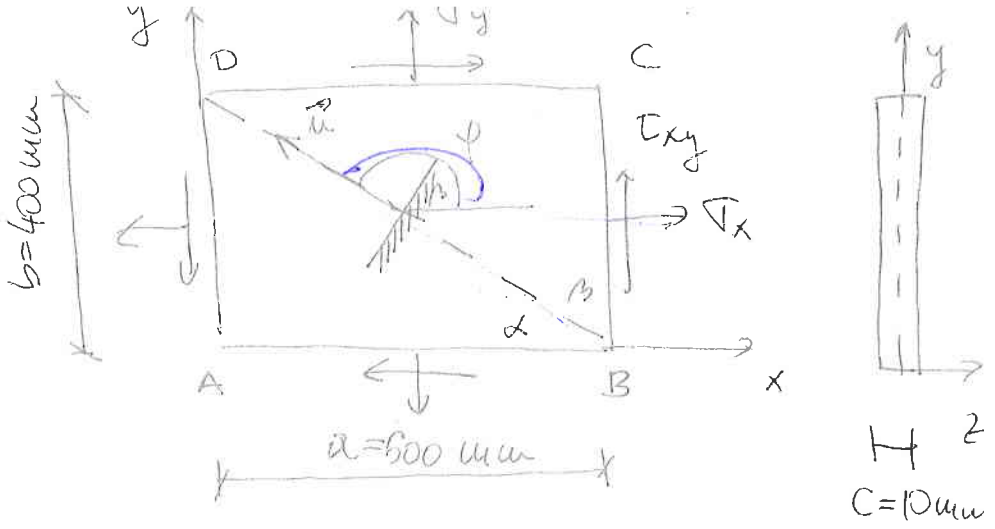
(1.) $\sigma_x = 100$ MPa

$\sigma_y = 80$ MPa

$\tau_{xy} = 90$ MPa

$E = 2 \cdot 10^5$ MPa

$\nu = 0,3$



$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) = \frac{1}{2\cdot10^5}(100 - 0.3\cdot 80) = 3,8\cdot10^{-4}$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) = \frac{1}{2\cdot10^5}(80 - 0.3\cdot 100) = 2,5\cdot10^{-4}$$

$$\varepsilon_{xy} = \frac{1+\nu}{E}\cdot\tau_{xy} = \frac{1+0.3}{2\cdot10^5}\cdot 90 = 5,85\cdot10^{-4}$$

$$\varepsilon_u = \varepsilon_x\cdot\cos^2\varphi + \varepsilon_y\sin^2\varphi + \varepsilon_{xy}\sin 2\varphi$$
$$= -2\cdot10^{-4}$$

$$\Delta_{DB} = \varepsilon_u \cdot l_{DB} = -0,144 \text{ mm (skrócenie)}$$

$$\operatorname{tg}\alpha = \frac{400}{600}$$

$\alpha = 33,69°$

$\beta = 56,31°$

$\varphi = 146,31°$

$l_{DB} = 721,11$ mm

$$\varepsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)] = \frac{-0.3}{2\cdot10^5}(100+80) = -2,7\cdot10^{-4}$$

$$\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z = 3,6\cdot10^{-4}$$

$$\Delta V = V\cdot\varepsilon_v = 400\cdot600\cdot10\cdot3,6\cdot10^{-4} = 864 \text{ mm}^3 \text{ (przyrost objętości)}$$

(2) $\Delta T = +40\,K$

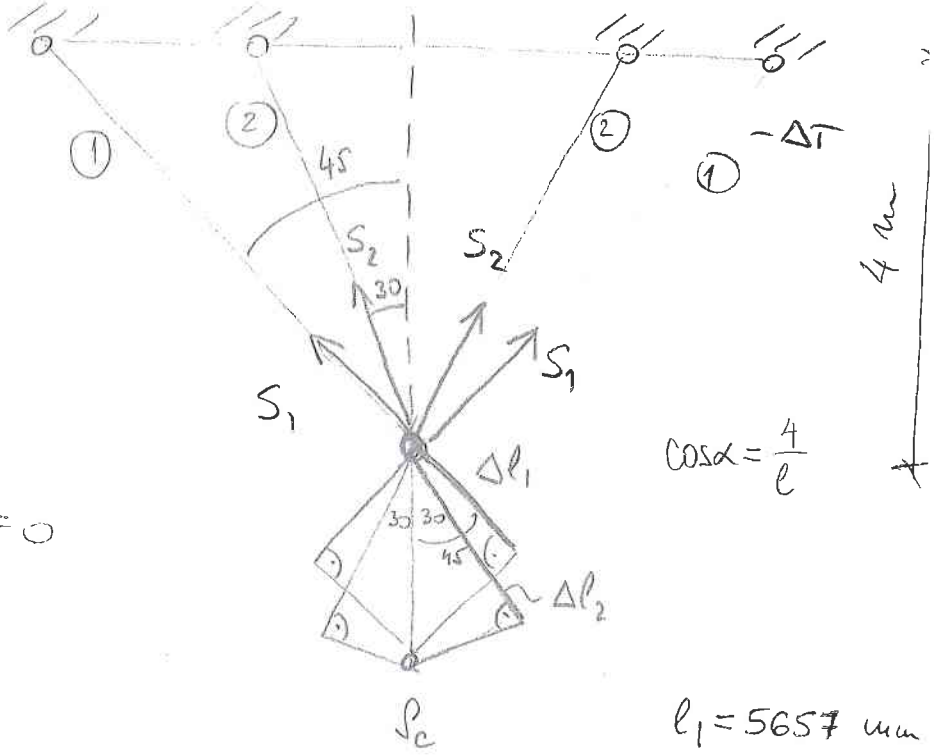
$E_1 = 1{,}6 \cdot 10^5\ MPa$

$E_2 = 2 \cdot 10^5\ MPa$

$A_1 = 100\ mm^2$

$A_2 = 70\ mm^2$

$\alpha_T = 1{,}2 \cdot 10^{-5}\ K^{-1}$



$\cos\alpha = \frac{4}{\ell}$

$2 S_1 \cos 45 + 2 S_2 \cos 30 = 0$

$S_1 = -1{,}225\ S_2$

$\frac{\Delta l_2}{\cos 30} = \frac{\Delta l_1}{\cos 45} = f_c$

$l_1 = 5657\ mm$

$l_2 = 4619\ mm$

$\Delta l_1 = \frac{S_1 l_1}{E_1 A_1} + \alpha \cdot \Delta T \cdot l_1 \qquad \Delta l_2 = \frac{S_2 \cdot l_2}{E_2 \cdot A_2}$

$\frac{S_2 l_2}{A_2 \cdot E_2 \cdot \cos 30} = \frac{S_1 l_1}{E_1 A_1 \cos 45} + \frac{\alpha \cdot \Delta T \cdot l_1}{\cos 45} \quad / \cdot \cos 45$

$S_2 \left(0{,}8165 \cdot \frac{l_2}{A_2 \cdot E_2} + \frac{1{,}225\ l_1}{E_1 A_1} \right) = \alpha \cdot \Delta T \cdot l_1$

$S_2 = \frac{1{,}2 \cdot 10^{-5} \cdot 40 \cdot 5658}{\frac{0{,}8165 \cdot 4619}{70 \cdot 2 \cdot 10^5} + \frac{1{,}225 \cdot 5658}{1{,}6 \cdot 10^5 \cdot 100}} = 3{,}86\ kN\ (Zug)$

$S_1 = -4{,}74\ kN\ (Druck)$

$\sigma_1 = \frac{S_1}{A_1} = -47{,}4\ MPa$

$\sigma_2 = \frac{S_2}{A_2} = 55{,}2\ MPa$

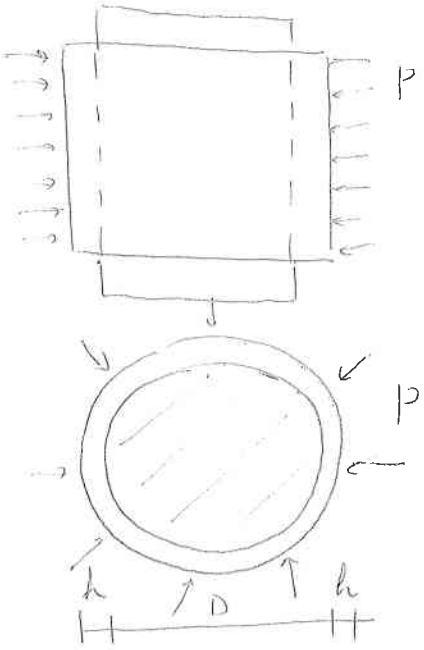
$f_c = \frac{\Delta l_2}{\cos 30} = \frac{S_2 \cdot l_2}{E_2 \cdot A_2 \cos 30}$

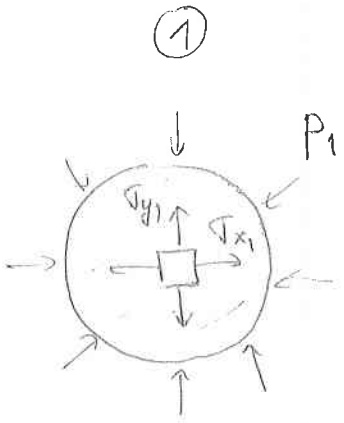
$f_c = 1{,}47\ mm$

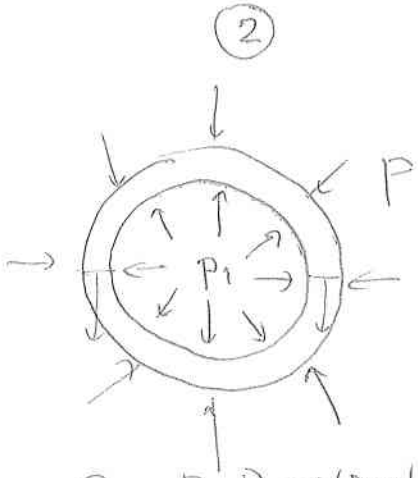
(3)

$D = 400$ mm
$h = 10$ mm
$p = 6$ MPa
$\Delta T = 60$ K
$\nu = 0.25$

$E_1 = 1{,}2 \cdot 10^5$ MPa
$E_2 = 2 \cdot 10^5$ MPa
$\alpha_T = 10^{-5}$







$\sigma_{x_1} = \sigma_{y1} = -P_1$

$\varepsilon_{1x} = \frac{1}{E_1}(\sigma_{x_1} - \nu \sigma_{y1}) + \alpha \cdot \Delta T$

$= \frac{1}{E_1}(-P_1 + \nu P_1) + \alpha \cdot \Delta T$

$\sigma_2 = \frac{P_1 \cdot D - p(D+2h)}{2h}$

$\varepsilon_2 = \frac{P_1 \cdot D - p(D+2h)}{2h \cdot E_2}$

U.D. $\varepsilon_2 = \varepsilon_{1x}$

$$\frac{-P_1 + \nu P_1}{E_1} + \alpha \cdot \Delta T = \frac{P_1 \cdot D - p(D+2h)}{2h E_2} \quad / \cdot E_1$$

$$P_1\left(\nu - 1 - \frac{D}{2h}\frac{E_1}{E_2}\right) = -\alpha \cdot \Delta T \cdot E_1 - \frac{p(D+2h)}{2h}\frac{E_1}{E_2}$$

$$P_1 = \frac{-72 - 75{,}6}{-12{,}75} = 11{,}58 \text{ MPa}$$

$\sigma_{x_1} = \sigma_{y1} = -11{,}58$ MPa

$\sigma_2 = \frac{+11{,}58 \cdot 400 - 6 \cdot 420}{20} = 105{,}6$ MPa