OTPORNOST MATERIJALA 1

1. kolokvij, 17. studenoga 2021.

Prezime i ime:

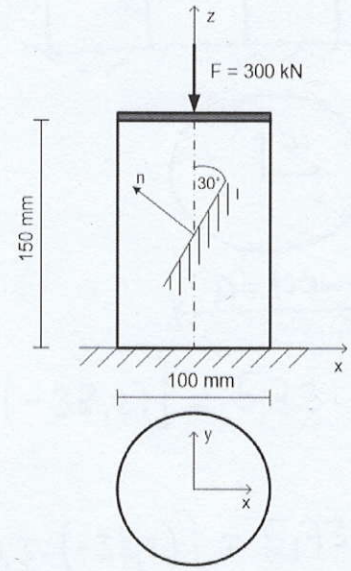
Grupa C

1.

Valjak od izotropnog i homogenog materijala opterećen je prema slici. Treba odrediti:

A) tenzor naprezanja i deformacija za koordinatni sustav xyz
B) apsolutnu i relativnu promjenu volumena valjka
C) vektor punog naprezanja za ravninu s normalom *n* označenom na slici

$E = 2 \cdot 10^5\ MPa$
$\nu = 0.3$

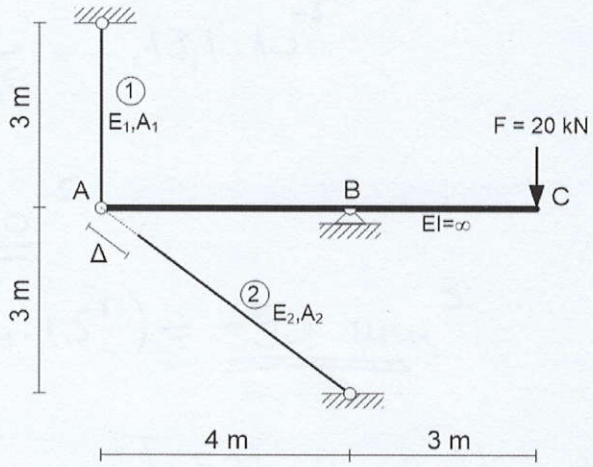


2.

Apsolutno kruta greda ABC pričvršćena je zglobno u točki B i pridržana štapovima **1** i **2** u točki A.
Štap **2** je izveden kraći od svoje potrebne duljine za $\Delta = 3\ mm$.
Treba odrediti <u>naprezanja u štapovima **1** i **2**</u> nakon spajanja štapa 2 i grede u točki A, te <u>pomak točke **C**</u>.

Zadano:

$E_1 = E_2 = 200$ GPa
$A_1 = 120$ mm^2
$A_2 = 200$ mm^2

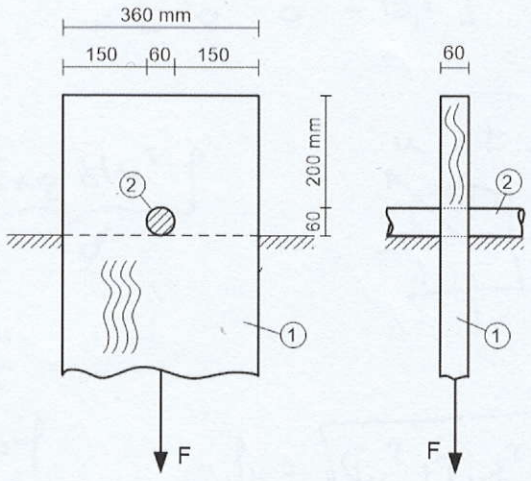


3.

Na spoj drvenog elementa 1 i čeličnog moždanika 2 djeluje sila F.
Treba odrediti dopuštenu veličinu sile F ako su zadane vrijednosti dopuštenih naprezanja.

Za drvo:
$\sigma^{\parallel}_{vl,dop} = 10\ MPa$
$\sigma^{\parallel}_{tl,dop} = 12\ MPa$
$\tau^{\parallel}_{dop} = 2\ MPa$

Za čelik:
$\sigma_{o,dop} = 210\ MPa$
$\tau_{dop} = 110\ MPa$

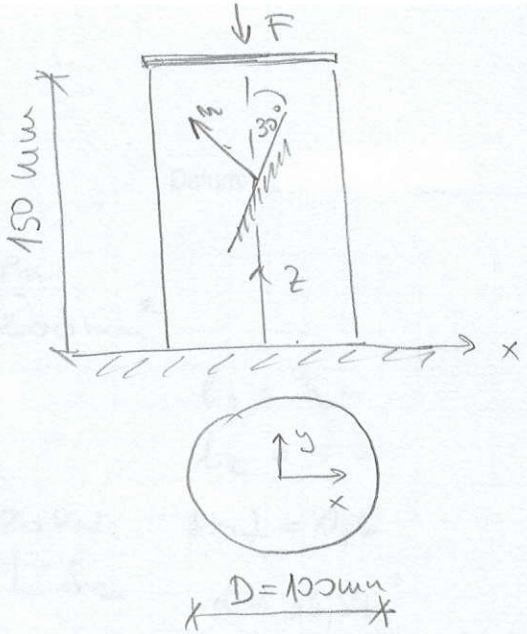


① $E = 2 \cdot 10^5$ MPa

$\nu = 0{,}3$

$F = 300$ kN

$$\sigma_z = \frac{F}{A} = \frac{-300 \cdot 10^3}{\frac{100^2 \pi}{4}} = -38{,}2 \text{ MPa}$$



$$\sigma_x = \sigma_y = 0$$

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right] = \frac{1}{2 \cdot 10^5}\left[-0{,}3 \cdot (-38{,}2)\right] = 5{,}73 \cdot 10^{-5}$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right] = \frac{1}{2 \cdot 10^5}\left[-0{,}3 \cdot (-38{,}2)\right] = 5{,}73 \cdot 10^{-5}$$

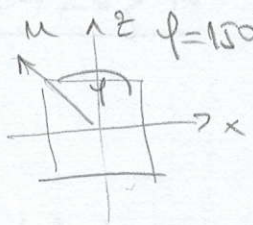
$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right] = \frac{-38{,}2}{2 \cdot 10^5} = -13{,}1 \cdot 10^{-5}$$

$$\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z = \underline{-7{,}64 \cdot 10^{-5}}$$

$$\Delta V = V \cdot \varepsilon_v = \frac{100^2 \pi}{4} \cdot 150 \cdot (-7{,}64 \cdot 10^{-5}) = \underline{-90 \text{ mm}^3}$$

$$\sigma_{ij} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -38{,}2 \end{bmatrix} \text{MPa} \qquad \varepsilon_{ij} = \begin{bmatrix} 5{,}73 & 0 & 0 \\ 0 & 5{,}73 & 0 \\ 0 & 0 & -13{,}1 \end{bmatrix} \cdot 10^{-5}$$

$$\sigma_n = \underbrace{\sigma_x \cdot \cos^2\varphi}_{0} + \sigma_z \sin^2\varphi + \underbrace{\tau_{xz} \sin^2\varphi}_{0}$$



$$= \sigma_z \cdot \sin^2\varphi = \underline{-9{,}55 \text{ MPa}}$$

$$\tau_{nt} = \frac{\sigma_z - \sigma_x}{2} \sin 2\varphi + \underbrace{\tau_{xz} \cdot \cos\varphi}_{0}$$

$$= \frac{\sigma_z}{2} \sin 2\varphi = \underline{16{,}54 \text{ MPa}}$$

$$f_n = \sqrt{\sigma_n^2 + \tau_{nt}^2}$$

$$= \underline{13{,}1 \text{ MPa}}$$

2.B

$E_1 = E_2 = E = 2 \cdot 10^5$ MPa

$A_1 = 120\ mm^2$ $A_2 = 200\ mm^2$

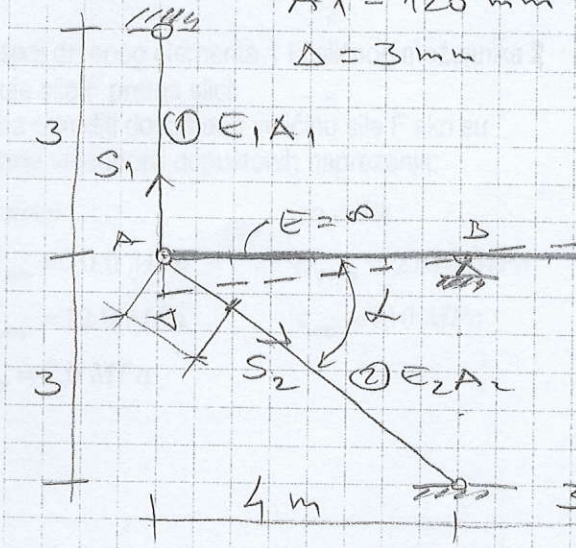
$\Delta = 3$ mm

$l_1 = 3$ m

$l_2 = 5$ m

$\sin\alpha = 0{,}6$

$\alpha = 36{,}86°$

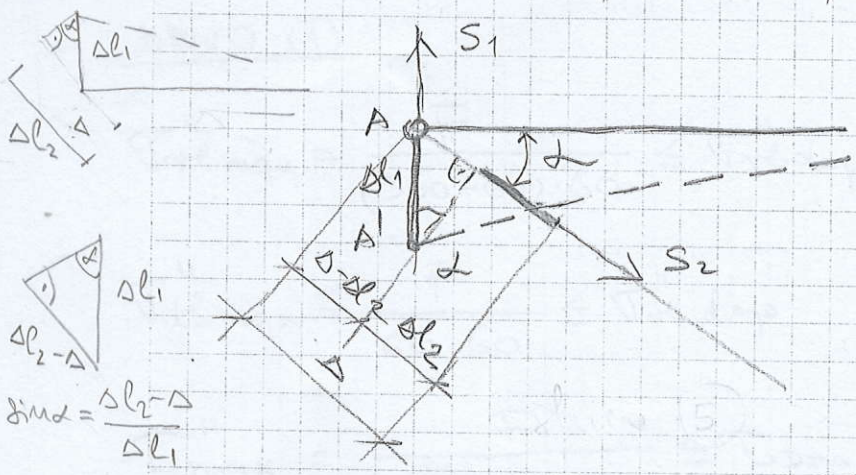


U.R. $\sum M_B = 0$

$S_1 \cdot 4 - S_2 \cdot \sin\alpha \cdot 4 + F \cdot 3 = 0$

$S_1 = S_2 \cdot \sin\alpha - \frac{3}{4} F$ (1)

14:



UVJET. DEF.

$\sin\alpha = \frac{\Delta - \Delta l_2}{\Delta l_1}$ (2)

$\Delta l_1 \cdot \sin\alpha = \Delta - \Delta l_2$ (2')

$\frac{S_1 \cdot l_1}{EA_1} \cdot \sin\alpha = \Delta - \frac{S_2 \cdot l_2}{EA_2}$ (2'')

(1) → (2'') $\left(S_2 \cdot \sin\alpha - \frac{3}{4} F\right) \frac{l_1 \cdot \sin\alpha}{EA_1} + \frac{S_2 \cdot l_2}{EA_2} = \Delta \Big/ \frac{EA_1}{l_1}$

$S_2 \cdot \sin^2\alpha - \frac{3}{4} F \cdot \sin\alpha + S_2 \frac{A_1}{A_2} \cdot \frac{l_2}{l_1} = \Delta \frac{EA_1}{l_1}$

$S_2 = \frac{\frac{3}{4} F \cdot \sin\alpha + \Delta \frac{EA_1}{l_1}}{\sin^2\alpha + \frac{A_1}{A_2} \frac{l_2}{l_1}} = \frac{9000 + 24000}{1{,}36} = 24265$ N

$S_2 = +24{,}265$ kN VLAK

$S_1 = S_2 \cdot \sin\alpha - \frac{3}{4} F = 14{,}559 - 15{,}00 = -0{,}441$ kN

TLAK

$\sigma_{①} = \frac{S_1}{A_1} = -3{,}675$ MPa

$\sigma_{②} = \frac{S_2}{A_2} = +121{,}325$ MPa

$\Delta l_1 = \frac{S_1 \cdot l_1}{EA_1} = -0{,}055$ mm

$f_c = \Delta l_1 \cdot \frac{3}{4} = +0{,}0413$ mm

SVEUČILIŠTE U ZAGREBU
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FACULTY OF CIVIL ENGINEERING

Datum: _____

3. C.

Na spoj drvenog elementa 1 i čeličnog moždanika 2 djeluje sila F prema slici.
Treba odrediti dopuštenu veličinu sile F ako su zadane vrijednosti dopuštenih naprezanja:

za drvo:

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za čelik:

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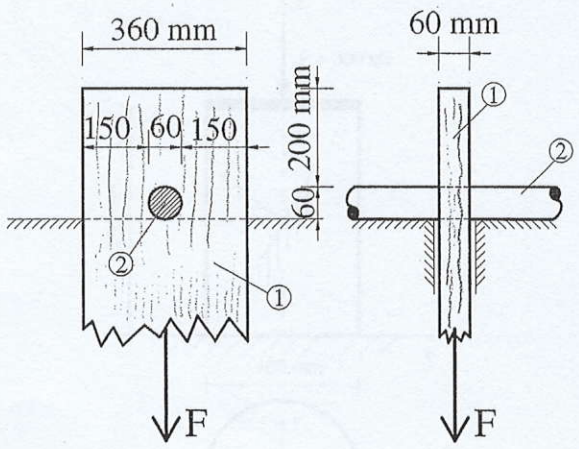

DRVO ①

$$\sigma''_{vl\,max} = \frac{F}{(360-60)\cdot 60} \le \sigma''_{vl\,dop} \rightarrow F \le 180{,}0\ kN$$

$$\sigma''_{tl\,max} = \frac{F}{60\cdot 60} \le \sigma''_{tl\,dop} \rightarrow F \le 43{,}20\ kN$$

MJERODAVNO !

$$\tau''_{max} = \frac{F}{(200+30)\cdot 60\cdot 2} \le \tau_{dop} \rightarrow F \le 55{,}20\ kN$$

ČELIK ②

$$\sigma_{o\,max} = \frac{F}{60\cdot 60} \le \sigma_{o\,dop} \rightarrow F \le 756{,}0\ kN$$

$$\tau_{max} = \frac{F}{2\cdot \frac{60^2\pi}{4}} \le \tau_{dop} \rightarrow F \le 622{,}04\ kN$$

$F_{dop} = 43{,}0\ kN$

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