Poglavlje 1
Matrice i linearni sustavi

ak. god. 2021./2022.

1.1 Osnovno o matricama

Definicija (3.1)

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Matrica A je tipa $m \times n \, (A \in \mathbb{R}^{m \times n})$ pa ima m redaka i n stupaca. S a_{ij} označavamo opći član matrice A.

Kvadratna matrica je matrica tipa $n \times n$ (ima jednak broj redaka i stupaca).
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Donje trokutasta kvadratna matrica je matrica kojoj su svi elementi iznad glavne dijagonale 0, tj. $a_{ij} = 0, i < j$.

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Nul matrica je matrica kojoj su svi elementi 0, tj. $a_{ij} = 0, \forall i, j$.

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1.2 Operacije s matricama

Množenje matrice skalarom $\lambda \in \mathbb{R}$

$$A = (a_{ij}), \lambda \in \mathbb{R} \Longrightarrow \lambda \cdot A = (\lambda \cdot a_{ij})$$

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$A, B \in \mathbb{R}^{m \times n}$

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$$A + B = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{bmatrix}$$

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$$
\begin{aligned}
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&= \begin{bmatrix} a_{11} + b_{11} & a_{12} + b_{12} & \cdots & a_{1n} + b_{1n} \\ a_{21} + b_{21} & a_{22} + b_{22} & \cdots & a_{2n} + b_{2n} \\ \vdots & \vdots & \ddots & \\ a_{m1} + b_{m1} & a_{m2} + b_{m2} & \cdots & a_{mn} + b_{mn} \end{bmatrix}
\end{aligned}
$$

Množenje matrica

Za $A \in \mathbb{R}^{m\times n}$ i $B \in \mathbb{R}^{n\times p}$ definiramo $A \cdot B = C = (c_{ik}) \in \mathbb{R}^{m\times p}$

$$c_{ik} = \sum_{j=1}^{n} a_{ij} \cdot b_{jk}$$

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$$= \begin{bmatrix} 1\cdot 2 + 5\cdot 0 & 1\cdot 1 + 5\cdot(-1) & 1\cdot 0 + 5\cdot 1 \\ 3\cdot 2 + 6\cdot 0 & 3\cdot 1 + 6\cdot(-1) & 3\cdot 0 + 6\cdot 1 \\ 1\cdot 2 + 0\cdot 0 & 1\cdot 1 + 0\cdot(-1) & 1\cdot 0 + 0\cdot 1 \\ 2\cdot 2 + 3\cdot 0 & 2\cdot 1 + 3\cdot(-1) & 2\cdot 0 + 3\cdot 1 \end{bmatrix}$$

Primjer: $A = \begin{bmatrix} 1 & 5 \\ 3 & 6 \\ 1 & 0 \\ 2 & 3 \end{bmatrix} \in \mathbb{R}^{4\times 2}$, $B = \begin{bmatrix} 2 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \in \mathbb{R}^{2\times 3}$

$$\begin{aligned} A \cdot B &= \begin{bmatrix} 1 & 5 \\ 3 & 6 \\ 1 & 0 \\ 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1\cdot 2 + 5\cdot 0 & 1\cdot 1 + 5\cdot(-1) & 1\cdot 0 + 5\cdot 1 \\ 3\cdot 2 + 6\cdot 0 & 3\cdot 1 + 6\cdot(-1) & 3\cdot 0 + 6\cdot 1 \\ 1\cdot 2 + 0\cdot 0 & 1\cdot 1 + 0\cdot(-1) & 1\cdot 0 + 0\cdot 1 \\ 2\cdot 2 + 3\cdot 0 & 2\cdot 1 + 3\cdot(-1) & 2\cdot 0 + 3\cdot 1 \end{bmatrix} \\ &= \begin{bmatrix} 2 & -4 & 5 \\ 6 & -3 & 6 \\ 2 & 1 & 0 \\ 4 & -1 & 3 \end{bmatrix} \in \mathbb{R}^{4\times 3} \end{aligned}$$

Zadatak (3.1.)

Zadane su matrice $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} \in \mathbb{R}^{3\times 2}$ i $B = \begin{bmatrix} 2 & 0 & 1 \\ -1 & 1 & 3 \end{bmatrix} \in \mathbb{R}^{2\times 3}$.

Izračunajte $2A + B^T$.

Zadatak (3.1.)

Zadane su matrice $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} \in \mathbb{R}^{3\times 2}$ i $B = \begin{bmatrix} 2 & 0 & 1 \\ -1 & 1 & 3 \end{bmatrix} \in \mathbb{R}^{2\times 3}$.

Izračunajte $2A + B^T$.

Rješenje:

Zadatak (3.1.)

Zadane su matrice $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} \in \mathbb{R}^{3\times 2}$ i $B = \begin{bmatrix} 2 & 0 & 1 \\ -1 & 1 & 3 \end{bmatrix} \in \mathbb{R}^{2\times 3}$.

Izračunajte $2A + B^T$.

Rješenje:

$$2 \cdot A + B^T \quad =$$

Zadatak (3.1.)

Zadane su matrice $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} \in \mathbb{R}^{3\times 2}$ i $B = \begin{bmatrix} 2 & 0 & 1 \\ -1 & 1 & 3 \end{bmatrix} \in \mathbb{R}^{2\times 3}$.

Izračunajte $2A + B^T$.

Rješenje:

$$2 \cdot A + B^T \quad = \quad 2 \cdot \begin{bmatrix} 3 & 1 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$$

Zadatak (3.1.)

Zadane su matrice $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} \in \mathbb{R}^{3\times 2}$ i $B = \begin{bmatrix} 2 & 0 & 1 \\ -1 & 1 & 3 \end{bmatrix} \in \mathbb{R}^{2\times 3}$.

Izračunajte $2A + B^T$.

Rješenje:

$$2 \cdot A + B^T \quad = \quad 2 \cdot \begin{bmatrix} 3 & 1 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 1 \\ -1 & 1 & 3 \end{bmatrix}^T$$

Zadatak (3.1.)

Zadane su matrice $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} \in \mathbb{R}^{3\times 2}$ i $B = \begin{bmatrix} 2 & 0 & 1 \\ -1 & 1 & 3 \end{bmatrix} \in \mathbb{R}^{2\times 3}$.

Izračunajte $2A + B^T$.

Rješenje:

$$\begin{aligned} 2\cdot A + B^T &= 2\cdot \begin{bmatrix} 3 & 1 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 1 \\ -1 & 1 & 3 \end{bmatrix}^T \\ &= \begin{bmatrix} 6 & 2 \\ -2 & 4 \\ 0 & 2 \end{bmatrix} \end{aligned}$$

Zadatak (3.1.)

Zadane su matrice $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} \in \mathbb{R}^{3\times 2}$ i $B = \begin{bmatrix} 2 & 0 & 1 \\ -1 & 1 & 3 \end{bmatrix} \in \mathbb{R}^{2\times 3}$.

Izračunajte $2A + B^T$.

Rješenje:

$$
\begin{aligned}
2 \cdot A + B^T &= 2 \cdot \begin{bmatrix} 3 & 1 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 1 \\ -1 & 1 & 3 \end{bmatrix}^T \\
&= \begin{bmatrix} 6 & 2 \\ -2 & 4 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ 0 & 1 \\ 1 & 3 \end{bmatrix}
\end{aligned}
$$

Zadatak (3.1.)

Zadane su matrice $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} \in \mathbb{R}^{3\times 2}$ i $B = \begin{bmatrix} 2 & 0 & 1 \\ -1 & 1 & 3 \end{bmatrix} \in \mathbb{R}^{2\times 3}$.

Izračunajte $2A + B^T$.

Rješenje:

$$
\begin{aligned}
2 \cdot A + B^T &= 2 \cdot \begin{bmatrix} 3 & 1 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 1 \\ -1 & 1 & 3 \end{bmatrix}^T \\
&= \begin{bmatrix} 6 & 2 \\ -2 & 4 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ 0 & 1 \\ 1 & 3 \end{bmatrix} \\
&= \begin{bmatrix} 8 & 1 \\ -2 & 5 \\ 1 & 5 \end{bmatrix}
\end{aligned}
$$

Zadatak (3.1.)

Zadane su matrice $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} \in \mathbb{R}^{3\times 2}$ i $B = \begin{bmatrix} 2 & 0 & 1 \\ -1 & 1 & 3 \end{bmatrix} \in \mathbb{R}^{2\times 3}$.

Izračunajte $2A + B^T$.

Rješenje:

$$
\begin{aligned}
2 \cdot A + B^T &= 2 \cdot \begin{bmatrix} 3 & 1 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 1 \\ -1 & 1 & 3 \end{bmatrix}^T \\
&= \begin{bmatrix} 6 & 2 \\ -2 & 4 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ 0 & 1 \\ 1 & 3 \end{bmatrix} \\
&= \begin{bmatrix} 8 & 1 \\ -2 & 5 \\ 1 & 5 \end{bmatrix}
\end{aligned}
$$

Zadatak (3.2.)

Zadane su kvadratne matrice $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$ i $B = \begin{bmatrix} 4 & 1 & 1 \\ -4 & 2 & 0 \\ 1 & 2 & 1 \end{bmatrix} \in \mathbb{R}^{3\times 3}$.

Izračunajte $A \cdot B$ i $B \cdot A$.

$$A \cdot B = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 4 & 1 & 1 \\ -4 & 2 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

Zadatak (3.2.)

Zadane su kvadratne matrice $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$ i $B = \begin{bmatrix} 4 & 1 & 1 \\ -4 & 2 & 0 \\ 1 & 2 & 1 \end{bmatrix} \in \mathbb{R}^{3\times 3}$.

Izračunajte $A \cdot B$ i $B \cdot A$.

$$
\begin{aligned}
A \cdot B &= \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 4 & 1 & 1 \\ -4 & 2 & 0 \\ 1 & 2 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 4 - 8 + 1 & & \\ & & \\ & & \end{bmatrix}
\end{aligned}
$$

Zadatak (3.2.)

Zadane su kvadratne matrice $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$ i $B = \begin{bmatrix} 4 & 1 & 1 \\ -4 & 2 & 0 \\ 1 & 2 & 1 \end{bmatrix} \in \mathbb{R}^{3\times 3}$.

Izračunajte $A \cdot B$ i $B \cdot A$.

$$\begin{aligned} A \cdot B &= \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 4 & 1 & 1 \\ -4 & 2 & 0 \\ 1 & 2 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 4-8+1 & 1+4+2 & \\ & & \\ & & \end{bmatrix} \end{aligned}$$

Zadatak (3.2.)

Zadane su kvadratne matrice $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$ i $B = \begin{bmatrix} 4 & 1 & 1 \\ -4 & 2 & 0 \\ 1 & 2 & 1 \end{bmatrix} \in \mathbb{R}^{3\times 3}$.

Izračunajte $A \cdot B$ i $B \cdot A$.

$$
\begin{aligned}
A \cdot B &= \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 4 & 1 & 1 \\ -4 & 2 & 0 \\ 1 & 2 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 4-8+1 & 1+4+2 & 1+0+1 \\ & & \\ & & \end{bmatrix}
\end{aligned}
$$

Zadatak (3.2.)

Zadane su kvadratne matrice $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$ i $B = \begin{bmatrix} 4 & 1 & 1 \\ -4 & 2 & 0 \\ 1 & 2 & 1 \end{bmatrix} \in \mathbb{R}^{3\times 3}$.

Izračunajte $A \cdot B$ i $B \cdot A$.

$$\begin{aligned} A \cdot B &= \begin{bmatrix} 1 & 2 & 1 \\ \boxed{2 \quad 1 \quad 2} \\ 1 & 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} \boxed{4} & 1 & 1 \\ \boxed{-4} & 2 & 0 \\ \boxed{1} & 2 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 4-8+1 & 1+4+2 & 1+0+1 \\ 8-4+2 & & \\ & & \end{bmatrix} \end{aligned}$$

Zadatak (3.2.)

Zadane su kvadratne matrice $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$ i $B = \begin{bmatrix} 4 & 1 & 1 \\ -4 & 2 & 0 \\ 1 & 2 & 1 \end{bmatrix} \in \mathbb{R}^{3\times 3}$.

Izračunajte $A \cdot B$ i $B \cdot A$.

$$
\begin{aligned}
A \cdot B &= \begin{bmatrix} 1 & 2 & 1 \\ \boxed{2 \quad 1 \quad 2} \\ 1 & 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 4 & 1 & 1 \\ -4 & 2 & 0 \\ 1 & 2 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 4-8+1 & 1+4+2 & 1+0+1 \\ 8-4+2 & 2+2+4 & \\ & & \end{bmatrix}
\end{aligned}
$$

Zadatak (3.2.)

Zadane su kvadratne matrice $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$ i $B = \begin{bmatrix} 4 & 1 & 1 \\ -4 & 2 & 0 \\ 1 & 2 & 1 \end{bmatrix} \in \mathbb{R}^{3\times 3}$.

Izračunajte $A \cdot B$ i $B \cdot A$.

$$\begin{aligned} A \cdot B &= \begin{bmatrix} 1 & 2 & 1 \\ \boxed{2 \quad 1 \quad 2} \\ 1 & 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 4 & 1 & 1 \\ -4 & 2 & 0 \\ 1 & 2 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 4-8+1 & 1+4+2 & 1+0+1 \\ 8-4+2 & 2+2+4 & 2+0+2 \\ & & \end{bmatrix} \end{aligned}$$

Zadatak (3.2.)

Zadane su kvadratne matrice $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$ i $B = \begin{bmatrix} 4 & 1 & 1 \\ -4 & 2 & 0 \\ 1 & 2 & 1 \end{bmatrix} \in \mathbb{R}^{3\times 3}$.

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$$
\begin{aligned}
A \cdot B &= \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 4 & 1 & 1 \\ -4 & 2 & 0 \\ 1 & 2 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 4-8+1 & 1+4+2 & 1+0+1 \\ 8-4+2 & 2+2+4 & 2+0+2 \\ 4-8+3 & & \end{bmatrix}
\end{aligned}
$$

Zadatak (3.2.)

Zadane su kvadratne matrice $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$ i $B = \begin{bmatrix} 4 & 1 & 1 \\ -4 & 2 & 0 \\ 1 & 2 & 1 \end{bmatrix} \in \mathbb{R}^{3\times 3}$.

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$$\begin{aligned} A \cdot B &= \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 4 & 1 & 1 \\ -4 & 2 & 0 \\ 1 & 2 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 4-8+1 & 1+4+2 & 1+0+1 \\ 8-4+2 & 2+2+4 & 2+0+2 \\ 4-8+3 & 1+4+6 & \end{bmatrix} \end{aligned}$$

Zadatak (3.2.)

Zadane su kvadratne matrice $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$ i $B = \begin{bmatrix} 4 & 1 & 1 \\ -4 & 2 & 0 \\ 1 & 2 & 1 \end{bmatrix} \in \mathbb{R}^{3\times 3}$.

Izračunajte $A \cdot B$ i $B \cdot A$.

$$
\begin{aligned}
A \cdot B &= \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 4 & 1 & 1 \\ -4 & 2 & 0 \\ 1 & 2 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 4-8+1 & 1+4+2 & 1+0+1 \\ 8-4+2 & 2+2+4 & 2+0+2 \\ 4-8+3 & 1+4+6 & 1+0+3 \end{bmatrix}
\end{aligned}
$$

Zadatak (3.2.)

Zadane su kvadratne matrice $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$ i $B = \begin{bmatrix} 4 & 1 & 1 \\ -4 & 2 & 0 \\ 1 & 2 & 1 \end{bmatrix} \in \mathbb{R}^{3\times 3}$.

Izračunajte $A \cdot B$ i $B \cdot A$.

$$\begin{aligned} A \cdot B &= \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 4 & 1 & 1 \\ -4 & 2 & 0 \\ 1 & 2 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 4-8+1 & 1+4+2 & 1+0+1 \\ 8-4+2 & 2+2+4 & 2+0+2 \\ 4-8+3 & 1+4+6 & 1+0+3 \end{bmatrix} \\ &= \begin{bmatrix} -3 & 7 & 2 \\ 6 & 8 & 4 \\ -1 & 11 & 4 \end{bmatrix} \in \mathbb{R}^{3\times 3} \end{aligned}$$

$$B \cdot A \quad = \quad \begin{bmatrix} 4 & 1 & 1 \\ -4 & 2 & 0 \\ 1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

$$
\begin{aligned}
B \cdot A &= \begin{bmatrix} \boxed{4 \quad 1 \quad 1} \\ -4 \quad 2 \quad 0 \\ 1 \quad 2 \quad 1 \end{bmatrix} \cdot \begin{bmatrix} \boxed{\begin{matrix} 1 \\ 2 \\ 1 \end{matrix}} & \begin{matrix} 2 \\ 1 \\ 2 \end{matrix} & \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} \end{bmatrix} \\
&= \begin{bmatrix} 4+2+1 & & \\ & & \\ & & \end{bmatrix}
\end{aligned}
$$

$$B \cdot A = \begin{bmatrix} \boxed{4 \quad 1 \quad 1} \\ -4 \quad 2 \quad 0 \\ 1 \quad 2 \quad 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & \boxed{2} & 1 \\ 2 & \boxed{1} & 2 \\ 1 & \boxed{2} & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4+2+1 & 8+1+2 & \\ & & \\ & & \end{bmatrix}$$

$$
\begin{aligned}
B \cdot A &= \begin{bmatrix} \boxed{4 \quad 1 \quad 1} \\ -4 \quad 2 \quad 0 \\ 1 \quad 2 \quad 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix} \\
&= \begin{bmatrix} 4+2+1 & 8+1+2 & 4+2+3 \\ & & \\ & & \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
B \cdot A &= \begin{bmatrix} 4 & 1 & 1 \\ -4 & 2 & 0 \\ 1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix} \\
&= \begin{bmatrix} 4+2+1 & 8+1+2 & 4+2+3 \\ -4+4+0 & & \\ & & \end{bmatrix}
\end{aligned}
$$

$$B \cdot A = \begin{bmatrix} 4 & 1 & 1 \\ -4 & 2 & 0 \\ 1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4+2+1 & 8+1+2 & 4+2+3 \\ -4+4+0 & -8+2+0 & \\ & & \end{bmatrix}$$

$$
\begin{aligned}
B \cdot A &= \begin{bmatrix} 4 & 1 & 1 \\ -4 & 2 & 0 \\ 1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix} \\
&= \begin{bmatrix} 4+2+1 & 8+1+2 & 4+2+3 \\ -4+4+0 & -8+2+0 & -4+4+0 \\ & & \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
B \cdot A &= \begin{bmatrix} 4 & 1 & 1 \\ -4 & 2 & 0 \\ 1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix} \\
&= \begin{bmatrix} 4+2+1 & 8+1+2 & 4+2+3 \\ -4+4+0 & -8+2+0 & -4+4+0 \\ 1+4+1 & & \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
B \cdot A &= \begin{bmatrix} 4 & 1 & 1 \\ -4 & 2 & 0 \\ 1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix} \\
&= \begin{bmatrix} 4+2+1 & 8+1+2 & 4+2+3 \\ -4+4+0 & -8+2+0 & -4+4+0 \\ 1+4+1 & 2+2+2 & \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
B \cdot A &= \begin{bmatrix} 4 & 1 & 1 \\ -4 & 2 & 0 \\ 1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix} \\
&= \begin{bmatrix} 4+2+1 & 8+1+2 & 4+2+3 \\ -4+4+0 & -8+2+0 & -4+4+0 \\ 1+4+1 & 2+2+2 & 1+4+3 \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
B \cdot A &= \begin{bmatrix} 4 & 1 & 1 \\ -4 & 2 & 0 \\ 1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix} \\
&= \begin{bmatrix} 4+2+1 & 8+1+2 & 4+2+3 \\ -4+4+0 & -8+2+0 & -4+4+0 \\ 1+4+1 & 2+2+2 & 1+4+3 \end{bmatrix} \\
&= \begin{bmatrix} 7 & 11 & 9 \\ 0 & -6 & 0 \\ 6 & 6 & 8 \end{bmatrix} \in \mathbb{R}^{3\times 3}
\end{aligned}
$$

$$A \cdot B = \begin{bmatrix} -3 & 7 & 2 \\ 6 & 8 & 4 \\ -1 & 11 & 4 \end{bmatrix} \in \mathbb{R}^{3\times 3}$$

$$B \cdot A = \begin{bmatrix} 7 & 11 & 9 \\ 0 & -6 & 0 \\ 6 & 6 & 8 \end{bmatrix} \in \mathbb{R}^{3\times 3}$$

$$A \cdot B = \begin{bmatrix} -3 & 7 & 2 \\ 6 & 8 & 4 \\ -1 & 11 & 4 \end{bmatrix} \in \mathbb{R}^{3\times 3}$$

$$B \cdot A = \begin{bmatrix} 7 & 11 & 9 \\ 0 & -6 & 0 \\ 6 & 6 & 8 \end{bmatrix} \in \mathbb{R}^{3\times 3}$$

$A \cdot B \neq B \cdot A \Longrightarrow$ množenje matrica nije komutativno.

Zadatak (3.3.)

Zadane su matrice $A = \begin{bmatrix} 3 & 1 & 2 \end{bmatrix}$ i $B = \begin{bmatrix} 2 & -1 & 1 \end{bmatrix} \in \mathbb{R}^{1\times 3}$. Izračunajte $A^T \cdot B$ i $A \cdot B^T$.

Zadatak (3.3.)

Zadane su matrice $A = \begin{bmatrix} 3 & 1 & 2 \end{bmatrix}$ i $B = \begin{bmatrix} 2 & -1 & 1 \end{bmatrix} \in \mathbb{R}^{1\times 3}$. Izračunajte $A^T \cdot B$ i $A \cdot B^T$.

Rješenje:

$A^T =$

Zadatak (3.3.)

Zadane su matrice $A = \begin{bmatrix} 3 & 1 & 2 \end{bmatrix}$ i $B = \begin{bmatrix} 2 & -1 & 1 \end{bmatrix} \in \mathbb{R}^{1\times 3}$. Izračunajte $A^T \cdot B$ i $A \cdot B^T$.

Rješenje:

$$A^T = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} \in \mathbb{R}^{3\times 1},$$

Zadatak (3.3.)

Zadane su matrice $A = \begin{bmatrix} 3 & 1 & 2 \end{bmatrix}$ i $B = \begin{bmatrix} 2 & -1 & 1 \end{bmatrix} \in \mathbb{R}^{1\times 3}$. Izračunajte $A^T \cdot B$ i $A \cdot B^T$.

Rješenje:

$$A^T = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} \in \mathbb{R}^{3\times 1}, B^T =$$

Zadatak (3.3.)

Zadane su matrice $A = \begin{bmatrix} 3 & 1 & 2 \end{bmatrix}$ i $B = \begin{bmatrix} 2 & -1 & 1 \end{bmatrix} \in \mathbb{R}^{1\times 3}$. Izračunajte $A^T \cdot B$ i $A \cdot B^T$.

Rješenje:

$$A^T = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} \in \mathbb{R}^{3\times 1}, B^T = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \in \mathbb{R}^{3\times 1}$$

Zadatak (3.3.)

Zadane su matrice $A = \begin{bmatrix} 3 & 1 & 2 \end{bmatrix}$ i $B = \begin{bmatrix} 2 & -1 & 1 \end{bmatrix} \in \mathbb{R}^{1\times 3}$. Izračunajte $A^T \cdot B$ i $A \cdot B^T$.

Rješenje:

$$A^T = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} \in \mathbb{R}^{3\times 1}, B^T = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \in \mathbb{R}^{3\times 1}$$

$$A^T \cdot B =$$

Zadatak (3.3.)

Zadane su matrice $A = \begin{bmatrix} 3 & 1 & 2 \end{bmatrix}$ i $B = \begin{bmatrix} 2 & -1 & 1 \end{bmatrix} \in \mathbb{R}^{1\times 3}$. Izračunajte $A^T \cdot B$ i $A \cdot B^T$.

Rješenje:

$$A^T = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} \in \mathbb{R}^{3\times 1}, B^T = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \in \mathbb{R}^{3\times 1}$$

$$A^T \cdot B = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 & 1 \end{bmatrix} =$$

Zadatak (3.3.)

Zadane su matrice $A = \begin{bmatrix} 3 & 1 & 2 \end{bmatrix}$ i $B = \begin{bmatrix} 2 & -1 & 1 \end{bmatrix} \in \mathbb{R}^{1\times 3}$. Izračunajte $A^T \cdot B$ i $A \cdot B^T$.

Rješenje:

$$A^T = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} \in \mathbb{R}^{3\times 1}, B^T = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \in \mathbb{R}^{3\times 1}$$

$$A^T \cdot B = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 6 & -3 & 3 \\ 2 & -1 & 1 \\ 4 & -2 & 2 \end{bmatrix} \in \mathbb{R}^{3\times 3}$$

Zadatak (3.3.)

Zadane su matrice $A = \begin{bmatrix} 3 & 1 & 2 \end{bmatrix}$ i $B = \begin{bmatrix} 2 & -1 & 1 \end{bmatrix} \in \mathbb{R}^{1\times 3}$. Izračunajte $A^T \cdot B$ i $A \cdot B^T$.

Rješenje:

$$A^T = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} \in \mathbb{R}^{3\times 1}, B^T = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \in \mathbb{R}^{3\times 1}$$

$$A^T \cdot B = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 6 & -3 & 3 \\ 2 & -1 & 1 \\ 4 & -2 & 2 \end{bmatrix} \in \mathbb{R}^{3\times 3}$$

$$A \cdot B^T =$$

Zadatak (3.3.)

Zadane su matrice $A = \begin{bmatrix} 3 & 1 & 2 \end{bmatrix}$ i $B = \begin{bmatrix} 2 & -1 & 1 \end{bmatrix} \in \mathbb{R}^{1\times 3}$. Izračunajte $A^T \cdot B$ i $A \cdot B^T$.

Rješenje:

$$A^T = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} \in \mathbb{R}^{3\times 1}, B^T = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \in \mathbb{R}^{3\times 1}$$

$$A^T \cdot B = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 6 & -3 & 3 \\ 2 & -1 & 1 \\ 4 & -2 & 2 \end{bmatrix} \in \mathbb{R}^{3\times 3}$$

$$A \cdot B^T = \begin{bmatrix} 3 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} =$$

Zadatak (3.3.)

Zadane su matrice $A = \begin{bmatrix} 3 & 1 & 2 \end{bmatrix}$ i $B = \begin{bmatrix} 2 & -1 & 1 \end{bmatrix} \in \mathbb{R}^{1\times 3}$. Izračunajte $A^T \cdot B$ i $A \cdot B^T$.

Rješenje:

$$A^T = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} \in \mathbb{R}^{3\times 1}, B^T = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \in \mathbb{R}^{3\times 1}$$

$$A^T \cdot B = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 6 & -3 & 3 \\ 2 & -1 & 1 \\ 4 & -2 & 2 \end{bmatrix} \in \mathbb{R}^{3\times 3}$$

$$A \cdot B^T = \begin{bmatrix} 3 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 7 \end{bmatrix} \in \mathbb{R}^{1\times 1}$$

Zadatak (3.4.)

Odredite $f(A)$, ako je $f(x) = 2x^2 + 3x - 4$ i $A = \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix} \in \mathbb{R}^{2\times 2}$.

Zadatak (3.4.)

Odredite $f(A)$, ako je $f(x) = 2x^2 + 3x - 4$ i $A = \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix} \in \mathbb{R}^{2\times 2}$.

Rješenje: $f(A) = 2A^2 + 3A - 4I$, gdje je $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ jedinična matrica i vrijedi $A \cdot I = I \cdot A = A$

Zadatak (3.4.)

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$$f(A) = 2A^2 + 3A - 4I$$

Zadatak (3.4.)

Odredite $f(A)$, ako je $f(x) = 2x^2 + 3x - 4$ i $A = \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix} \in \mathbb{R}^{2\times 2}$.

Rješenje: $f(A) = 2A^2 + 3A - 4I$, gdje je $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ jedinična matrica i vrijedi $A \cdot I = I \cdot A = A$

$$\begin{aligned} f(A) &= 2A^2 + 3A - 4I \\ &= 2 \cdot \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix} \end{aligned}$$

Zadatak (3.4.)

Odredite $f(A)$, ako je $f(x) = 2x^2 + 3x - 4$ i $A = \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix} \in \mathbb{R}^{2\times 2}$.

Rješenje: $f(A) = 2A^2 + 3A - 4I$, gdje je $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ jedinična matrica i vrijedi $A \cdot I = I \cdot A = A$

$$\begin{aligned} f(A) &= 2A^2 + 3A - 4I \\ &= 2 \cdot \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix} + 3 \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix} \end{aligned}$$

Zadatak (3.4.)

Odredite $f(A)$, ako je $f(x) = 2x^2 + 3x - 4$ i $A = \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix} \in \mathbb{R}^{2\times 2}$.

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$$\begin{aligned} f(A) &= 2A^2 + 3A - 4I \\ &= 2 \cdot \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix} + 3 \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix} - 4 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{aligned}$$

Zadatak (3.4.)

Odredite $f(A)$, ako je $f(x) = 2x^2 + 3x - 4$ i $A = \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix} \in \mathbb{R}^{2\times 2}$.

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$$
\begin{aligned}
f(A) &= 2A^2 + 3A - 4I \\
&= 2 \cdot \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix} + 3 \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix} - 4 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
&= 2 \cdot \begin{bmatrix} 4 & -6 \\ 0 & 16 \end{bmatrix}
\end{aligned}
$$

Zadatak (3.4.)

Odredite $f(A)$, ako je $f(x) = 2x^2 + 3x - 4$ i $A = \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix} \in \mathbb{R}^{2\times 2}$.

Rješenje: $f(A) = 2A^2 + 3A - 4I$, gdje je $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ jedinična matrica i vrijedi $A \cdot I = I \cdot A = A$

$$\begin{aligned} f(A) &= 2A^2 + 3A - 4I \\ &= 2 \cdot \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix} + 3 \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix} - 4 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= 2 \cdot \begin{bmatrix} 4 & -6 \\ 0 & 16 \end{bmatrix} + \begin{bmatrix} 6 & -3 \\ 0 & 12 \end{bmatrix} \end{aligned}$$

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Odredite $f(A)$, ako je $f(x) = 2x^2 + 3x - 4$ i $A = \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix} \in \mathbb{R}^{2\times 2}$.

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$$\begin{aligned} f(A) &= 2A^2 + 3A - 4I \\ &= 2 \cdot \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix} + 3 \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix} - 4 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= 2 \cdot \begin{bmatrix} 4 & -6 \\ 0 & 16 \end{bmatrix} + \begin{bmatrix} 6 & -3 \\ 0 & 12 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} \end{aligned}$$

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Rješenje: $f(A) = 2A^2 + 3A - 4I$, gdje je $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ jedinična matrica i vrijedi $A \cdot I = I \cdot A = A$

$$
\begin{aligned}
f(A) &= 2A^2 + 3A - 4I \\
&= 2 \cdot \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix} + 3 \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix} - 4 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
&= 2 \cdot \begin{bmatrix} 4 & -6 \\ 0 & 16 \end{bmatrix} + \begin{bmatrix} 6 & -3 \\ 0 & 12 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} \\
&= \begin{bmatrix} 8 & -12 \\ 0 & 32 \end{bmatrix}
\end{aligned}
$$

Zadatak (3.4.)

Odredite $f(A)$, ako je $f(x) = 2x^2 + 3x - 4$ i $A = \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix} \in \mathbb{R}^{2\times 2}$.

Rješenje: $f(A) = 2A^2 + 3A - 4I$, gdje je $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ jedinična matrica i vrijedi $A \cdot I = I \cdot A = A$

$$\begin{aligned}
f(A) &= 2A^2 + 3A - 4I \\
&= 2 \cdot \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix} + 3 \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix} - 4 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
&= 2 \cdot \begin{bmatrix} 4 & -6 \\ 0 & 16 \end{bmatrix} + \begin{bmatrix} 6 & -3 \\ 0 & 12 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} \\
&= \begin{bmatrix} 8 & -12 \\ 0 & 32 \end{bmatrix} + \begin{bmatrix} 6 & -3 \\ 0 & 12 \end{bmatrix}
\end{aligned}$$

Zadatak (3.4.)

Odredite $f(A)$, ako je $f(x) = 2x^2 + 3x - 4$ i $A = \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix} \in \mathbb{R}^{2\times 2}$.

Rješenje: $f(A) = 2A^2 + 3A - 4I$, gdje je $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ jedinična matrica i vrijedi $A \cdot I = I \cdot A = A$

$$
\begin{aligned}
f(A) &= 2A^2 + 3A - 4I \\
&= 2 \cdot \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix} + 3 \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix} - 4 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
&= 2 \cdot \begin{bmatrix} 4 & -6 \\ 0 & 16 \end{bmatrix} + \begin{bmatrix} 6 & -3 \\ 0 & 12 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} \\
&= \begin{bmatrix} 8 & -12 \\ 0 & 32 \end{bmatrix} + \begin{bmatrix} 6 & -3 \\ 0 & 12 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}
\end{aligned}
$$

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Odredite $f(A)$, ako je $f(x) = 2x^2 + 3x - 4$ i $A = \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix} \in \mathbb{R}^{2\times 2}$.

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$$\begin{aligned}
f(A) &= 2A^2 + 3A - 4I \\
&= 2 \cdot \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix} + 3 \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix} - 4 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
&= 2 \cdot \begin{bmatrix} 4 & -6 \\ 0 & 16 \end{bmatrix} + \begin{bmatrix} 6 & -3 \\ 0 & 12 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} \\
&= \begin{bmatrix} 8 & -12 \\ 0 & 32 \end{bmatrix} + \begin{bmatrix} 6 & -3 \\ 0 & 12 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} \\
&= \begin{bmatrix} 10 & -15 \\ 0 & 40 \end{bmatrix}
\end{aligned}$$

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f(A) &= 2A^2 + 3A - 4I \\
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&= 2 \cdot \begin{bmatrix} 4 & -6 \\ 0 & 16 \end{bmatrix} + \begin{bmatrix} 6 & -3 \\ 0 & 12 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} \\
&= \begin{bmatrix} 8 & -12 \\ 0 & 32 \end{bmatrix} + \begin{bmatrix} 6 & -3 \\ 0 & 12 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} \\
&= \begin{bmatrix} 10 & -15 \\ 0 & 40 \end{bmatrix}
\end{aligned}$$

Zadatak (3.5.)

Za matricu $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \in \mathbb{R}^{2\times 2}$ odredite matricu $B \in \mathbb{R}^{2\times 2}$ tako da vrijedi $A \cdot B = 0$.

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Rješenje:

$$B = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in \mathbb{R}^{2\times 2}$$

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Rješenje:

$$B = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in \mathbb{R}^{2\times 2}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} =$$

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Rješenje:

$$B = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in \mathbb{R}^{2\times 2}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$$

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Rješenje:

$$B = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in \mathbb{R}^{2\times 2}$$

$$\begin{aligned}
\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} &= \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} \\
&= \begin{bmatrix} a + 2c & b + 2d \\ 3a + 6c & 3b + 6d \end{bmatrix}
\end{aligned}$$

Zadatak (3.5.)

Za matricu $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \in \mathbb{R}^{2\times 2}$ odredite matricu $B \in \mathbb{R}^{2\times 2}$ tako da vrijedi $A \cdot B = 0$.

Rješenje:

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$$\left.\begin{aligned} a + 2c &= 0 \\ b + 2d &= 0 \\ 3a + 6c &= 0 \\ 3b + 6d &= 0 \end{aligned}\right\}$$

Zadatak (3.5.)

Za matricu $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \in \mathbb{R}^{2\times 2}$ odredite matricu $B \in \mathbb{R}^{2\times 2}$ tako da vrijedi $A \cdot B = 0$.

Rješenje:

$$B = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in \mathbb{R}^{2\times 2}$$

$$\begin{aligned} \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} &= \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} \\ &= \begin{bmatrix} a+2c & b+2d \\ 3a+6c & 3b+6d \end{bmatrix} \end{aligned}$$

$$\left.\begin{aligned} a+2c &= 0 \\ b+2d &= 0 \\ 3a+6c &= 0 \\ 3b+6d &= 0 \end{aligned}\right\} \implies \begin{aligned} a &= -2c \\ b &= -2d \end{aligned}$$

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Za matricu $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \in \mathbb{R}^{2\times 2}$ odredite matricu $B \in \mathbb{R}^{2\times 2}$ tako da vrijedi $A \cdot B = 0$.

Rješenje:

$$B = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in \mathbb{R}^{2\times 2}$$

$$\begin{aligned} \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} &= \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} \\ &= \begin{bmatrix} a+2c & b+2d \\ 3a+6c & 3b+6d \end{bmatrix} \end{aligned}$$

$$\left.\begin{aligned} a+2c &= 0 \\ b+2d &= 0 \\ 3a+6c &= 0 \\ 3b+6d &= 0 \end{aligned}\right\} \implies \begin{aligned} a &= -2c \\ b &= -2d \end{aligned} \implies B = \begin{bmatrix} -2c & -2d \\ c & d \end{bmatrix}, \forall c, d \in \mathbb{R}$$

1.3 Vektorski prostori matrica

Svojstva zbrajanja matrica

Za sve matrice $A, B, C \in M_{mn}$ vrijedi:

z1 **asocijativnost:** $(A + B) + C = A + (B + C)$,

Svojstva zbrajanja matrica

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z3 **nul matrica je neutralni element:** $A + 0_{mn} = 0_{mn} + A = A$.

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Za sve matrice $A, B, C \in M_{mn}$ vrijedi:

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z3 **nul matrica je neutralni element:** $A + 0_{mn} = 0_{mn} + A = A$.

z4 **postojanje inverza:** Za svaku matricu A postoji matrica A' takva da vrijedi $A + A' = A' + A = 0_{mn}$. Matricu $\overrightarrow{A'}$ zovemo suprotna matrica i označavamo je s $A' = -A$.

Osnovne operacije – množenje matrice skalarom

Definicija

Za matricu $A = [a_{ij}] \in M_{mn}$ i skalar $\lambda \in \mathbb{R}$ definiramo matricu $\lambda A = [b_{ij}] \in M_{mn}$ s elementima $b_{ij} = \lambda a_{ij}$ za sve $i = 1, \dots, m$ $j = 1, \dots, n$.

Za sve matrice $A, B \in M_{mn}$ i sve skalare $\alpha, \beta \in \mathbb{R}$ vrijedi:

m1 **distributivnost na zbrajanje matrica:** $\alpha(A + B) = \alpha A + \alpha B$

Osnovne operacije – množenje matrice skalarom

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Za matricu $A = [a_{ij}] \in M_{mn}$ i skalar $\lambda \in \mathbb{R}$ definiramo matricu $\lambda A = [b_{ij}] \in M_{mn}$ s elementima $b_{ij} = \lambda a_{ij}$ za sve $i = 1, \ldots, m$ $j = 1, \ldots, n$.

Za sve matrice $A, B \in M_{mn}$ i sve skalare $\alpha, \beta \in \mathbb{R}$ vrijedi:

m1 **distributivnost na zbrajanje matrica:** $\alpha(A + B) = \alpha A + \alpha B$

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m3 **homogenost:** $\alpha(\beta A) = (\alpha\beta)A$.

Osnovne operacije – množenje matrice skalarom

Definicija

Za matricu $A = [a_{ij}] \in M_{mn}$ i skalar $\lambda \in \mathbb{R}$ definiramo matricu $\lambda A = [b_{ij}] \in M_{mn}$ s elementima $b_{ij} = \lambda a_{ij}$ za sve $i = 1, \ldots, m$ $j = 1, \ldots, n$.

Za sve matrice $A, B \in M_{mn}$ i sve skalare $\alpha, \beta \in \mathbb{R}$ vrijedi:

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m2 **distributivnost na zbrajanje skalara:** $(\alpha + \beta)A = \alpha A + \beta A$

m3 **homogenost:** $\alpha(\beta A) = (\alpha\beta)A$.

m4 **postojanje jedinice:** $1 \cdot A = A$.

Definicija vektorskog prostora

Neka je S neprazan skup na kojem imamo operacije zbrajanja $+$ i množenja sa skalarima $\cdot$ tako da vrijedi :

Definicija vektorskog prostora

Neka je S neprazan skup na kojem imamo operacije zbrajanja $+$ i množenja sa skalarima $\cdot$ tako da vrijedi :

1. za sve $a, b \in S$ je $a + b \in S$,

Definicija vektorskog prostora

Neka je S neprazan skup na kojem imamo operacije zbrajanja $+$ i množenja sa skalarima $\cdot$ tako da vrijedi :

1. za sve $a, b \in S$ je $a + b \in S$,
2. za svaki $a \in S$ i svaki $\lambda \in \mathbb{R}$ je $\lambda \cdot a \in S$,

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2. za svaki $a \in S$ i svaki $\lambda \in \mathbb{R}$ je $\lambda \cdot a \in S$,
3. operacije zbrajanja i množenja sa skalarom zadovoljavaju svojstva z1-z4 i m1-m4

Tada kažemo da je skup S **vektorski prostor**. Elemente skupa S ponekad zovemo **vektori**, iako to ne moraju biti vektori u geometrijskom smislu.

1.4 Linearna nezavisnost

Linearna nezavisnost matrica

Definicija

Za matrice $A_1, A_2, \ldots, A_n$ kažemo da su **linearno zavisne** ako postoje skalari $\lambda_1, \lambda_2, \ldots, \lambda_n \in \mathbb{R}$ takvi da je $\lambda_1 A_1 + \lambda_2 A_2 + \cdots + \lambda_n A_n = 0$ i bar jedan od skalara $\lambda_1, \lambda_2, \ldots, \lambda_n$ je različit od nule.

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Matrice $A_1, A_2, \ldots, A_n$ koje nisu linearno zavisne su **linearno nezavisne**.

Primjer: Provjerimo linearnu zavisnost idućeg skupa matrica:

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

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Pitamo se postoji li neka linearna relacija među matricama, dakle postoje li skalari $\alpha, \beta, \gamma \in \mathbb{R}$ t.d je

$$\alpha A + \beta B + \gamma C = 0,$$

a da nisu svi skalari nula. Ovdje 0 predstavlja nul-matricu $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

Računamo:

$$
\alpha \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \beta \begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix} + \gamma \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}
$$

$$
\begin{bmatrix} \alpha & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 2\beta \\ 3\beta & 0 \end{bmatrix} + \begin{bmatrix} \gamma & \gamma \\ 0 & \gamma \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}
$$

$$
\begin{bmatrix} \alpha + \gamma & 2\beta + \gamma \\ 3\beta & \gamma \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}
$$

Kako su dvije matrice jednake samo ako su im elementi jednaki, zaključujemo odmah da je $\beta = 0$ te $\gamma = 0$. Odmah dobivamo i $\alpha = 0$. Zaključujemo da su matrice A, B, C linearno nezavisne.

1.5 Determinanta kvadratne matrice

Definicija (3.2.)

Determinanta kvadratne matrice je preslikavanje $\det : \mathbb{R}^{n\times n} \to \mathbb{R}$ koje matrici pridružuje realni broj na sljedeći način:

Definicija (3.2.)

Determinanta kvadratne matrice je preslikavanje $\det : \mathbb{R}^{n\times n} \to \mathbb{R}$ koje matrici pridružuje realni broj na sljedeći način:

1. $n = 2$:
   Za $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

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   Za matricu $A \in \mathbb{R}^{n\times n}$ definiramo $A_{ij}, i, j \in \{1, 2, ..., n\}$ kao matricu dobivenu od A nakon izbacivanja i-tog retka i j-tog stupca.

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$$\det A = \sum_{j=1}^{n} (-1)^{i+j} \cdot a_{ij} \cdot \det(A_{ij})$$

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$$\alpha \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \beta \begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix} + \gamma \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} \alpha & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 2\beta \\ 3\beta & 0 \end{bmatrix} + \begin{bmatrix} \gamma & \gamma \\ 0 & \gamma \end{bmatrix} =$$

$$\begin{bmatrix} \alpha + \gamma & 2\beta + \gamma \\ 3\beta & \gamma \end{bmatrix}$$

Zadatak (3.6.)

Izračunajte sljedeće determinante:

a)

$$\begin{vmatrix} 2 & 1 & -1 \\ 3 & 1 & -2 \\ 1 & 0 & 1 \end{vmatrix} =$$

Zadatak (3.6.)

Izračunajte sljedeće determinante:

a)

$$\begin{vmatrix} 2 & 1 & -1 \\ 3 & 1 & -2 \\ 1 & 0 & 1 \end{vmatrix} = 2$$

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Izračunajte sljedeće determinante:

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$$\begin{vmatrix} 2 & 1 & -1 \\ 3 & 1 & -2 \\ 1 & 0 & 1 \end{vmatrix} = 2 \cdot \begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix}$$

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&= 2(1-0) - 1(3+2) - 1(0-1) = 2 - 5 + 1 = -2
\end{aligned}$$

2. način:

$$\begin{vmatrix} 2 & 1 & -1 \\ 3 & 1 & -2 \\ 1 & 0 & 1 \end{vmatrix}$$

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\begin{vmatrix} 2 & 1 & -1 \\ 3 & 1 & -2 \\ \boxed{1 \quad 0 \quad 1} \end{vmatrix} &= (-1)^{3+1} \cdot 1 \cdot \begin{vmatrix} 1 & -1 \\ 1 & -2 \end{vmatrix} + (-1)^{3+3} \cdot 1 \cdot \begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix} \\
&= 1(-2+1)
\end{aligned}$$

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&= 1(-2+1) + 1(2-3)
\end{aligned}$$

2. način:

$$\begin{vmatrix} 2 & 1 & -1 \\ 3 & 1 & -2 \\ \boxed{1 \quad 0 \quad 1} \end{vmatrix} \begin{aligned} &= (-1)^{3+1} \cdot 1 \cdot \begin{vmatrix} 1 & -1 \\ 1 & -2 \end{vmatrix} + (-1)^{3+3} \cdot 1 \cdot \begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix} \\ &= 1(-2+1) + 1(2-3) \\ &= -1 - 1 \end{aligned}$$

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$$\begin{aligned}
\begin{vmatrix} 2 & 1 & -1 \\ 3 & 1 & -2 \\ \boxed{1 \quad 0 \quad 1} \end{vmatrix} &= (-1)^{3+1} \cdot 1 \cdot \begin{vmatrix} 1 & -1 \\ 1 & -2 \end{vmatrix} + (-1)^{3+3} \cdot 1 \cdot \begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix} \\
&= 1(-2+1) + 1(2-3) \\
&= -1 - 1 \\
&= -2
\end{aligned}$$

b)

$$\begin{vmatrix} -3 & 1 & 0 & 2 \\ 0 & 1 & 3 & -4 \\ 2 & 0 & 1 & 0 \\ 1 & -5 & 0 & -3 \end{vmatrix} =$$

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$$= (-1)^{3+1} \cdot 2 \cdot \begin{vmatrix} 1 & 0 & 2 \\ 1 & 3 & -4 \\ -5 & 0 & -3 \end{vmatrix} + (-1)^{3+3} \cdot 1 \cdot \begin{vmatrix} -3 & 1 & 2 \\ 0 & 1 & -4 \\ 1 & -5 & -3 \end{vmatrix}$$

b)

$$\begin{vmatrix} -3 & 1 & \boxed{0} & 2 \\ 0 & 1 & \boxed{3} & -4 \\ \boxed{2} & \boxed{0} & \boxed{1} & \boxed{0} \\ 1 & -5 & \boxed{0} & -3 \end{vmatrix} =$$

$$= (-1)^{3+1} \cdot 2 \cdot \begin{vmatrix} 1 & 0 & 2 \\ 1 & 3 & -4 \\ -5 & 0 & -3 \end{vmatrix} + (-1)^{3+3} \cdot 1 \cdot \begin{vmatrix} -3 & 1 & 2 \\ 0 & 1 & -4 \\ 1 & -5 & -3 \end{vmatrix} = (*)$$

$$\begin{vmatrix} 1 & 0 & 2 \\ 1 & 3 & -4 \\ -5 & 0 & -3 \end{vmatrix} =$$

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$$\begin{vmatrix} 1 & \boxed{0} & 2 \\ 1 & \boxed{3} & -4 \\ -5 & \boxed{0} & -3 \end{vmatrix} = (-1)^{2+2}$$

$$\begin{vmatrix} 1 & \boxed{0} & 2 \\ 1 & \boxed{3} & -4 \\ -5 & \boxed{0} & -3 \end{vmatrix} = (-1)^{2+2} \cdot 3$$

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$$\begin{vmatrix} 1 & 0 & 2 \\ 1 & 3 & -4 \\ -5 & 0 & -3 \end{vmatrix} = (-1)^{2+2} \cdot 3 \cdot \begin{vmatrix} 1 & 2 \\ -5 & -3 \end{vmatrix}$$

$$
\begin{aligned}
\begin{vmatrix} 1 & 0 & 2 \\ 1 & 3 & -4 \\ -5 & 0 & -3 \end{vmatrix} &= (-1)^{2+2} \cdot 3 \cdot \begin{vmatrix} 1 & 2 \\ -5 & -3 \end{vmatrix} \\
&= 3 \cdot (-3 + 10)
\end{aligned}
$$

$$
\begin{aligned}
\begin{vmatrix} 1 & 0 & 2 \\ 1 & 3 & -4 \\ -5 & 0 & -3 \end{vmatrix} &= (-1)^{2+2} \cdot 3 \cdot \begin{vmatrix} 1 & 2 \\ -5 & -3 \end{vmatrix} \\
&= 3 \cdot (-3 + 10) \\
&= 21
\end{aligned}
$$

$$\begin{vmatrix} -3 & 1 & 2 \\ 0 & 1 & -4 \\ 1 & -5 & -3 \end{vmatrix} =$$

$$\begin{vmatrix} -3 & 1 & 2 \\ \boxed{0 \quad 1 \quad -4} \\ 1 & -5 & -3 \end{vmatrix} =$$

$$\begin{vmatrix} -3 & 1 & 2 \\ \boxed{0 \quad 1 \quad -4} \\ 1 & -5 & -3 \end{vmatrix} = (-1)^{2+2}$$

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$$\begin{vmatrix} -3 & 1 & 2 \\ 0 & 1 & -4 \\ 1 & -5 & -3 \end{vmatrix} = (-1)^{2+2} \cdot 1$$

$$\begin{vmatrix} -3 & 1 & 2 \\ 0 & 1 & -4 \\ 1 & -5 & -3 \end{vmatrix} = (-1)^{2+2} \cdot 1 \cdot \begin{vmatrix} -3 & 2 \\ 1 & -3 \end{vmatrix}$$

$$\begin{vmatrix} -3 & 1 & 2 \\ \boxed{0 \quad 1 \quad -4} \\ 1 & -5 & -3 \end{vmatrix} = (-1)^{2+2} \cdot 1 \cdot \begin{vmatrix} -3 & 2 \\ 1 & -3 \end{vmatrix} + (-1)^{2+3}$$

$$\begin{vmatrix} -3 & 1 & 2 \\ 0 & 1 & -4 \\ 1 & -5 & -3 \end{vmatrix} = (-1)^{2+2} \cdot 1 \cdot \begin{vmatrix} -3 & 2 \\ 1 & -3 \end{vmatrix} + (-1)^{2+3} \cdot (-4)$$

$$\begin{vmatrix} -3 & 1 & \boxed{2} \\ \boxed{0 \quad 1 \quad -4} & & \\ 1 & -5 & \boxed{-3} \end{vmatrix} = (-1)^{2+2} \cdot 1 \cdot \begin{vmatrix} -3 & 2 \\ 1 & -3 \end{vmatrix} + (-1)^{2+3} \cdot (-4)$$

$$\begin{vmatrix} -3 & 1 & 2 \\ 0 & 1 & -4 \\ 1 & -5 & -3 \end{vmatrix} = (-1)^{2+2} \cdot 1 \cdot \begin{vmatrix} -3 & 2 \\ 1 & -3 \end{vmatrix} + (-1)^{2+3} \cdot (-4) \cdot \begin{vmatrix} -3 & 1 \\ 1 & -5 \end{vmatrix}$$

$$
\begin{aligned}
\begin{vmatrix} -3 & 1 & 2 \\ \boxed{0 \quad 1 \quad -4} \\ 1 & -5 & -3 \end{vmatrix} &= (-1)^{2+2} \cdot 1 \cdot \begin{vmatrix} -3 & 2 \\ 1 & -3 \end{vmatrix} + (-1)^{2+3} \cdot (-4) \cdot \begin{vmatrix} -3 & 1 \\ 1 & -5 \end{vmatrix} \\
&= 1 \cdot (9-2) + 4(15-1)
\end{aligned}
$$

$$
\begin{aligned}
\begin{vmatrix} -3 & 1 & 2 \\ \boxed{0 \quad 1 \quad -4} \\ 1 & -5 & -3 \end{vmatrix} &= (-1)^{2+2} \cdot 1 \cdot \begin{vmatrix} -3 & 2 \\ 1 & -3 \end{vmatrix} + (-1)^{2+3} \cdot (-4) \cdot \begin{vmatrix} -3 & 1 \\ 1 & -5 \end{vmatrix} \\
&= 1 \cdot (9-2) + 4(15-1) \\
&= 63
\end{aligned}
$$

b)

$$\begin{vmatrix} -3 & 1 & 0 & 2 \\ 0 & 1 & 3 & -4 \\ 2 & 0 & 1 & 0 \\ 1 & -5 & 0 & -3 \end{vmatrix}$$

$$= (-1)^{3+1} \cdot 2 \cdot \begin{vmatrix} 1 & 0 & 2 \\ 1 & 3 & -4 \\ -5 & 0 & -3 \end{vmatrix} + (-1)^{3+3} \cdot 1 \cdot \begin{vmatrix} -3 & 1 & 2 \\ 0 & 1 & -4 \\ 1 & -5 & -3 \end{vmatrix} = (*)$$

b)

$$\begin{vmatrix} -3 & 1 & 0 & 2 \\ 0 & 1 & 3 & -4 \\ 2 & 0 & 1 & 0 \\ 1 & -5 & 0 & -3 \end{vmatrix}$$

$$= (-1)^{3+1} \cdot 2 \cdot \begin{vmatrix} 1 & 0 & 2 \\ 1 & 3 & -4 \\ -5 & 0 & -3 \end{vmatrix} + (-1)^{3+3} \cdot 1 \cdot \begin{vmatrix} -3 & 1 & 2 \\ 0 & 1 & -4 \\ 1 & -5 & -3 \end{vmatrix}$$

$$= 2 \cdot 21 + 1 \cdot 63$$

b)

$$\begin{vmatrix} -3 & 1 & 0 & 2 \\ 0 & 1 & 3 & -4 \\ 2 & 0 & 1 & 0 \\ 1 & -5 & 0 & -3 \end{vmatrix}$$

$$= (-1)^{3+1} \cdot 2 \cdot \begin{vmatrix} 1 & 0 & 2 \\ 1 & 3 & -4 \\ -5 & 0 & -3 \end{vmatrix} + (-1)^{3+3} \cdot 1 \cdot \begin{vmatrix} -3 & 1 & 2 \\ 0 & 1 & -4 \\ 1 & -5 & -3 \end{vmatrix}$$

$$= 2 \cdot 21 + 1 \cdot 63$$

$$= 105$$

c) (sami)

$$
\begin{vmatrix} 2 & 8 & 0 \\ \mathbf{0} & \mathbf{0} & \mathbf{-6} \\ 1 & -4 & 2 \end{vmatrix}
\begin{aligned}
&= (-1)^{2+3} \cdot (-6) \cdot \begin{vmatrix} 2 & 8 \\ 1 & -4 \end{vmatrix} \\
&= (-1) \cdot (-6) \cdot (2 \cdot (-4) - 1 \cdot 8) \\
&= 6 \cdot (-16) \\
&= -96
\end{aligned}
$$

d) (sami)

$$\begin{aligned}
\begin{vmatrix} \mathbf{5} & 2 & -1 & 3 \\ \mathbf{0} & 1 & -1 & 2 \\ \mathbf{0} & 3 & 1 & 2 \\ \mathbf{0} & 0 & 1 & 3 \end{vmatrix} &= (-1)^{1+1} \cdot 5 \cdot \begin{vmatrix} 1 & -1 & 2 \\ 3 & 1 & 2 \\ \mathbf{0} & \mathbf{1} & \mathbf{3} \end{vmatrix} \\
&= 5 \cdot \left((-1)^{2+3} \cdot 1 \cdot \begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix} + (-1)^{3+3} \cdot 3 \cdot \begin{vmatrix} 1 & -1 \\ 3 & 1 \end{vmatrix} \right) \\
&= 5 \cdot (-1 \cdot (1 \cdot 2 - 3 \cdot 2) + 3 \cdot (1 \cdot 1 - 3 \cdot (-1))) \\
&= 5 \cdot (-1 \cdot (-4) + 3 \cdot 4) \\
&= 5 \cdot 16 \\
&= 80
\end{aligned}$$

e)

$$\begin{vmatrix} 3 & 1 & 2 & 4 \\ \mathbf{0} & \mathbf{0} & \mathbf{-1} & \mathbf{6} \\ 2 & 1 & 3 & 1 \\ 2 & -2 & 3 & 1 \end{vmatrix} = (-1)^{2+3} \cdot (-1) \cdot \begin{vmatrix} 3 & 1 & 4 \\ 2 & 1 & 1 \\ 2 & -2 & 1 \end{vmatrix} + (-1)^{2+4} \cdot 6 \cdot \begin{vmatrix} 3 & 1 & 2 \\ 2 & 1 & 3 \\ 2 & -2 & 3 \end{vmatrix} = (*)$$

$$\begin{aligned} \begin{vmatrix} 3 & 1 & 4 \\ 2 & 1 & 1 \\ 2 & -2 & 1 \end{vmatrix} &= (-1)^{1+1} \cdot 3 \cdot \begin{vmatrix} 1 & 1 \\ -2 & 1 \end{vmatrix} + (-1)^{1+2} \cdot 1 \cdot \begin{vmatrix} 2 & 1 \\ 2 & 1 \end{vmatrix} + (-1)^{1+3} \cdot 4 \cdot \begin{vmatrix} 2 & 1 \\ 2 & -2 \end{vmatrix} \\ &= 3 \cdot (1 \cdot 1 - (-2) \cdot 1) - (2 \cdot 1 - 2 \cdot 1) + 4 \cdot (2 \cdot (-2) - 2 \cdot 1) \\ &= 3 \cdot 3 - 0 + 4 \cdot (-6) \\ &= -15 \end{aligned}$$

$$\begin{aligned} \begin{vmatrix} 3 & 1 & 2 \\ 2 & 1 & 3 \\ 2 & -2 & 3 \end{vmatrix} &= (-1)^{1+1} \cdot 3 \cdot \begin{vmatrix} 1 & 3 \\ -2 & 3 \end{vmatrix} + (-1)^{1+2} \cdot 1 \cdot \begin{vmatrix} 2 & 3 \\ 2 & 3 \end{vmatrix} + (-1)^{1+3} \cdot 2 \cdot \begin{vmatrix} 2 & 1 \\ 2 & -2 \end{vmatrix} \\ &= 3 \cdot (1 \cdot 3 - (-2) \cdot 3) - (2 \cdot 1 - 2 \cdot 3) + 2 \cdot (2 \cdot (-2) - 2 \cdot 1) \\ &= 3 \cdot 9 - 0 + 2 \cdot (-6) \\ &= 15 \end{aligned}$$

$$\begin{aligned} (*) &= -15 + 6 \cdot 15 \\ &= 75 \end{aligned}$$

f) (sami)

$$
\begin{aligned}
\begin{vmatrix} 1 & 0 & 2 & a \\ 2 & 0 & b & 0 \\ 3 & c & 4 & 5 \\ \mathbf{d} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{vmatrix} &= (-1)^{4+1} \cdot d \cdot \begin{vmatrix} \mathbf{0} & 2 & a \\ \mathbf{0} & b & 0 \\ \mathbf{c} & 4 & 5 \end{vmatrix} \\
&= -d \cdot (-1)^{3+1} \cdot c \cdot \begin{vmatrix} 2 & a \\ b & 0 \end{vmatrix} \\
&= -dc \cdot (2 \cdot 0 - a \cdot b) \\
&= -dc \cdot (-ab) \\
&= abcd
\end{aligned}
$$

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8. $\det A = \det A^T$

1.6 Rang matrice

Rang matrice je broj linearno nezavisnih redaka (stupaca) matrice.

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Za $A \in \mathbb{R}^{n \times n}$ vrijedi: $r(A) = n \Longleftrightarrow \det A \neq 0$.

Zadatak (3.7.)

Svedite na reducirani oblik matricu

$$A = \begin{bmatrix} 1 & 2 & 1 & 1 \\ 2 & 0 & 1 & -1 \\ 0 & 0 & 2 & 0 \end{bmatrix}$$

i odredite joj rang.

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Zadatak (3.8.)

Odredite rang matrice

$$\begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 1 & -2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 & 4 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 0 & 3 \end{bmatrix}.$$

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$$\sim \begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 & 4 \\ 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & -3 & -1 \end{bmatrix} \begin{array}{l} \text{I}-2\text{II} \\ \\ \\ /:(-1) \\ \\ \end{array} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 & 4 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & -3 & -1 \end{bmatrix} \begin{array}{l} \\ \\ \text{III}-3\text{IV} \\ \\ \\ \end{array}$$

Rješenje:

$$\begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 1 & -2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 & 4 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 0 & 3 \end{bmatrix} \begin{array}{l} \\ \text{II}-\text{I} \\ \\ \\ \\ \end{array} \sim \begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 0 & -4 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 & 4 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 0 & 3 \end{bmatrix} \begin{array}{l} \\ /:(-4) \\ \\ \text{IV}-\text{III} \\ \text{V}-\text{III} \end{array}$$

$$\sim \begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 & 4 \\ 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & -3 & -1 \end{bmatrix} \begin{array}{l} \text{I}-2\text{II} \\ \\ \\ /:(-1) \\ \\ \end{array} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 & 4 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & -3 & -1 \end{bmatrix} \begin{array}{l} \\ \\ \text{III}-3\text{IV} \\ \\ \text{V}+3\text{IV} \end{array}$$

Rješenje:

$$\begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 1 & -2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 & 4 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 0 & 3 \end{bmatrix} \begin{array}{l} \\ \text{II}-\text{I} \\ \\ \\ \\ \end{array} \sim \begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 0 & -4 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 & 4 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 0 & 3 \end{bmatrix} \begin{array}{l} \\ /:(-4) \\ \\ \text{IV}-\text{III} \\ \text{V}-\text{III} \end{array}$$

$$\sim \begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 & 4 \\ 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & -3 & -1 \end{bmatrix} \begin{array}{l} \text{I}-2\text{II} \\ \\ \\ /:(-1) \\ \\ \end{array} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 & 4 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & -3 & -1 \end{bmatrix} \begin{array}{l} \\ \\ \text{III}-3\text{IV} \\ \\ \text{V}+3\text{IV} \end{array}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

Rješenje:

$$\begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 1 & -2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 & 4 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 0 & 3 \end{bmatrix} \begin{array}{l} \\ \text{II}-\text{I} \\ \\ \\ \\ \end{array} \sim \begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 0 & -4 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 & 4 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 0 & 3 \end{bmatrix} \begin{array}{l} \\ /:(-4) \\ \\ \text{IV}-\text{III} \\ \text{V}-\text{III} \end{array}$$

$$\sim \begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 & 4 \\ 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & -3 & -1 \end{bmatrix} \begin{array}{l} \text{I}-2\text{II} \\ \\ \\ /:(-1) \\ \\ \end{array} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 & 4 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & -3 & -1 \end{bmatrix} \begin{array}{l} \\ \\ \text{III}-3\text{IV} \\ \\ \text{V}+3\text{IV} \end{array}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix} \begin{array}{l} \\ \\ \text{III}-\frac{1}{2}\text{V} \\ \\ \\ \end{array}$$

Rješenje:

$$\begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 1 & -2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 & 4 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 0 & 3 \end{bmatrix} \begin{array}{l} \\ \mathrm{II}-\mathrm{I} \\ \\ \\ \\ \end{array} \sim \begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 0 & -4 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 & 4 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 0 & 3 \end{bmatrix} \begin{array}{l} \\ /:(-4) \\ \\ \mathrm{IV}-\mathrm{III} \\ \mathrm{V}-\mathrm{III} \end{array}$$

$$\sim \begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 & 4 \\ 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & -3 & -1 \end{bmatrix} \begin{array}{l} \mathrm{I}-2\mathrm{II} \\ \\ \\ /:(-1) \\ \\ \end{array} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 & 4 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & -3 & -1 \end{bmatrix} \begin{array}{l} \\ \\ \mathrm{III}-3\mathrm{IV} \\ \\ \mathrm{V}+3\mathrm{IV} \end{array}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix} \begin{array}{l} \\ \\ \mathrm{III}-\frac{1}{2}\mathrm{V} \\ \mathrm{IV}-\frac{1}{2}\mathrm{V} \\ \\ \end{array}$$

Rješenje:

$$\begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 1 & -2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 & 4 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 0 & 3 \end{bmatrix} \begin{array}{l} \\ \mathrm{II}-\mathrm{I} \\ \\ \\ \\ \end{array} \sim \begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 0 & -4 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 & 4 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 0 & 3 \end{bmatrix} \begin{array}{l} \\ / : (-4) \\ \\ \mathrm{IV}-\mathrm{III} \\ \mathrm{V}-\mathrm{III} \end{array}$$

$$\sim \begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 & 4 \\ 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & -3 & -1 \end{bmatrix} \begin{array}{l} \mathrm{I}-2\mathrm{II} \\ \\ \\ / : (-1) \\ \\ \end{array} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 & 4 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & -3 & -1 \end{bmatrix} \begin{array}{l} \\ \\ \mathrm{III}-3\mathrm{IV} \\ \\ \mathrm{V}+3\mathrm{IV} \end{array}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix} \begin{array}{l} \\ \\ \mathrm{III}-\frac{1}{2}\mathrm{V} \\ \mathrm{IV}-\frac{1}{2}\mathrm{V} \\ / : 2 \end{array} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Longrightarrow r(A) = 5$$

Zadatak (3.9.)

U ovisnosti o parametru λ odredite rang matrice

$$A = \begin{bmatrix} 3 & 1 & 1 & 4 \\ \lambda & 4 & 10 & 1 \\ 1 & 7 & 17 & 3 \\ 2 & 2 & 4 & 3 \end{bmatrix}$$

Rješenje: $r(A) \leq 4$

Rješenje: $r(A) \leq 4$

$$A = \begin{bmatrix} 3 & 1 & 1 & 4 \\ \lambda & 4 & 10 & 1 \\ 1 & 7 & 17 & 3 \\ 2 & 2 & 4 & 3 \end{bmatrix}$$

Rješenje: $r(A) \leq 4$

$$A = \begin{bmatrix} 3 & 1 & 1 & 4 \\ \lambda & 4 & 10 & 1 \\ 1 & 7 & 17 & 3 \\ 2 & 2 & 4 & 3 \end{bmatrix} \quad \mathrm{I}S \leftrightarrow \mathrm{IV}S$$

Rješenje: $r(A) \leq 4$

$$A = \begin{bmatrix} 3 & 1 & 1 & 4 \\ \lambda & 4 & 10 & 1 \\ 1 & 7 & 17 & 3 \\ 2 & 2 & 4 & 3 \end{bmatrix} \; \mathrm{I}S \leftrightarrow \mathrm{IV}S \qquad \sim \begin{bmatrix} 1 & 1 & 4 & 3 \\ 4 & 10 & 1 & \lambda \\ 7 & 17 & 3 & 1 \\ 2 & 4 & 3 & 2 \end{bmatrix}$$

Rješenje: $r(A) \leq 4$

$$A = \begin{bmatrix} 3 & 1 & 1 & 4 \\ \lambda & 4 & 10 & 1 \\ 1 & 7 & 17 & 3 \\ 2 & 2 & 4 & 3 \end{bmatrix} \begin{matrix} \mathsf{I}S \leftrightarrow \mathsf{IV}S \\ \\ \\ \\ \end{matrix} \sim \begin{bmatrix} 1 & 1 & 4 & 3 \\ 4 & 10 & 1 & \lambda \\ 7 & 17 & 3 & 1 \\ 2 & 4 & 3 & 2 \end{bmatrix} \begin{matrix} \\ \mathsf{II}-4\mathsf{I} \\ \\ \\ \end{matrix}$$

Rješenje: $r(A) \leq 4$

$$A = \begin{bmatrix} 3 & 1 & 1 & 4 \\ \lambda & 4 & 10 & 1 \\ 1 & 7 & 17 & 3 \\ 2 & 2 & 4 & 3 \end{bmatrix} \begin{array}{l} \mathsf{I}S \leftrightarrow \mathsf{IV}S \\ \\ \\ \\ \end{array} \sim \begin{bmatrix} 1 & 1 & 4 & 3 \\ 4 & 10 & 1 & \lambda \\ 7 & 17 & 3 & 1 \\ 2 & 4 & 3 & 2 \end{bmatrix} \begin{array}{l} \\ \mathsf{II} - 4\mathsf{I} \\ \mathsf{III} - 7\mathsf{I} \\ \\ \end{array}$$

Rješenje: $r(A) \leq 4$

$$A = \begin{bmatrix} 3 & 1 & 1 & 4 \\ \lambda & 4 & 10 & 1 \\ 1 & 7 & 17 & 3 \\ 2 & 2 & 4 & 3 \end{bmatrix} \begin{array}{l} \mathsf{I}S \leftrightarrow \mathsf{IV}S \\ \\ \\ \\ \end{array} \sim \begin{bmatrix} 1 & 1 & 4 & 3 \\ 4 & 10 & 1 & \lambda \\ 7 & 17 & 3 & 1 \\ 2 & 4 & 3 & 2 \end{bmatrix} \begin{array}{l} \\ \mathsf{II}-4\mathsf{I} \\ \mathsf{III}-7\mathsf{I} \\ \mathsf{IV}-2\mathsf{I} \end{array}$$

Rješenje: $r(A) \leq 4$

$$A = \begin{bmatrix} 3 & 1 & 1 & 4 \\ \lambda & 4 & 10 & 1 \\ 1 & 7 & 17 & 3 \\ 2 & 2 & 4 & 3 \end{bmatrix} \quad \begin{array}{l} \mathrm{I}S \leftrightarrow \mathrm{IV}S \\ \\ \\ \\ \end{array} \qquad \sim \begin{bmatrix} 1 & 1 & 4 & 3 \\ 4 & 10 & 1 & \lambda \\ 7 & 17 & 3 & 1 \\ 2 & 4 & 3 & 2 \end{bmatrix} \begin{array}{l} \\ \mathrm{II}-4\mathrm{I} \\ \mathrm{III}-7\mathrm{I} \\ \mathrm{IV}-2\mathrm{I} \end{array}$$

$$\sim \begin{bmatrix} 1 & 1 & 4 & 3 \\ 0 & 6 & -15 & \lambda - 12 \\ 0 & 10 & -25 & -20 \\ 0 & 2 & -5 & -4 \end{bmatrix}$$

Rješenje: $r(A) \leq 4$

$$A = \begin{bmatrix} 3 & 1 & 1 & 4 \\ \lambda & 4 & 10 & 1 \\ 1 & 7 & 17 & 3 \\ 2 & 2 & 4 & 3 \end{bmatrix} \quad \mathrm{I}S \leftrightarrow \mathrm{IV}S \qquad \sim \begin{bmatrix} 1 & 1 & 4 & 3 \\ 4 & 10 & 1 & \lambda \\ 7 & 17 & 3 & 1 \\ 2 & 4 & 3 & 2 \end{bmatrix} \begin{array}{l} \\ \mathrm{II}-4\mathrm{I} \\ \mathrm{III}-7\mathrm{I} \\ \mathrm{IV}-2\mathrm{I} \end{array}$$

$$\sim \begin{bmatrix} 1 & 1 & 4 & 3 \\ 0 & 6 & -15 & \lambda - 12 \\ 0 & 10 & -25 & -20 \\ 0 & 2 & -5 & -4 \end{bmatrix} \quad \mathrm{II} \leftrightarrow \mathrm{IV}$$

Rješenje: $r(A) \leq 4$

$$A = \begin{bmatrix} 3 & 1 & 1 & 4 \\ \lambda & 4 & 10 & 1 \\ 1 & 7 & 17 & 3 \\ 2 & 2 & 4 & 3 \end{bmatrix} \begin{array}{l} \mathsf{I}S \leftrightarrow \mathsf{IV}S \\ \\ \\ \\ \end{array} \sim \begin{bmatrix} 1 & 1 & 4 & 3 \\ 4 & 10 & 1 & \lambda \\ 7 & 17 & 3 & 1 \\ 2 & 4 & 3 & 2 \end{bmatrix} \begin{array}{l} \\ \mathsf{II}-4\mathsf{I} \\ \mathsf{III}-7\mathsf{I} \\ \mathsf{IV}-2\mathsf{I} \end{array}$$

$$\sim \begin{bmatrix} 1 & 1 & 4 & 3 \\ 0 & 6 & -15 & \lambda - 12 \\ 0 & 10 & -25 & -20 \\ 0 & 2 & -5 & -4 \end{bmatrix} \begin{array}{l} \\ \mathsf{II} \leftrightarrow \mathsf{IV} \\ \\ \\ \end{array} \sim \begin{bmatrix} 1 & 1 & 4 & 3 \\ 0 & 2 & -5 & -4 \\ 0 & 10 & -25 & -20 \\ 0 & 6 & -15 & \lambda - 12 \end{bmatrix}$$

Rješenje: $r(A) \leq 4$

$$A = \begin{bmatrix} 3 & 1 & 1 & 4 \\ \lambda & 4 & 10 & 1 \\ 1 & 7 & 17 & 3 \\ 2 & 2 & 4 & 3 \end{bmatrix} \begin{array}{l} \mathrm{I}S \leftrightarrow \mathrm{IV}S \\ \\ \\ \\ \end{array} \sim \begin{bmatrix} 1 & 1 & 4 & 3 \\ 4 & 10 & 1 & \lambda \\ 7 & 17 & 3 & 1 \\ 2 & 4 & 3 & 2 \end{bmatrix} \begin{array}{l} \\ \mathrm{II}-4\mathrm{I} \\ \mathrm{III}-7\mathrm{I} \\ \mathrm{IV}-2\mathrm{I} \end{array}$$

$$\sim \begin{bmatrix} 1 & 1 & 4 & 3 \\ 0 & 6 & -15 & \lambda-12 \\ 0 & 10 & -25 & -20 \\ 0 & 2 & -5 & -4 \end{bmatrix} \mathrm{II} \leftrightarrow \mathrm{IV} \sim \begin{bmatrix} 1 & 1 & 4 & 3 \\ 0 & 2 & -5 & -4 \\ 0 & 10 & -25 & -20 \\ 0 & 6 & -15 & \lambda-12 \end{bmatrix} \begin{array}{l} \\ \\ \mathrm{III}-5\mathrm{II} \\ \\ \end{array}$$

Rješenje: $r(A) \leq 4$

$$A = \begin{bmatrix} 3 & 1 & 1 & 4 \\ \lambda & 4 & 10 & 1 \\ 1 & 7 & 17 & 3 \\ 2 & 2 & 4 & 3 \end{bmatrix} \begin{array}{l} \mathrm{I}S \leftrightarrow \mathrm{IV}S \\ \\ \\ \\ \end{array} \sim \begin{bmatrix} 1 & 1 & 4 & 3 \\ 4 & 10 & 1 & \lambda \\ 7 & 17 & 3 & 1 \\ 2 & 4 & 3 & 2 \end{bmatrix} \begin{array}{l} \\ \mathrm{II}-4\mathrm{I} \\ \mathrm{III}-7\mathrm{I} \\ \mathrm{IV}-2\mathrm{I} \end{array}$$

$$\sim \begin{bmatrix} 1 & 1 & 4 & 3 \\ 0 & 6 & -15 & \lambda - 12 \\ 0 & 10 & -25 & -20 \\ 0 & 2 & -5 & -4 \end{bmatrix} \begin{array}{l} \\ \mathrm{II} \leftrightarrow \mathrm{IV} \\ \\ \\ \end{array} \sim \begin{bmatrix} 1 & 1 & 4 & 3 \\ 0 & 2 & -5 & -4 \\ 0 & 10 & -25 & -20 \\ 0 & 6 & -15 & \lambda - 12 \end{bmatrix} \begin{array}{l} \\ \\ \mathrm{III}-5\mathrm{II} \\ \mathrm{IV}-3\mathrm{II} \end{array}$$

Rješenje: $r(A) \leq 4$

$$A = \begin{bmatrix} 3 & 1 & 1 & 4 \\ \lambda & 4 & 10 & 1 \\ 1 & 7 & 17 & 3 \\ 2 & 2 & 4 & 3 \end{bmatrix} \quad \mathrm{I}S \leftrightarrow \mathrm{IV}S \qquad \sim \begin{bmatrix} 1 & 1 & 4 & 3 \\ 4 & 10 & 1 & \lambda \\ 7 & 17 & 3 & 1 \\ 2 & 4 & 3 & 2 \end{bmatrix} \begin{array}{l} \\ \mathrm{II}-4\mathrm{I} \\ \mathrm{III}-7\mathrm{I} \\ \mathrm{IV}-2\mathrm{I} \end{array}$$

$$\sim \begin{bmatrix} 1 & 1 & 4 & 3 \\ 0 & 6 & -15 & \lambda - 12 \\ 0 & 10 & -25 & -20 \\ 0 & 2 & -5 & -4 \end{bmatrix} \quad \mathrm{II} \leftrightarrow \mathrm{IV} \qquad \sim \begin{bmatrix} 1 & 1 & 4 & 3 \\ 0 & 2 & -5 & -4 \\ 0 & 10 & -25 & -20 \\ 0 & 6 & -15 & \lambda - 12 \end{bmatrix} \begin{array}{l} \\ \\ \mathrm{III}-5\mathrm{II} \\ \mathrm{IV}-3\mathrm{II} \end{array}$$

$$\sim \begin{bmatrix} 1 & 1 & 4 & 3 \\ 0 & 2 & -5 & -4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix}$$

Rješenje: $r(A) \leq 4$

$$A = \begin{bmatrix} 3 & 1 & 1 & 4 \\ \lambda & 4 & 10 & 1 \\ 1 & 7 & 17 & 3 \\ 2 & 2 & 4 & 3 \end{bmatrix} \begin{array}{l} \mathrm{I}S \leftrightarrow \mathrm{IV}S \\ \\ \\ \\ \end{array} \sim \begin{bmatrix} 1 & 1 & 4 & 3 \\ 4 & 10 & 1 & \lambda \\ 7 & 17 & 3 & 1 \\ 2 & 4 & 3 & 2 \end{bmatrix} \begin{array}{l} \\ \mathrm{II}-4\mathrm{I} \\ \mathrm{III}-7\mathrm{I} \\ \mathrm{IV}-2\mathrm{I} \end{array}$$

$$\sim \begin{bmatrix} 1 & 1 & 4 & 3 \\ 0 & 6 & -15 & \lambda-12 \\ 0 & 10 & -25 & -20 \\ 0 & 2 & -5 & -4 \end{bmatrix} \begin{array}{l} \\ \mathrm{II} \leftrightarrow \mathrm{IV} \\ \\ \\ \end{array} \sim \begin{bmatrix} 1 & 1 & 4 & 3 \\ 0 & 2 & -5 & -4 \\ 0 & 10 & -25 & -20 \\ 0 & 6 & -15 & \lambda-12 \end{bmatrix} \begin{array}{l} \\ \\ \mathrm{III}-5\mathrm{II} \\ \mathrm{IV}-3\mathrm{II} \end{array}$$

$$\sim \begin{bmatrix} 1 & 1 & 4 & 3 \\ 0 & 2 & -5 & -4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix} \qquad \Longrightarrow r(A) = \begin{cases} 2, & \\ & \end{cases}$$

Rješenje: $r(A) \leq 4$

$$A = \begin{bmatrix} 3 & 1 & 1 & 4 \\ \lambda & 4 & 10 & 1 \\ 1 & 7 & 17 & 3 \\ 2 & 2 & 4 & 3 \end{bmatrix} \begin{matrix} \mathrm{I}S \leftrightarrow \mathrm{IV}S \\ \\ \\ \\ \end{matrix} \sim \begin{bmatrix} 1 & 1 & 4 & 3 \\ 4 & 10 & 1 & \lambda \\ 7 & 17 & 3 & 1 \\ 2 & 4 & 3 & 2 \end{bmatrix} \begin{matrix} \\ \mathrm{II}-4\mathrm{I} \\ \mathrm{III}-7\mathrm{I} \\ \mathrm{IV}-2\mathrm{I} \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 4 & 3 \\ 0 & 6 & -15 & \lambda-12 \\ 0 & 10 & -25 & -20 \\ 0 & 2 & -5 & -4 \end{bmatrix} \begin{matrix} \\ \mathrm{II} \leftrightarrow \mathrm{IV} \\ \\ \\ \end{matrix} \sim \begin{bmatrix} 1 & 1 & 4 & 3 \\ 0 & 2 & -5 & -4 \\ 0 & 10 & -25 & -20 \\ 0 & 6 & -15 & \lambda-12 \end{bmatrix} \begin{matrix} \\ \\ \mathrm{III}-5\mathrm{II} \\ \mathrm{IV}-3\mathrm{II} \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 4 & 3 \\ 0 & 2 & -5 & -4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix} \qquad \Longrightarrow r(A) = \begin{cases} 2, & \lambda = 0 \\ & \end{cases}$$

Rješenje: $r(A) \leq 4$

$$A = \begin{bmatrix} 3 & 1 & 1 & 4 \\ \lambda & 4 & 10 & 1 \\ 1 & 7 & 17 & 3 \\ 2 & 2 & 4 & 3 \end{bmatrix} \; \text{I}S \leftrightarrow \text{IV}S \quad \sim \begin{bmatrix} 1 & 1 & 4 & 3 \\ 4 & 10 & 1 & \lambda \\ 7 & 17 & 3 & 1 \\ 2 & 4 & 3 & 2 \end{bmatrix} \begin{array}{l} \\ \text{II}-4\text{I} \\ \text{III}-7\text{I} \\ \text{IV}-2\text{I} \end{array}$$

$$\sim \begin{bmatrix} 1 & 1 & 4 & 3 \\ 0 & 6 & -15 & \lambda - 12 \\ 0 & 10 & -25 & -20 \\ 0 & 2 & -5 & -4 \end{bmatrix} \; \text{II} \leftrightarrow \text{IV} \quad \sim \begin{bmatrix} 1 & 1 & 4 & 3 \\ 0 & 2 & -5 & -4 \\ 0 & 10 & -25 & -20 \\ 0 & 6 & -15 & \lambda - 12 \end{bmatrix} \begin{array}{l} \\ \\ \text{III}-5\text{II} \\ \text{IV}-3\text{II} \end{array}$$

$$\sim \begin{bmatrix} 1 & 1 & 4 & 3 \\ 0 & 2 & -5 & -4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix} \quad \Longrightarrow r(A) = \begin{cases} 2, & \lambda = 0 \\ 3, & \end{cases}$$

Rješenje: $r(A) \leq 4$

$$A = \begin{bmatrix} 3 & 1 & 1 & 4 \\ \lambda & 4 & 10 & 1 \\ 1 & 7 & 17 & 3 \\ 2 & 2 & 4 & 3 \end{bmatrix} \begin{array}{l} \mathrm{I}S \leftrightarrow \mathrm{IV}S \\ \\ \\ \\ \end{array} \sim \begin{bmatrix} 1 & 1 & 4 & 3 \\ 4 & 10 & 1 & \lambda \\ 7 & 17 & 3 & 1 \\ 2 & 4 & 3 & 2 \end{bmatrix} \begin{array}{l} \\ \mathrm{II}-4\mathrm{I} \\ \mathrm{III}-7\mathrm{I} \\ \mathrm{IV}-2\mathrm{I} \end{array}$$

$$\sim \begin{bmatrix} 1 & 1 & 4 & 3 \\ 0 & 6 & -15 & \lambda-12 \\ 0 & 10 & -25 & -20 \\ 0 & 2 & -5 & -4 \end{bmatrix} \mathrm{II} \leftrightarrow \mathrm{IV} \quad \sim \begin{bmatrix} 1 & 1 & 4 & 3 \\ 0 & 2 & -5 & -4 \\ 0 & 10 & -25 & -20 \\ 0 & 6 & -15 & \lambda-12 \end{bmatrix} \begin{array}{l} \\ \\ \mathrm{III}-5\mathrm{II} \\ \mathrm{IV}-3\mathrm{II} \end{array}$$

$$\sim \begin{bmatrix} 1 & 1 & 4 & 3 \\ 0 & 2 & -5 & -4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix} \quad \Longrightarrow r(A) = \begin{cases} 2, & \lambda = 0 \\ 3, & \lambda \neq 0 \end{cases}$$

Zadatak (3.10.)

U ovisnosti o parametru $\alpha \in \mathbb{R}$ odredite rang matrice

$$A = \begin{bmatrix} 3 & 6 & 9 \\ 4 & 8 & 12 \\ 2 & 7 & \alpha \end{bmatrix}$$

Zadatak (3.10.)

U ovisnosti o parametru $\alpha \in \mathbb{R}$ odredite rang matrice

$$A = \begin{bmatrix} 3 & 6 & 9 \\ 4 & 8 & 12 \\ 2 & 7 & \alpha \end{bmatrix}$$

Rješenje:

$$A = \begin{bmatrix} 3 & 6 & 9 \\ 4 & 8 & 12 \\ 2 & 7 & \alpha \end{bmatrix} \begin{array}{l} /:3 \\ /:4 \\ \end{array} \sim \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 2 & 7 & \alpha \end{bmatrix} \begin{array}{l} \\ \mathrm{II}-\mathrm{I} \\ \mathrm{III}-2\mathrm{I} \end{array}$$

$$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 3 & \alpha-6 \end{bmatrix} \begin{array}{l} \\ \mathrm{II} \leftrightarrow \mathrm{III} \\ /:3 \end{array} \sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & \frac{\alpha-6}{3} \\ 0 & 0 & 0 \end{bmatrix}$$

$\Longrightarrow r(A) = 2$

1.7 Inverzna matrica

Definicija

Za matricu $A \in \mathbb{R}^{n\times n}$ kažemo da je **regularna** ako postoji A^{-1} za koju vrijedi da je $A \cdot A^{-1} = A^{-1} \cdot A = I$. Za A^{-1} kažemo da je **inverzna matrica** matrice A.

Vrijedi: $A \in \mathbb{R}^{n\times n}$ je regularna $\iff r(A) = n \iff \det(A) \neq 0$.

Zadatak (3.11.)

Odredite inverz matrice $A = \begin{bmatrix} -3 & 1 & -1 \\ 1 & 0 & 1 \\ -2 & 2 & 3 \end{bmatrix}$.

Rješenje:

$$\left[\begin{array}{rrr:rrr} -3 & 1 & -1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ -2 & 2 & 3 & 0 & 0 & 1 \end{array}\right]$$

Rješenje:

$$\left[\begin{array}{rrr:rrr} -3 & 1 & -1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ -2 & 2 & 3 & 0 & 0 & 1 \end{array}\right] \quad \mathsf{II} \leftrightarrow \mathsf{I}$$

Rješenje:

$$\left[\begin{array}{ccc:ccc} -3 & 1 & -1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ -2 & 2 & 3 & 0 & 0 & 1 \end{array}\right] \text{II} \leftrightarrow \text{I} \qquad \sim \qquad \left[\begin{array}{ccc:ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ -3 & 1 & -1 & 1 & 0 & 0 \\ -2 & 2 & 3 & 0 & 0 & 1 \end{array}\right]$$

Rješenje:

$$\left[\begin{array}{rrr:rrr} -3 & 1 & -1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ -2 & 2 & 3 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \mathsf{II} \leftrightarrow \mathsf{I} \\ \\ \\ \end{array} \sim \left[\begin{array}{rrr:rrr} 1 & 0 & 1 & 0 & 1 & 0 \\ -3 & 1 & -1 & 1 & 0 & 0 \\ -2 & 2 & 3 & 0 & 0 & 1 \end{array}\right]$$

Rješenje:

$$\left[\begin{array}{ccc:ccc} -3 & 1 & -1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ -2 & 2 & 3 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \mathsf{II} \leftrightarrow \mathsf{I} \\ \\ \\ \end{array} \sim \left[\begin{array}{ccc:ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ -3 & 1 & -1 & 1 & 0 & 0 \\ -2 & 2 & 3 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ \mathsf{II}+3\mathsf{I} \\ \\ \end{array}$$

Rješenje:

$$\left[\begin{array}{rrr|rrr} -3 & 1 & -1 & 1 & 0 & 0\\ 1 & 0 & 1 & 0 & 1 & 0\\ -2 & 2 & 3 & 0 & 0 & 1\end{array}\right]\begin{array}{l} \mathrm{II}\leftrightarrow\mathrm{I}\\ \\ \end{array} \sim \left[\begin{array}{rrr|rrr} 1 & 0 & 1 & 0 & 1 & 0\\ -3 & 1 & -1 & 1 & 0 & 0\\ -2 & 2 & 3 & 0 & 0 & 1\end{array}\right]\begin{array}{l} \\ \mathrm{II}+3\mathrm{I}\\ \mathrm{III}+2\mathrm{I}\end{array}$$

Rješenje:

$$\begin{array}{cl}\left[\begin{array}{rrr|rrr} -3 & 1 & -1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ -2 & 2 & 3 & 0 & 0 & 1 \end{array}\right] & \begin{array}{l} \mathrm{II} \leftrightarrow \mathrm{I} \\ \\ \\ \end{array}\end{array} \sim \begin{array}{cl}\left[\begin{array}{rrr|rrr} 1 & 0 & 1 & 0 & 1 & 0 \\ -3 & 1 & -1 & 1 & 0 & 0 \\ -2 & 2 & 3 & 0 & 0 & 1 \end{array}\right] & \begin{array}{l} \\ \mathrm{II}+3\mathrm{I} \\ \mathrm{III}+2\mathrm{I} \end{array}\end{array}$$

$$\sim \left[\begin{array}{rrr|rrr} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 3 & 0 \\ 0 & 2 & 5 & 0 & 2 & 1 \end{array}\right]$$

Rješenje:

$$\left[\begin{array}{rrr|rrr} -3 & 1 & -1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ -2 & 2 & 3 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \text{II}\leftrightarrow\text{I} \\ \\ \\ \end{array} \sim \left[\begin{array}{rrr|rrr} 1 & 0 & 1 & 0 & 1 & 0 \\ -3 & 1 & -1 & 1 & 0 & 0 \\ -2 & 2 & 3 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ \text{II}+3\text{I} \\ \text{III}+2\text{I} \end{array}$$

$$\sim \left[\begin{array}{rrr|rrr} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 3 & 0 \\ 0 & 2 & 5 & 0 & 2 & 1 \end{array}\right] \begin{array}{l} \\ \\ \text{III}-2\text{II} \end{array}$$

Rješenje:

$$\left[\begin{array}{rrr|rrr} -3 & 1 & -1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ -2 & 2 & 3 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \mathrm{II} \leftrightarrow \mathrm{I} \\ \\ \\ \end{array} \sim \left[\begin{array}{rrr|rrr} 1 & 0 & 1 & 0 & 1 & 0 \\ -3 & 1 & -1 & 1 & 0 & 0 \\ -2 & 2 & 3 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ \mathrm{II}+3\mathrm{I} \\ \mathrm{III}+2\mathrm{I} \end{array}$$

$$\sim \left[\begin{array}{rrr|rrr} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 3 & 0 \\ 0 & 2 & 5 & 0 & 2 & 1 \end{array}\right] \begin{array}{l} \\ \\ \mathrm{III}-2\mathrm{II} \end{array}$$

$$\sim \left[\begin{array}{rrr|rrr} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 3 & 0 \\ 0 & 0 & 1 & -2 & -4 & 1 \end{array}\right]$$

Rješenje:

$$\left[\begin{array}{rrr|rrr} -3 & 1 & -1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ -2 & 2 & 3 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \mathsf{II}\leftrightarrow\mathsf{I} \\ \\ \\ \end{array}$$

$$\sim \left[\begin{array}{rrr|rrr} 1 & 0 & 1 & 0 & 1 & 0 \\ -3 & 1 & -1 & 1 & 0 & 0 \\ -2 & 2 & 3 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ \mathsf{II}+3\mathsf{I} \\ \mathsf{III}+2\mathsf{I} \end{array}$$

$$\sim \left[\begin{array}{rrr|rrr} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 3 & 0 \\ 0 & 2 & 5 & 0 & 2 & 1 \end{array}\right] \begin{array}{l} \\ \\ \mathsf{III}-2\mathsf{II} \end{array}$$

$$\sim \left[\begin{array}{rrr|rrr} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 3 & 0 \\ 0 & 0 & 1 & -2 & -4 & 1 \end{array}\right] \begin{array}{l} \mathsf{I}-\mathsf{III} \\ \\ \\ \end{array}$$

Rješenje:

$$\left[\begin{array}{rrr|rrr} -3 & 1 & -1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ -2 & 2 & 3 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \mathrm{II}\leftrightarrow\mathrm{I} \\ \\ \\ \end{array} \sim \left[\begin{array}{rrr|rrr} 1 & 0 & 1 & 0 & 1 & 0 \\ -3 & 1 & -1 & 1 & 0 & 0 \\ -2 & 2 & 3 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ \mathrm{II}+3\mathrm{I} \\ \mathrm{III}+2\mathrm{I} \end{array}$$

$$\sim \left[\begin{array}{rrr|rrr} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 3 & 0 \\ 0 & 2 & 5 & 0 & 2 & 1 \end{array}\right] \begin{array}{l} \\ \\ \mathrm{III}-2\mathrm{II} \end{array}$$

$$\sim \left[\begin{array}{rrr|rrr} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 3 & 0 \\ 0 & 0 & 1 & -2 & -4 & 1 \end{array}\right] \begin{array}{l} \mathrm{I}-\mathrm{III} \\ \mathrm{II}-2\mathrm{III} \\ \\ \end{array}$$

Rješenje:

$$\left[\begin{array}{rrr|rrr} -3 & 1 & -1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ -2 & 2 & 3 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \mathrm{II}\leftrightarrow\mathrm{I} \\ \\ \\ \end{array} \sim \left[\begin{array}{rrr|rrr} 1 & 0 & 1 & 0 & 1 & 0 \\ -3 & 1 & -1 & 1 & 0 & 0 \\ -2 & 2 & 3 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ \mathrm{II}+3\mathrm{I} \\ \mathrm{III}+2\mathrm{I} \end{array}$$

$$\sim \left[\begin{array}{rrr|rrr} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 3 & 0 \\ 0 & 2 & 5 & 0 & 2 & 1 \end{array}\right] \begin{array}{l} \\ \\ \mathrm{III}-2\mathrm{II} \end{array}$$

$$\sim \left[\begin{array}{rrr|rrr} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 3 & 0 \\ 0 & 0 & 1 & -2 & -4 & 1 \end{array}\right] \begin{array}{l} \mathrm{I}-\mathrm{III} \\ \mathrm{II}-2\mathrm{III} \\ \\ \end{array}$$

$$\sim \left[\begin{array}{rrr|rrr} 1 & 0 & 0 & 2 & 5 & -1 \\ 0 & 1 & 0 & 5 & 11 & -2 \\ 0 & 0 & 1 & -2 & -4 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 2 & 5 & -1 \\ 5 & 11 & -2 \\ -2 & -4 & 1 \end{bmatrix}$$

Zadatak (3.12.)

Za matricu $A \in \mathbb{R}^{2\times 2} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ pokažite da je $A^{-1} = \dfrac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

Zadatak (3.12.)

Za matricu $A \in \mathbb{R}^{2\times 2} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ pokažite da je $A^{-1} = \dfrac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

Rješenje:

Zadatak (3.12.)

Za matricu $A \in \mathbb{R}^{2\times 2} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ pokažite da je $A^{-1} = \dfrac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

Rješenje:

$$A \cdot A^{-1}$$

Zadatak (3.12.)

Za matricu $A \in \mathbb{R}^{2\times 2} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ pokažite da je $A^{-1} = \dfrac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

Rješenje:

$$A \cdot A^{-1} = \frac{1}{\det A}$$

Zadatak (3.12.)

Za matricu $A \in \mathbb{R}^{2\times 2} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ pokažite da je $A^{-1} = \dfrac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

Rješenje:

$$A \cdot A^{-1} \quad = \quad \frac{1}{\det A} \cdot$$

Zadatak (3.12.)

Za matricu $A \in \mathbb{R}^{2\times 2} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ pokažite da je $A^{-1} = \dfrac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

Rješenje:

$$A \cdot A^{-1} = \frac{1}{\det A} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Zadatak (3.12.)

Za matricu $A \in \mathbb{R}^{2\times 2} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ pokažite da je $A^{-1} = \dfrac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

Rješenje:

$$A \cdot A^{-1} \quad = \quad \frac{1}{\det A} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot$$

Zadatak (3.12.)

Za matricu $A \in \mathbb{R}^{2\times 2} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ pokažite da je $A^{-1} = \dfrac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

Rješenje:

$$A \cdot A^{-1} = \frac{1}{\det A} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Zadatak (3.12.)

Za matricu $A \in \mathbb{R}^{2\times 2} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ pokažite da je $A^{-1} = \dfrac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

Rješenje:

$$
\begin{aligned}
A \cdot A^{-1} &= \frac{1}{\det A} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \\
&= \frac{1}{ad - bc}
\end{aligned}
$$

Zadatak (3.12.)

Za matricu $A \in \mathbb{R}^{2\times 2} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ pokažite da je $A^{-1} = \frac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

Rješenje:

$$\begin{aligned} A \cdot A^{-1} &= \frac{1}{\det A} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \\ &= \frac{1}{ad - bc} \cdot \begin{bmatrix} ad - bc & 0 \\ 0 & ad - bc \end{bmatrix} \end{aligned}$$

Zadatak (3.12.)

Za matricu $A \in \mathbb{R}^{2\times 2} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ pokažite da je $A^{-1} = \dfrac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

Rješenje:

$$
\begin{aligned}
A \cdot A^{-1} &= \frac{1}{\det A} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \\
&= \frac{1}{ad - bc} \cdot \begin{bmatrix} ad - bc & 0 \\ 0 & ad - bc \end{bmatrix} \\
&= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \mathbf{I}
\end{aligned}
$$

Zadatak (3.13.)

Odredite inverz matrice $A = \begin{bmatrix} 3 & 8 \\ 1 & 7 \end{bmatrix}$.

Rješenje:

1. način:

$$\left[\begin{array}{cc|cc} 3 & 8 & 1 & 0 \\ 1 & 7 & 0 & 1 \end{array}\right] \text{II}\leftrightarrow\text{I} \sim \left[\begin{array}{cc|cc} 1 & 7 & 0 & 1 \\ 3 & 8 & 1 & 0 \end{array}\right] \text{II}-3\text{I}$$

$$\sim \left[\begin{array}{cc|cc} 1 & 7 & 0 & 1 \\ 0 & -13 & 1 & -3 \end{array}\right] \; / : (-13)$$

$$\sim \left[\begin{array}{cc|cc} 1 & 7 & 0 & 1 \\ 0 & 1 & \frac{-1}{13} & \frac{3}{13} \end{array}\right] \text{I}-7\text{II}$$

$$\sim \left[\begin{array}{cc|cc} 1 & 0 & \frac{7}{13} & \frac{-8}{13} \\ 0 & 1 & \frac{-1}{13} & \frac{3}{13} \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} \frac{7}{13} & \frac{-8}{13} \\ \frac{-1}{13} & \frac{3}{13} \end{bmatrix}$$

2.način:

$\det A = 13$

$$A^{-1} = \frac{1}{13}\begin{bmatrix} 7 & -8 \\ -1 & 3 \end{bmatrix}$$

Zadatak (3.14.)

Odredite inverz matrice $A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$.

Zadatak (3.14.)

Odredite inverz matrice $A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$.

Rješenje:

$$\left[\begin{array}{cccc|cccc} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \mathrm{I} \leftrightarrow \mathrm{II}$$

$$\sim \left[\begin{array}{cccc|cccc} 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ \\ \mathrm{III}-\mathrm{I} \\ \mathrm{IV}-\mathrm{I} \end{array}$$

$$\sim \left[\begin{array}{cccc|cccc} 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 1 & 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 & 0 & 1 \end{array}\right] \begin{array}{l} \\ \\ \mathrm{III}-\mathrm{II} \\ \mathrm{IV}-\mathrm{II} \end{array}$$

$$\sim \left[\begin{array}{cccc|cccc} 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -2 & -1 & -1 & -1 & 1 & 0 \\ 0 & 0 & -1 & -2 & -1 & -1 & 0 & 1 \end{array}\right] \begin{array}{l} \\ \\ \\ \mathrm{IV}-\frac{1}{2}\mathrm{III} \end{array}$$

$$\sim \left[\begin{array}{cccc:cccc} 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -2 & -1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 0 & -\frac{3}{2} & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & 1 \end{array}\right] \begin{array}{l} \\ \\ /:(-2) \\ \cdot\frac{-2}{3} \end{array}$$

$$\sim \left[\begin{array}{cccc:cccc} 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & -\frac{2}{3} \end{array}\right] \begin{array}{l} \mathrm{I}-\mathrm{IV} \\ \mathrm{II}-\mathrm{IV} \\ \mathrm{I}-\frac{1}{2}\mathrm{IV} \\ \\ \end{array}$$

$$\sim \left[\begin{array}{cccc:cccc} 1 & 0 & 1 & 0 & -\frac{1}{3} & \frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \\ 0 & 1 & 1 & 0 & \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 1 & 0 & \frac{1}{3} & \frac{1}{3} & -\frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 1 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & -\frac{2}{3} \end{array}\right] \begin{array}{l} \mathrm{I}-\mathrm{III} \\ \mathrm{II}-\mathrm{III} \\ \\ \\ \end{array}$$

$$\sim \quad \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -\frac{2}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 1 & 0 & 0 & \frac{1}{3} & -\frac{2}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 1 & 0 & \frac{1}{3} & \frac{1}{3} & -\frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 1 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & -\frac{2}{3} \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -\frac{2}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & -\frac{2}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & -\frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & -\frac{2}{3} \end{bmatrix}$$

Zadatak (3.15.)

Riješite matričnu jednadžbu $A \cdot X = B$ ako je $A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$ i $B = \begin{bmatrix} 20 & 30 \\ 35 & 40 \end{bmatrix}$.

Rješenje:

$$A \cdot X = B$$

Zadatak (3.15.)

Riješite matričnu jednadžbu $A \cdot X = B$ ako je $A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$ i $B = \begin{bmatrix} 20 & 30 \\ 35 & 40 \end{bmatrix}$.

Rješenje:

$$A^{-1} \cdot / \qquad A \cdot X = B$$

Zadatak (3.15.)

Riješite matričnu jednadžbu $A \cdot X = B$ ako je $A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$ i $B = \begin{bmatrix} 20 & 30 \\ 35 & 40 \end{bmatrix}$.

Rješenje:

$$\begin{aligned} A^{-1} \cdot / \qquad\qquad A \cdot X &= B \\ A^{-1} \cdot A \cdot X &= A^{-1} \cdot B \end{aligned}$$

Zadatak (3.15.)

Riješite matričnu jednadžbu $A \cdot X = B$ ako je $A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$ i $B = \begin{bmatrix} 20 & 30 \\ 35 & 40 \end{bmatrix}$.

Rješenje:

$$A^{-1} \cdot / \qquad \begin{aligned} A \cdot X &= B \\ A^{-1} \cdot A \cdot X &= A^{-1} \cdot B \end{aligned} \qquad A^{-1} \cdot A \cdot X = I \cdot X = X$$

Zadatak (3.15.)

Riješite matričnu jednadžbu $A \cdot X = B$ ako je $A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$ i $B = \begin{bmatrix} 20 & 30 \\ 35 & 40 \end{bmatrix}$.

Rješenje:

$$
\begin{array}{lrcl}
A^{-1} \cdot / & A \cdot X & = & B \\
 & A^{-1} \cdot A \cdot X & = & A^{-1} \cdot B \\
 & X & = & A^{-1} \cdot B
\end{array}
$$

$$A^{-1} \cdot A \cdot X = I \cdot X = X$$

Zadatak (3.15.)

Riješite matričnu jednadžbu $A \cdot X = B$ ako je $A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$ i $B = \begin{bmatrix} 20 & 30 \\ 35 & 40 \end{bmatrix}$.

Rješenje:

$$\begin{aligned} A^{-1} \cdot / \qquad\qquad A \cdot X &= B \\ A^{-1} \cdot A \cdot X &= A^{-1} \cdot B \qquad\qquad A^{-1} \cdot A \cdot X = I \cdot X = X \\ X &= A^{-1} \cdot B \end{aligned}$$

$\det A = -5$

Zadatak (3.15.)

Riješite matričnu jednadžbu $A \cdot X = B$ ako je $A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$ i $B = \begin{bmatrix} 20 & 30 \\ 35 & 40 \end{bmatrix}$.

Rješenje:

$$A^{-1} \cdot / \qquad \begin{aligned} A \cdot X &= B \\ A^{-1} \cdot A \cdot X &= A^{-1} \cdot B \\ X &= A^{-1} \cdot B \end{aligned}$$

$$A^{-1} \cdot A \cdot X = I \cdot X = X$$

$$\det A = -5 \Longrightarrow A^{-1} = -\tfrac{1}{5} \cdot \begin{bmatrix} 4 & -3 \\ -3 & 1 \end{bmatrix}$$

$$X = A^{-1} \cdot B$$

$$X = A^{-1} \cdot B$$

$$= -\frac{1}{5} \cdot \begin{bmatrix} 4 & -3 \\ -3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 20 & 30 \\ 35 & 40 \end{bmatrix}$$

$$X = A^{-1} \cdot B$$
$$= -\frac{1}{5} \cdot \begin{bmatrix} 4 & -3 \\ -3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 20 & 30 \\ 35 & 40 \end{bmatrix}$$
$$= \begin{bmatrix} 4 & -3 \\ -3 & 1 \end{bmatrix} \cdot \begin{bmatrix} -4 & -6 \\ -7 & -8 \end{bmatrix}$$

$$
\begin{aligned}
X &= A^{-1} \cdot B \\
&= -\tfrac{1}{5} \cdot \begin{bmatrix} 4 & -3 \\ -3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 20 & 30 \\ 35 & 40 \end{bmatrix} \\
&= \begin{bmatrix} 4 & -3 \\ -3 & 1 \end{bmatrix} \cdot \begin{bmatrix} -4 & -6 \\ -7 & -8 \end{bmatrix} \\
&= \begin{bmatrix} 5 & 0 \\ 5 & 10 \end{bmatrix}
\end{aligned}
$$

Zadatak (3.16.)

Riješite matričnu jednadžbu:

i) $X \cdot A = B$, $A = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$ i $B = \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$

ii) $B \cdot X = I - A$, $A = \begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix}$ i $B = \begin{bmatrix} 1 & 1 \\ -3 & 2 \end{bmatrix}$

iii) $A \cdot X \cdot B = I$, $A = \begin{bmatrix} 1 & -3 \\ 2 & 5 \end{bmatrix}$ i $B = \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix}$

1.8 Sustavi linearnih jednadžbi

Definicija (3.4.)

Linearna jednadžba s varijablama $x_1, x_2, ..., x_n$ je jednadžba oblika $a_1x_1 + a_2x_2 + \cdots + a_nx_n = d$, gdje su $a_1, a_2, ..., a_n$ koeficijenti, a d je konstanta, $a_i, d \in \mathbb{R}, \forall i$.

Zadatak (3.17.)

Riješite sustav: $$\left\{\begin{array}{rcrcrcrcr} x_1 & & & - & x_3 & + & 2x_4 & = & 0 \\ & & 2x_2 & + & x_3 & + & x_4 & = & 0 \\ x_1 & + & x_2 & - & x_3 & & & = & 0 \\ 2x_1 & + & x_2 & + & x_3 & + & x_4 & = & 0 \end{array}\right.$$

Zadatak (3.17.)

Riješite sustav:
$$\left\{\begin{array}{rcrcrcrcl} x_1 & & & - & x_3 & + & 2x_4 & = & 0 \\ & & 2x_2 & + & x_3 & + & x_4 & = & 0 \\ x_1 & + & x_2 & - & x_3 & & & = & 0 \\ 2x_1 & + & x_2 & + & x_3 & + & x_4 & = & 0 \end{array}\right.$$

Rješenje:

Neka je $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$,

Zadatak (3.17.)

Riješite sustav: $\begin{cases} x_1 - x_3 + 2x_4 = 0 \\ 2x_2 + x_3 + x_4 = 0 \\ x_1 + x_2 - x_3 = 0 \\ 2x_1 + x_2 + x_3 + x_4 = 0 \end{cases}$

Rješenje:

Neka je $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, b = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \in \mathbb{R}^{4\times 1}$

Zadatak (3.17.)

Riješite sustav:

$$\begin{cases} x_1 - x_3 + 2x_4 = 0 \\ 2x_2 + x_3 + x_4 = 0 \\ x_1 + x_2 - x_3 = 0 \\ 2x_1 + x_2 + x_3 + x_4 = 0 \end{cases}$$

Rješenje:

Neka je $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, b = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \in \mathbb{R}^{4\times 1}$ i $A = \begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 2 & 1 & 1 \\ 1 & 1 & -1 & 0 \\ 2 & 1 & 1 & 1 \end{bmatrix} \in \mathbb{R}^{4\times 4}$

Zadatak (3.17.)

Riješite sustav: $\begin{cases} x_1 \quad\quad\quad - x_3 + 2x_4 = 0 \\ \quad\quad 2x_2 + x_3 + x_4 = 0 \\ x_1 + x_2 - x_3 \quad\quad = 0 \\ 2x_1 + x_2 + x_3 + x_4 = 0 \end{cases}$

Rješenje:

Neka je $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, b = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \in \mathbb{R}^{4\times 1}$ i $A = \begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 2 & 1 & 1 \\ 1 & 1 & -1 & 0 \\ 2 & 1 & 1 & 1 \end{bmatrix} \in \mathbb{R}^{4\times 4}$

Općenito, $A \in \mathbb{R}^{m\times n}$, $x \in \mathbb{R}^{n\times 1}$ i $b \in \mathbb{R}^{m\times 1}$ i $A \cdot x = b$.

Elementarnim transformacijama dovodimo matricu A do reduciranog oblika.

$$\left[\begin{array}{cccc:c} 1 & 0 & -1 & 2 & 0 \\ 0 & 2 & 1 & 1 & 0 \\ 1 & 1 & -1 & 0 & 0 \\ 2 & 1 & 1 & 1 & 0 \end{array}\right]$$

$$\left[\begin{array}{cccc:c} 1 & 0 & -1 & 2 & 0 \\ 0 & 2 & 1 & 1 & 0 \\ 1 & 1 & -1 & 0 & 0 \\ 2 & 1 & 1 & 1 & 0 \end{array}\right] \quad \text{III}-\text{I}$$

$$\left[\begin{array}{rrrr|r} 1 & 0 & -1 & 2 & 0 \\ 0 & 2 & 1 & 1 & 0 \\ 1 & 1 & -1 & 0 & 0 \\ 2 & 1 & 1 & 1 & 0 \end{array}\right] \begin{array}{l} \\ \\ \mathrm{III}-\mathrm{I} \\ \mathrm{IV}-2\mathrm{I} \end{array} \qquad \sim$$

$$
\begin{array}{cc}
\left[\begin{array}{rrrr|r}
1 & 0 & -1 & 2 & 0 \\
0 & 2 & 1 & 1 & 0 \\
1 & 1 & -1 & 0 & 0 \\
2 & 1 & 1 & 1 & 0
\end{array}\right] &
\begin{array}{l}
\\
\\
\mathsf{III}-\mathsf{I} \\
\mathsf{IV}-2\mathsf{I}
\end{array}
\end{array}
\sim
\left[\begin{array}{rrrr|r}
1 & 0 & -1 & 2 & 0 \\
0 & 2 & 1 & 1 & 0 \\
0 & 1 & 0 & -2 & 0 \\
0 & 1 & 3 & -3 & 0
\end{array}\right]
$$

$$\begin{array}{c} \left[\begin{array}{cccc:c} 1 & 0 & -1 & 2 & 0 \\ 0 & 2 & 1 & 1 & 0 \\ 1 & 1 & -1 & 0 & 0 \\ 2 & 1 & 1 & 1 & 0 \end{array}\right] \end{array} \begin{array}{l} \\ \\ \mathsf{III}-\mathsf{I} \\ \mathsf{IV}-2\mathsf{I} \end{array} \quad \sim \quad \left[\begin{array}{cccc:c} 1 & 0 & -1 & 2 & 0 \\ 0 & 2 & 1 & 1 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 1 & 3 & -3 & 0 \end{array}\right] \begin{array}{l} \\ \mathsf{II}\leftrightarrow\mathsf{III} \\ \\ \\ \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & 2 & 0 \\ 0 & 2 & 1 & 1 & 0 \\ 1 & 1 & -1 & 0 & 0 \\ 2 & 1 & 1 & 1 & 0 \end{array}\right] \begin{array}{l} \\ \\ \text{III}-\text{I} \\ \text{IV}-2\text{I} \end{array} \sim \left[\begin{array}{cccc|c} 1 & 0 & -1 & 2 & 0 \\ 0 & 2 & 1 & 1 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 1 & 3 & -3 & 0 \end{array}\right] \begin{array}{l} \\ \text{II}\leftrightarrow\text{III} \\ \\ \\ \end{array}$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & -1 & 2 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 2 & 1 & 1 & 0 \\ 0 & 1 & 3 & -3 & 0 \end{array}\right]$$

$$
\left[\begin{array}{rrrr|r} 1 & 0 & -1 & 2 & 0 \\ 0 & 2 & 1 & 1 & 0 \\ 1 & 1 & -1 & 0 & 0 \\ 2 & 1 & 1 & 1 & 0 \end{array}\right] \begin{array}{l} \\ \\ \mathrm{III}-\mathrm{I} \\ \mathrm{IV}-2\mathrm{I} \end{array} \sim \left[\begin{array}{rrrr|r} 1 & 0 & -1 & 2 & 0 \\ 0 & 2 & 1 & 1 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 1 & 3 & -3 & 0 \end{array}\right] \begin{array}{l} \\ \mathrm{II} \leftrightarrow \mathrm{III} \\ \\ \\ \end{array}
$$

$$
\sim \left[\begin{array}{rrrr|r} 1 & 0 & -1 & 2 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 2 & 1 & 1 & 0 \\ 0 & 1 & 3 & -3 & 0 \end{array}\right] \begin{array}{l} \\ \\ \mathrm{III}-2\mathrm{II} \\ \\ \end{array}
$$

$$\left[\begin{array}{rrrr:r} 1 & 0 & -1 & 2 & 0 \\ 0 & 2 & 1 & 1 & 0 \\ 1 & 1 & -1 & 0 & 0 \\ 2 & 1 & 1 & 1 & 0 \end{array}\right] \begin{array}{l} \\ \\ \mathrm{III}-\mathrm{I} \\ \mathrm{IV}-2\mathrm{I} \end{array} \sim \left[\begin{array}{rrrr:r} 1 & 0 & -1 & 2 & 0 \\ 0 & 2 & 1 & 1 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 1 & 3 & -3 & 0 \end{array}\right] \mathrm{II} \leftrightarrow \mathrm{III}$$

$$\sim \left[\begin{array}{rrrr:r} 1 & 0 & -1 & 2 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 2 & 1 & 1 & 0 \\ 0 & 1 & 3 & -3 & 0 \end{array}\right] \begin{array}{l} \\ \\ \mathrm{III}-2\mathrm{II} \\ \mathrm{IV}-\mathrm{II} \end{array} \sim$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & 2 & 0 \\ 0 & 2 & 1 & 1 & 0 \\ 1 & 1 & -1 & 0 & 0 \\ 2 & 1 & 1 & 1 & 0 \end{array}\right] \begin{array}{l} \\ \\ \mathrm{III}-\mathrm{I} \\ \mathrm{IV}-2\mathrm{I} \end{array} \sim \left[\begin{array}{cccc|c} 1 & 0 & -1 & 2 & 0 \\ 0 & 2 & 1 & 1 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 1 & 3 & -3 & 0 \end{array}\right] \begin{array}{l} \\ \mathrm{II}\leftrightarrow\mathrm{III} \\ \\ \\ \end{array}$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & -1 & 2 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 2 & 1 & 1 & 0 \\ 0 & 1 & 3 & -3 & 0 \end{array}\right] \begin{array}{l} \\ \\ \mathrm{III}-2\mathrm{II} \\ \mathrm{IV}-\mathrm{II} \end{array} \sim \left[\begin{array}{cccc|c} 1 & 0 & -1 & 2 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & 5 & 0 \\ 0 & 0 & 3 & -1 & 0 \end{array}\right]$$

$$\left[\begin{array}{rrrr|r} 1 & 0 & -1 & 2 & 0 \\ 0 & 2 & 1 & 1 & 0 \\ 1 & 1 & -1 & 0 & 0 \\ 2 & 1 & 1 & 1 & 0 \end{array}\right] \begin{array}{l} \\ \\ \mathrm{III}-\mathrm{I} \\ \mathrm{IV}-2\mathrm{I} \end{array} \sim \left[\begin{array}{rrrr|r} 1 & 0 & -1 & 2 & 0 \\ 0 & 2 & 1 & 1 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 1 & 3 & -3 & 0 \end{array}\right] \begin{array}{l} \\ \mathrm{II}\leftrightarrow\mathrm{III} \\ \\ \\ \end{array}$$

$$\sim \left[\begin{array}{rrrr|r} 1 & 0 & -1 & 2 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 2 & 1 & 1 & 0 \\ 0 & 1 & 3 & -3 & 0 \end{array}\right] \begin{array}{l} \\ \\ \mathrm{III}-2\mathrm{II} \\ \mathrm{IV}-\mathrm{II} \end{array} \sim \left[\begin{array}{rrrr|r} 1 & 0 & -1 & 2 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & 5 & 0 \\ 0 & 0 & 3 & -1 & 0 \end{array}\right] \begin{array}{l} \\ \\ \\ \mathrm{IV}-3\mathrm{III} \end{array}$$

$$\left[\begin{array}{rrrr:r} 1 & 0 & -1 & 2 & 0 \\ 0 & 2 & 1 & 1 & 0 \\ 1 & 1 & -1 & 0 & 0 \\ 2 & 1 & 1 & 1 & 0 \end{array}\right] \begin{array}{l} \\ \\ \mathrm{III}-\mathrm{I} \\ \mathrm{IV}-2\mathrm{I} \end{array} \sim \left[\begin{array}{rrrr:r} 1 & 0 & -1 & 2 & 0 \\ 0 & 2 & 1 & 1 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 1 & 3 & -3 & 0 \end{array}\right] \begin{array}{l} \\ \mathrm{II} \leftrightarrow \mathrm{III} \\ \\ \\ \end{array}$$

$$\sim \left[\begin{array}{rrrr:r} 1 & 0 & -1 & 2 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 2 & 1 & 1 & 0 \\ 0 & 1 & 3 & -3 & 0 \end{array}\right] \begin{array}{l} \\ \\ \mathrm{III}-2\mathrm{II} \\ \mathrm{IV}-\mathrm{II} \end{array} \sim \left[\begin{array}{rrrr:r} 1 & 0 & -1 & 2 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & 5 & 0 \\ 0 & 0 & 3 & -1 & 0 \end{array}\right] \begin{array}{l} \\ \\ \\ \mathrm{IV}-3\mathrm{III} \end{array}$$

$$\sim \left[\begin{array}{rrrr:r} 1 & 0 & -1 & 2 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & -16 & 0 \end{array}\right]$$

$$x_1 - x_3 + 2x_4 \quad = \quad 0$$

$$\begin{aligned} x_1 - x_3 + 2x_4 &= 0 \\ x_2 - x_4 &= 0 \end{aligned}$$

$$
\begin{aligned}
x_1 - x_3 + 2x_4 &= 0\\
x_2 - x_4 &= 0\\
x_3 + 5x_4 &= 0
\end{aligned}
$$

$$
\begin{aligned}
x_1 - x_3 + 2x_4 &= 0\\
x_2 - x_4 &= 0\\
x_3 + 5x_4 &= 0\\
-16x_4 &= 0
\end{aligned}
$$

$$\left.\begin{array}{rcl} x_1 - x_3 + 2x_4 & = & 0 \\ x_2 - x_4 & = & 0 \\ x_3 + 5x_4 & = & 0 \\ -16x_4 & = & 0 \end{array}\right\}$$

$$\left.\begin{array}{rcl} x_1 - x_3 + 2x_4 & = & 0 \\ x_2 - x_4 & = & 0 \\ x_3 + 5x_4 & = & 0 \\ -16x_4 & = & 0 \end{array}\right\} \Longrightarrow x = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Zadatak (3.18.)

Riješite sustav:
$$\begin{cases} 3x_1 + 2x_2 + 4x_3 &= 0 \\ 2x_1 + x_2 + x_3 &= 0 \\ x_1 + x_2 + 3x_3 &= 0 \end{cases}$$

Rješenje:

Zadatak (3.18.)

Riješite sustav: $\left\{\begin{array}{rcl} 3x_1 + 2x_2 + 4x_3 & = & 0 \\ 2x_1 + x_2 + x_3 & = & 0 \\ x_1 + x_2 + 3x_3 & = & 0 \end{array}\right.$

Rješenje: Elementarnim transformacijama dovodimo matricu $\left[\begin{array}{c:c} A & b \end{array}\right]$ do reduciranog oblika.

Zadatak (3.18.)

Riješite sustav: $\left\{\begin{array}{rcl} 3x_1 + 2x_2 + 4x_3 & = & 0 \\ 2x_1 + x_2 + x_3 & = & 0 \\ x_1 + x_2 + 3x_3 & = & 0 \end{array}\right.$

Rješenje: Elementarnim transformacijama dovodimo matricu $\left[\begin{array}{c|c} A & b \end{array}\right]$ do reduciranog oblika.

$$\left[\begin{array}{cccc} 3 & 2 & 4 & 0 \\ & & & \\ & & & \end{array}\right.$$

Zadatak (3.18.)

Riješite sustav: $\left\{\begin{array}{rcl} 3x_1 + 2x_2 + 4x_3 & = & 0 \\ 2x_1 + x_2 + x_3 & = & 0 \\ x_1 + x_2 + 3x_3 & = & 0 \end{array}\right.$

Rješenje: Elementarnim transformacijama dovodimo matricu $\left[\begin{array}{c:c} A & b \end{array}\right]$ do reduciranog oblika.

$$\left[\begin{array}{cccc} 3 & 2 & 4 & 0 \\ 2 & 1 & 1 & 0 \\ & & & \end{array}\right.$$

Zadatak (3.18.)

Riješite sustav: $\left\{\begin{array}{rcl} 3x_1 + 2x_2 + 4x_3 & = & 0 \\ 2x_1 + x_2 + x_3 & = & 0 \\ x_1 + x_2 + 3x_3 & = & 0 \end{array}\right.$

Rješenje: Elementarnim transformacijama dovodimo matricu $\left[\begin{array}{c|c} A & b \end{array}\right]$ do reduciranog oblika.

$$\left[\begin{array}{cccc} 3 & 2 & 4 & 0 \\ 2 & 1 & 1 & 0 \\ 1 & 1 & 3 & 0 \end{array}\right]$$

Zadatak (3.18.)

Riješite sustav: $\left\{\begin{array}{rcl} 3x_1 + 2x_2 + 4x_3 & = & 0 \\ 2x_1 + x_2 + x_3 & = & 0 \\ x_1 + x_2 + 3x_3 & = & 0 \end{array}\right.$

Rješenje: Elementarnim transformacijama dovodimo matricu $\left[\begin{array}{c:c} A & b \end{array}\right]$ do reduciranog oblika.

$$\left[\begin{array}{ccc:c} 3 & 2 & 4 & 0 \\ 2 & 1 & 1 & 0 \\ 1 & 1 & 3 & 0 \end{array}\right] \begin{array}{c} \mathrm{I} \leftrightarrow \mathrm{III} \\ \\ \\ \end{array} \sim$$

Zadatak (3.18.)

Riješite sustav: $\begin{cases} 3x_1 + 2x_2 + 4x_3 &= 0 \\ 2x_1 + x_2 + x_3 &= 0 \\ x_1 + x_2 + 3x_3 &= 0 \end{cases}$

Rješenje: Elementarnim transformacijama dovodimo matricu $\left[\begin{array}{c:c} A & b \end{array}\right]$ do reduciranog oblika.

$$\left[\begin{array}{ccc:c} 3 & 2 & 4 & 0 \\ 2 & 1 & 1 & 0 \\ 1 & 1 & 3 & 0 \end{array}\right] \overset{\mathrm{I} \leftrightarrow \mathrm{III}}{\sim} \left[\begin{array}{ccc:c} 1 & 1 & 3 & 0 \\ 2 & 1 & 1 & 0 \\ 3 & 2 & 4 & 0 \end{array}\right]$$

Zadatak (3.18.)

Riješite sustav: $\left\{\begin{array}{rcl} 3x_1 + 2x_2 + 4x_3 & = & 0 \\ 2x_1 + x_2 + x_3 & = & 0 \\ x_1 + x_2 + 3x_3 & = & 0 \end{array}\right.$

Rješenje: Elementarnim transformacijama dovodimo matricu $\left[\begin{array}{c:c} A & b \end{array}\right]$ do reduciranog oblika.

$$\left[\begin{array}{ccc:c} 3 & 2 & 4 & 0 \\ 2 & 1 & 1 & 0 \\ 1 & 1 & 3 & 0 \end{array}\right] \begin{array}{l} \mathrm{I} \leftrightarrow \mathrm{III} \\ \\ \\ \end{array} \sim \left[\begin{array}{ccc:c} 1 & 1 & 3 & 0 \\ 2 & 1 & 1 & 0 \\ 3 & 2 & 4 & 0 \end{array}\right] \begin{array}{l} \\ \mathrm{II}-2\mathrm{I} \\ \\ \end{array}$$

Zadatak (3.18.)

Riješite sustav: $\left\{\begin{array}{rcl} 3x_1 + 2x_2 + 4x_3 & = & 0 \\ 2x_1 + x_2 + x_3 & = & 0 \\ x_1 + x_2 + 3x_3 & = & 0 \end{array}\right.$

Rješenje: Elementarnim transformacijama dovodimo matricu $\left[\begin{array}{c|c} A & b \end{array}\right]$ do reduciranog oblika.

$$\left[\begin{array}{ccc|c} 3 & 2 & 4 & 0 \\ 2 & 1 & 1 & 0 \\ 1 & 1 & 3 & 0 \end{array}\right] \begin{array}{l} \mathrm{I}\leftrightarrow\mathrm{III} \\ \\ \\ \end{array} \sim \left[\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 2 & 1 & 1 & 0 \\ 3 & 2 & 4 & 0 \end{array}\right] \begin{array}{l} \\ \mathrm{II}-2\mathrm{I} \\ \mathrm{III}-3\mathrm{I} \end{array}$$

$\sim$

Zadatak (3.18.)

Riješite sustav: $\left\{\begin{array}{rcl} 3x_1 + 2x_2 + 4x_3 & = & 0 \\ 2x_1 + x_2 + x_3 & = & 0 \\ x_1 + x_2 + 3x_3 & = & 0 \end{array}\right.$

Rješenje: Elementarnim transformacijama dovodimo matricu $\left[\begin{array}{c:c} A & b \end{array}\right]$ do reduciranog oblika.

$$\left[\begin{array}{ccc:c} 3 & 2 & 4 & 0 \\ 2 & 1 & 1 & 0 \\ 1 & 1 & 3 & 0 \end{array}\right] \begin{array}{l} \text{I}\leftrightarrow\text{III} \\ \\ \\ \end{array} \sim \left[\begin{array}{ccc:c} 1 & 1 & 3 & 0 \\ 2 & 1 & 1 & 0 \\ 3 & 2 & 4 & 0 \end{array}\right] \begin{array}{l} \\ \text{II}-2\text{I} \\ \text{III}-3\text{I} \end{array}$$

$$\sim \left[\begin{array}{ccc:c} 1 & 1 & 3 & 0 \\ 0 & -1 & -5 & 0 \\ 0 & -1 & -5 & 0 \end{array}\right]$$

Zadatak (3.18.)

Riješite sustav: $\left\{\begin{array}{rcl} 3x_1 + 2x_2 + 4x_3 & = & 0 \\ 2x_1 + x_2 + x_3 & = & 0 \\ x_1 + x_2 + 3x_3 & = & 0 \end{array}\right.$

Rješenje: Elementarnim transformacijama dovodimo matricu $\left[\begin{array}{c|c} A & b \end{array}\right]$ do reduciranog oblika.

$$\left[\begin{array}{ccc|c} 3 & 2 & 4 & 0 \\ 2 & 1 & 1 & 0 \\ 1 & 1 & 3 & 0 \end{array}\right] \begin{array}{l} \mathrm{I} \leftrightarrow \mathrm{III} \\ \\ \\ \end{array} \sim \left[\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 2 & 1 & 1 & 0 \\ 3 & 2 & 4 & 0 \end{array}\right] \begin{array}{l} \\ \mathrm{II}-2\mathrm{I} \\ \mathrm{III}-3\mathrm{I} \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 0 & -1 & -5 & 0 \\ 0 & -1 & -5 & 0 \end{array}\right] \begin{array}{l} \\ \\ \mathrm{III}-\mathrm{II} \end{array}$$

Zadatak (3.18.)

Riješite sustav: $\left\{ \begin{aligned} 3x_1 + 2x_2 + 4x_3 &= 0 \\ 2x_1 + x_2 + x_3 &= 0 \\ x_1 + x_2 + 3x_3 &= 0 \end{aligned} \right.$

Rješenje: Elementarnim transformacijama dovodimo matricu $\left[\begin{array}{c|c} A & b \end{array}\right]$ do reduciranog oblika.

$$\left[\begin{array}{ccc|c} 3 & 2 & 4 & 0 \\ 2 & 1 & 1 & 0 \\ 1 & 1 & 3 & 0 \end{array}\right] \begin{array}{l} \mathrm{I}\leftrightarrow\mathrm{III} \\ \\ \\ \end{array} \sim \left[\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 2 & 1 & 1 & 0 \\ 3 & 2 & 4 & 0 \end{array}\right] \begin{array}{l} \\ \mathrm{II}-2\mathrm{I} \\ \mathrm{III}-3\mathrm{I} \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 0 & -1 & -5 & 0 \\ 0 & -1 & -5 & 0 \end{array}\right] \begin{array}{l} \\ \\ \mathrm{III}-\mathrm{II} \end{array} \sim \left[\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 0 & -1 & -5 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$x_1 + x_2 + 3x_3 = 0$$

$$
\begin{aligned}
x_1 + x_2 + 3x_3 &= 0 \\
- x_2 - 5x_3 &= 0
\end{aligned}
$$

$$
\begin{aligned}
x_1 + x_2 + 3x_3 &= 0 \\
- x_2 - 5x_3 &= 0
\end{aligned}
$$

$$
(2) \quad \Longrightarrow \quad x_2 = -5x_3
$$

$$
\begin{aligned}
x_1 + x_2 + 3x_3 &= 0 \\
- x_2 - 5x_3 &= 0
\end{aligned}
$$

$$
\begin{aligned}
(2) \quad &\Longrightarrow \quad x_2 = -5x_3 \\
(1) \quad &\Longrightarrow \quad x_1 = 2x_3
\end{aligned}
$$

$$\begin{aligned} x_1 + x_2 + 3x_3 &= 0 \\ -x_2 - 5x_3 &= 0 \end{aligned}$$

$$(2) \quad \Longrightarrow \quad x_2 = -5x_3$$

$$(1) \quad \Longrightarrow \quad x_1 = 2x_3$$

Kako imamo dvije linearno nezavisne jednadžbe, a tri nepoznanice, rješenje sustava će biti parametarsko. Uvedimo parametar t i neka je $x_3 = t$. Zapišimo rješenje u matričnom obliku.

$x_1 = 2t$

$$x_1 = 2t$$

$$x_2 = -5t$$

$$x_1 = 2t$$
$$x_2 = -5t$$
$$x_3 = t$$

$$x_1 = 2t$$
$$x_2 = -5t$$
$$x_3 = t$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} =$$

$$x_1 = 2t$$
$$x_2 = -5t$$
$$x_3 = t$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2t \\ -5t \\ t \end{bmatrix} = t \cdot \begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix}, \; t \in \mathbb{R}$$

$$x_1 = 2t$$
$$x_2 = -5t$$
$$x_3 = t$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2t \\ -5t \\ t \end{bmatrix} = t \cdot \begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix}, \; t \in \mathbb{R}$$

Skup rješenja sustava je $\{(2t, -5t, t) : t \in \mathbb{R}\}$. U ovom slučaju kažemo da sustav ima jednoparametarski skup rješenja.

$$x_1 = 2t$$
$$x_2 = -5t$$
$$x_3 = t$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2t \\ -5t \\ t \end{bmatrix} = t \cdot \begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix}, \ t \in \mathbb{R}$$

Skup rješenja sustava je $\{(2t, -5t, t) : t \in \mathbb{R}\}$. U ovom slučaju kažemo da sustav ima jednoparametarski skup rješenja.

Homogeni sustavi ($b = 0$) uvijek imaju barem trivijalno rješenje.

Zadatak (3.19.)

Riješite sustav:
$$\left\{\begin{array}{rcr} x_1 - x_3 + 2x_4 & = & -3 \\ 2x_2 + x_3 + x_4 & = & 1 \\ x_1 + x_2 - x_3 & = & -1 \\ 2x_1 + x_2 + x_3 + x_4 & = & 3 \end{array}\right.$$

Zadatak (3.19.)

Riješite sustav: $\begin{cases} x_1 - x_3 + 2x_4 = -3 \\ 2x_2 + x_3 + x_4 = 1 \\ x_1 + x_2 - x_3 = -1 \\ 2x_1 + x_2 + x_3 + x_4 = 3 \end{cases}$

Rješenje:

$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & 2 & -3 \\ 0 & 2 & 1 & 1 & 1 \\ 1 & 1 & -1 & 0 & -1 \\ 2 & 1 & 1 & 1 & 3 \end{array}\right] \begin{array}{l} \\ \\ III-I \\ IV-2I \end{array} \sim \left[\begin{array}{cccc|c} 1 & 0 & -1 & 2 & -3 \\ 0 & 2 & 1 & 1 & 1 \\ 0 & 1 & 0 & -2 & 2 \\ 0 & 1 & 3 & -3 & 9 \end{array}\right] \begin{array}{l} \\ II \leftrightarrow III \\ \\ \\ \end{array}$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & -1 & 2 & -3 \\ 0 & 1 & 0 & -2 & 2 \\ 0 & 2 & 1 & 1 & 1 \\ 0 & 1 & 3 & -3 & 9 \end{array}\right] \begin{array}{l} \\ \\ III-2II \\ IV-II \end{array} \sim \left[\begin{array}{cccc|c} 1 & 0 & -1 & 2 & -3 \\ 0 & 1 & 0 & -2 & 2 \\ 0 & 0 & 1 & 5 & -3 \\ 0 & 0 & 3 & -1 & 7 \end{array}\right] \begin{array}{l} \\ \\ \\ IV-3III \end{array}$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & -1 & 2 & -3 \\ 0 & 1 & 0 & -2 & 2 \\ 0 & 0 & 1 & 5 & -3 \\ 0 & 0 & 0 & -16 & 16 \end{array}\right]$$

$$
\begin{aligned}
x_1 - x_3 + 2x_4 &= -3 \\
x_2 - 2x_4 &= 0 \\
x_3 + 5x_4 &= 0 \\
-16x_4 &= 16
\end{aligned}
\Longrightarrow x = \begin{bmatrix} 1 \\ 0 \\ 2 \\ -1 \end{bmatrix}
$$

Zadatak (3.20.)

Riješite sustav: $\left\{ \begin{array}{rcl} x + y & = & 0 \\ 2x - y + 3z & = & 3 \\ x - 2y - z & = & 0 \end{array} \right.$

Zadatak (3.21.)

Riješite sustav: $\left\{\begin{aligned} x+y+z &= 9\\ 2x+4y-3z &= 1\\ 3x+6y-5z &= 0\end{aligned}\right.$

Rješenje:

$$\left[\begin{array}{rrr|r} 1 & 1 & 1 & 9\\ 2 & 4 & -3 & 1\\ 3 & 6 & -5 & 0\end{array}\right] \begin{array}{l} \\ \mathrm{II}-2\mathrm{I}\\ \mathrm{III}-3\mathrm{I}\end{array} \sim \left[\begin{array}{rrr|r} 1 & 1 & 1 & 9\\ 0 & 2 & -5 & -17\\ 0 & 3 & -8 & -27\end{array}\right] \begin{array}{l} \\ \\ \mathrm{III}-\frac{3}{2}\mathrm{II}\end{array}$$

$$\sim \left[\begin{array}{rrr|r} 1 & 1 & 1 & 9\\ 0 & 2 & -5 & -17\\ 0 & 0 & -\frac{1}{2} & -\frac{3}{2}\end{array}\right]$$

$$\left.\begin{aligned} x+y+z &= 9\\ 2y-5z &= -17\\ \frac{1}{2}z &= -\frac{3}{2}\end{aligned}\right\} \Longrightarrow \begin{bmatrix} x\\ y\\ z\end{bmatrix} = \begin{bmatrix} 7\\ -1\\ 3\end{bmatrix}$$

Zadatak (3.22.)

Riješi sustav: $\begin{cases} 2x - 2z = 6 \\ y + z = 1 \\ 2x + 2y - z = 7 \\ 3y + 3z = 0 \end{cases}$

Zadatak (3.22.)

Riješi sustav: $\left\{\begin{array}{rcl} 2x-2z & = & 6 \\ y+z & = & 1 \\ 2x+2y-z & = & 7 \\ 3y+3z & = & 0 \end{array}\right.$

Rješenje:

$$\left[\begin{array}{rrr|r} 2 & 0 & -2 & 6 \\ 0 & 1 & 1 & 1 \\ 2 & 1 & -1 & 7 \\ 0 & 3 & 3 & 0 \end{array}\right] \; / : (2) \quad \sim \left[\begin{array}{rrr|r} 1 & 0 & -1 & 3 \\ 0 & 1 & 1 & 1 \\ 2 & 1 & -1 & 7 \\ 0 & 3 & 3 & 0 \end{array}\right] \begin{array}{l} \\ \\ \mathrm{III}-2\mathrm{I} \\ \\ \end{array}$$

$$\sim \left[\begin{array}{rrr|r} 1 & 0 & -1 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 3 & 3 & 0 \end{array}\right] \begin{array}{l} \\ \\ \mathrm{III}-\mathrm{II} \\ \mathrm{IV}-3\mathrm{II} \end{array} \quad \sim \left[\begin{array}{rrr|r} 1 & 0 & -1 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3 \end{array}\right]$$

$$\left.\begin{array}{rcr} x - z & = & 3 \\ y + z & = & 1 \\ 0 & = & 0 \\ 0 & = & -3 \end{array}\right\} \Longrightarrow \text{Sustav nema rješenja (zbog } 0 = -3)$$

Zadatak (3.23.)

Riješite sustav: $\left\{\begin{array}{rcr} x+3y & = & 1 \\ 2x+y & = & -3 \\ 2x+2y & = & -2 \end{array}\right.$

Zadatak (3.23.)

Riješite sustav: $\left\{\begin{array}{rcr} x+3y & = & 1 \\ 2x+y & = & -3 \\ 2x+2y & = & -2 \end{array}\right.$

Rješenje:

$$\left[\begin{array}{rr|r} 1 & 3 & 1 \\ 2 & 1 & -3 \\ 2 & 2 & -2 \end{array}\right] \begin{array}{l} \\ \mathrm{II}-2\mathrm{I} \\ \mathrm{III}-2\mathrm{I} \end{array} \sim \left[\begin{array}{rr|r} 1 & 3 & 1 \\ 0 & -5 & -5 \\ 0 & -4 & -4 \end{array}\right] \begin{array}{l} \\ /:(-5) \\ /:(-4) \end{array}$$

$$\sim \left[\begin{array}{rr|r} 1 & 3 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{array}\right] \begin{array}{l} \\ \\ \mathrm{III}-\mathrm{II} \end{array} \sim \left[\begin{array}{rr|r} 1 & 3 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{array}\right]$$

$$\begin{array}{rcr} x+3y & = & 1 \\ y & = & 1 \end{array} \Longrightarrow x = -2$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

Zadatak (3.24.)

Riješite sustav: $\begin{cases} 2x + z &= 3 \\ x - y - z &= 1 \\ 3x - y &= 4 \end{cases}$

Zadatak (3.24.)

Riješite sustav: $\begin{cases} 2x + z &= 3 \\ x - y - z &= 1 \\ 3x - y &= 4 \end{cases}$

Rješenje:

$$\left[\begin{array}{rrr|r} 2 & 0 & 1 & 3 \\ 1 & -1 & -1 & 1 \\ 3 & -1 & 0 & 4 \end{array}\right] \begin{array}{l} \text{I}\leftrightarrow\text{II} \\ \\ \\ \end{array} \sim \left[\begin{array}{rrr|r} 1 & -1 & -1 & 1 \\ 2 & 0 & 1 & 3 \\ 3 & -1 & 0 & 4 \end{array}\right] \begin{array}{l} \\ \text{II}-2\text{I} \\ \text{III}-3\text{I} \end{array}$$

$$\sim \left[\begin{array}{rrr|r} 1 & -1 & -1 & 1 \\ 0 & 2 & 3 & 1 \\ 0 & 2 & 3 & 1 \end{array}\right] \begin{array}{l} \\ \\ \text{III}-\text{II} \end{array} \sim \left[\begin{array}{rrr|r} 1 & -1 & -1 & 1 \\ 0 & 2 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{aligned} x - y - z &= 1 \\ 2y + 3z &= 1 \implies z = t \\ 0 &= 0 \end{aligned}$$

$$2y + 3z = 1 \implies y = \tfrac{1}{2} - \tfrac{3}{2}z = \tfrac{1}{2} - \tfrac{3}{2}t$$

$$x - y - z = 1 \implies x = 1 + y + z = 1 + \tfrac{1}{2} - \tfrac{3}{2}t + t = \tfrac{3}{2} - \tfrac{1}{2}t$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{3}{2} - \frac{1}{2}t \\ \frac{1}{2} - \frac{3}{2}t \\ t \end{bmatrix} = \begin{bmatrix} \frac{3}{2} \\ \frac{1}{2} \\ 0 \end{bmatrix} + t \cdot \begin{bmatrix} -\frac{1}{2} \\ -\frac{3}{2} \\ 1 \end{bmatrix}, t \in \mathbb{R}$$

Teorem (3.1.)

Sustav linearnih jednadžbi je rješiv ako i samo ako matrica tog sustava i njegova proširena matrica imaju isti rang, tj. $r(A|b) = r(A)$.

Zadatak (3.25.)

Gaussovom metodom riješite sustav:
$$\begin{cases} x + 2y + 3z &= 5 \\ 2x - y - z &= 1 \\ x + 3y + 4z &= 6 \end{cases}$$

Zadatak (3.25.)

Gaussovom metodom riješite sustav: $\left\{\begin{array}{rcl} x+2y+3z & = & 5 \\ 2x-y-z & = & 1 \\ x+3y+4z & = & 6 \end{array}\right.$

Rješenje:

$$\left[\begin{array}{rrr|r} 1 & 2 & 3 & 5 \\ 2 & -1 & -1 & 1 \\ 1 & 3 & 4 & 6 \end{array}\right] \begin{array}{l} \\ \mathrm{II}-2\mathrm{I} \\ \mathrm{III}-\mathrm{I} \end{array} \sim \left[\begin{array}{rrr|r} 1 & 2 & 3 & 5 \\ 0 & -5 & -7 & -9 \\ 0 & 1 & 1 & 1 \end{array}\right] \ \mathrm{II}\leftrightarrow\mathrm{I}$$

$$\sim \left[\begin{array}{rrr|r} 1 & 2 & 3 & 5 \\ 0 & 1 & 1 & 1 \\ 0 & -5 & -7 & -9 \end{array}\right] \begin{array}{l} \\ \\ \mathrm{III}+5\mathrm{II} \end{array} \sim \left[\begin{array}{rrr|r} 1 & 2 & 3 & 5 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & -2 & -4 \end{array}\right] \begin{array}{l} \\ \\ /:(-2) \end{array}$$

$$\sim \left[\begin{array}{rrr|r} 1 & 2 & 3 & 5 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right] \qquad r(A) = r(A|b) = 3.$$

$$\begin{aligned} x + 2y + 3z &= 5 \\ y + z &= 1 \\ z &= 2 \end{aligned}$$

$$y + z = 1 \Longrightarrow y = -1$$

$$x + 2y + 3z = 5 \Longrightarrow x = 5 - 3z - 2y = 5 - 6 + 2 = 1$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

Zadatak (3.26.)

Gaussovom metodom riješite sustav: $\left\{ \begin{array}{rcl} 3x - y + 3z & = & 4 \\ 6x - 2y + 6z & = & 1 \\ 5x + 4y & = & 2 \end{array} \right.$

Zadatak (3.26.)

Gaussovom metodom riješite sustav: $\left\{ \begin{array}{rcl} 3x - y + 3z & = & 4 \\ 6x - 2y + 6z & = & 1 \\ 5x + 4y & = & 2 \end{array} \right.$

Rješenje:

$$\left[\begin{array}{ccc|c} 3 & -1 & 3 & 4 \\ 6 & -2 & 6 & 1 \\ 5 & 4 & 0 & 2 \end{array} \right] \begin{array}{l} \\ \mathrm{II}-2\mathrm{I} \\ \mathrm{III}-\frac{5}{3}\mathrm{I} \end{array} \sim \left[\begin{array}{ccc|c} 3 & -1 & 3 & 4 \\ 0 & 0 & 0 & -7 \\ 0 & \frac{17}{3} & -5 & -\frac{14}{3} \end{array} \right] \mathrm{II} \leftrightarrow \mathrm{III}$$

$$\sim \left[\begin{array}{ccc|c} 3 & -1 & 3 & 4 \\ 0 & \frac{17}{3} & -5 & -\frac{14}{3} \\ 0 & 0 & 0 & -7 \end{array} \right] \qquad r(A) = 2 \neq r(A|b) = 3$$

Sustav nema rješenje.

Zadatak (3.27.)

Gaussovom metodom riješite sustav:

$$\left\{\begin{array}{rcr} x_1 + 2x_2 + 2x_3 + 3x_4 + x_5 & = & 3 \\ 2x_1 - x_3 - x_4 + 5x_5 & = & 2 \\ x_1 + 2x_2 + 6x_3 - x_4 + 5x_5 & = & 3 \\ x_1 - 2x_2 + 5x_3 - 12x_4 + 12x_5 & = & -1 \end{array}\right.$$

Zadatak (3.27.)

Gaussovom metodom riješite sustav:

$$\begin{cases} x_1 + 2x_2 + 2x_3 + 3x_4 + x_5 & = & 3 \\ 2x_1 - x_3 - x_4 + 5x_5 & = & 2 \\ x_1 + 2x_2 + 6x_3 - x_4 + 5x_5 & = & 3 \\ x_1 - 2x_2 + 5x_3 - 12x_4 + 12x_5 & = & -1 \end{cases}$$

Rješenje:

$$\left[\begin{array}{rrrrr|r} 1 & 2 & 2 & 3 & 1 & 3 \\ 2 & 0 & -1 & -1 & 5 & 2 \\ 1 & 2 & 6 & -1 & 5 & 3 \\ 1 & -2 & 5 & -12 & 12 & -1 \end{array}\right] \begin{array}{l} \\ \text{II}-2\text{I} \\ \text{III}-\text{I} \\ \text{IV}-\text{I} \end{array} \sim \left[\begin{array}{rrrrr|r} 1 & 2 & 2 & 3 & 1 & 3 \\ 0 & -4 & -5 & -7 & 3 & -4 \\ 0 & 0 & 4 & -4 & 4 & 0 \\ 0 & -4 & 3 & -15 & 11 & -4 \end{array}\right] \begin{array}{l} \\ \text{II}-2\text{I} \\ /:4 \\ \text{IV}-\text{II} \end{array}$$

$$\sim \left[\begin{array}{rrrrr|r} 1 & 2 & 2 & 3 & 1 & 3 \\ 0 & -4 & -5 & -7 & 3 & -4 \\ 0 & 0 & 1 & -1 & 1 & 0 \\ 0 & 0 & 8 & -8 & 8 & 0 \end{array}\right] \begin{array}{l} \\ \\ \\ \text{IV}-8\text{III} \end{array} \sim \left[\begin{array}{rrrrr|r} 1 & 2 & 2 & 3 & 1 & 3 \\ 0 & -4 & -5 & -7 & 3 & -4 \\ 0 & 0 & 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{aligned} x_1+2x_2+2x_3+3x_4+x_5 &= 3 \\ -4x_2-5x_3-7x_4+3x_5 &= -4 \\ x_3-x_4+x_5 &= 0 \\ 0 &= 0 \end{aligned}$$

$x_4 = t_1,\ x_5 = t_2,\ t_1, t_2 \in \mathbb{R}$

$$\begin{aligned} x_3-x_4+x_5 &= 0 \\ x_3 &= x_4-x_5 \\ x_3 &= t_1-t_2 \end{aligned}$$

$$\begin{aligned} -4x_2-5x_3-7x_4+3x_5 &= -4 \\ -4x_2 &= -4+5x_3+7x_4-3x_5 \\ -4x_2 &= -4+5t_1-5t_2+7t_1-3t_2 \\ -4x_2 &= -4+12t_1-8t_2 / : (-4) \\ x_2 &= 1-3t_1+2t_2 \end{aligned}$$

$$
\begin{aligned}
x_1 + 2x_2 + 2x_3 + 3x_4 + x_5 &= 3 \\
x_1 &= 3 - 2x_2 - 2x_3 - 3x_4 - x_5 \\
x_1 &= 3 - 2(1 - 3t_1 + 2t_2) - 2(t_1 - t_2) - 3t_1 - t_2 \\
x_1 &= 1 + 6t_1 - 4t_2 - 2t_1 + 2t_2 - 3t_1 - t_2 \\
x_1 &= 1 + t_1 - 3t_2
\end{aligned}
$$

$$
x = \begin{bmatrix} 1 + t_1 - 3t_2 \\ 1 - 3t_1 + 2t_2 \\ t_1 - t_2 \\ t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + t_1 \begin{bmatrix} 1 \\ -3 \\ 1 \\ 1 \\ 0 \end{bmatrix} + t_2 \begin{bmatrix} -3 \\ 2 \\ -1 \\ 0 \\ 1 \end{bmatrix}, t_1, t_2 \in \mathbb{R}
$$

Dvoparametarski skup rješenja.

1.9 Svojstveni vektori i svojstvene vrijednosti

Definicija (3.5.)

Svojstvena vrijednost matrice $A \in \mathbb{R}^{n\times n}$ je skalar $\lambda \in \mathbb{R}$ za koji postoji vektor $v \neq \vec{0}$ tako da je $A \cdot v = \lambda \cdot v$.

Definicija (3.6.)

Svojstveni vektor matrice A je vektor v koji pripada svojstvenoj vrijednosti λ.

Definicija (3.7.)

Polinom $k(\lambda) = \det(\lambda I - A)$ nazivamo **karakteristični polinom** matrice A.

Teorem (3.2.)

Nultočke karakterističnom polinoma su svojstvene vrijednosti matrice.

Zadatak (3.28.)

Odredite svojstvene vrijednosti i svojstvene vektore matrice $A = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$.

Zadatak (3.28.)

Odredite svojstvene vrijednosti i svojstvene vektore matrice $A = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$.

Rješenje:

$$\begin{aligned} \lambda I - A &= \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix} \\ &= \begin{bmatrix} \lambda - 1 & 1 \\ -2 & \lambda - 4 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} k(\lambda) &= \det(\lambda I - A) \\ &= \begin{vmatrix} \lambda - 1 & 1 \\ -2 & \lambda - 4 \end{vmatrix} \\ &= (\lambda - 1)(\lambda - 4) + 2 \\ &= \lambda^2 - 4\lambda - \lambda + 4 + 2 \\ &= \lambda^2 - 5\lambda + 6 \end{aligned}$$

$$\begin{aligned} \lambda^2 - 5\lambda + 6 &= 0 \\ \lambda_1, \lambda_2 &= \tfrac{5 \pm \sqrt{5^2 - 4 \cdot 1 \cdot 6}}{2} \\ \lambda_1, \lambda_2 &= \tfrac{5 \pm 1}{2} \\ \lambda_1 = 2 \quad &, \quad \lambda_2 = 3 \end{aligned}$$

Sada tražimo pripadne svojstvene vektore:

$\lambda = 2$

$$\begin{aligned} Av &= \lambda v \\ \lambda I v - Av &= 0 \\ (\lambda I - A)v &= 0 \end{aligned} \qquad \begin{aligned} \left(\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix} \right) \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} &= \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 1 & 1 \\ -2 & -2 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} &= \begin{bmatrix} 0 \\ 0 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} x_1 + x_2 &= 0 \\ -2x_1 - 2x_2 &= 0 \end{aligned} \Longrightarrow x_1 = -x_2, x_2 = t \in \mathbb{R} \Longrightarrow v = \begin{bmatrix} -t \\ t \end{bmatrix}, t \in \mathbb{R} \setminus \{0\}$$

$\lambda = 3$

$(\lambda I - A)v = 0$

$$\left(\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}\right) \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ -2 & -1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{aligned} 2x_1 + x_2 &= 0 \\ -2x_1 - x_2 &= 0 \end{aligned}$$

$x_2 = -2x_1,\ x_1 = t \in \mathbb{R}$

$v = \begin{bmatrix} t \\ -2t \end{bmatrix}, t \in \mathbb{R} \setminus \{0\}$

Zadatak (3.29.)

Odredite svojstvene vrijednosti i svojstvene vektore matrice $A = \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}$.

Zadatak (3.29.)

Odredite svojstvene vrijednosti i svojstvene vektore matrice $A = \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}$.

Rješenje:

$$\begin{aligned} \lambda I - A &= \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix} \\ &= \begin{bmatrix} \lambda - 3 & -2 \\ 1 & \lambda \end{bmatrix} \end{aligned}$$

$$\begin{aligned} k(\lambda) &= \det(\lambda I - A) \\ &= \begin{vmatrix} \lambda - 3 & -2 \\ 1 & \lambda \end{vmatrix} \\ &= (\lambda - 3)\lambda + 2 \\ &= \lambda^2 - 3\lambda + 2 \end{aligned}$$

$$\begin{aligned} \lambda^2 - 3\lambda + 2 &= 0 \\ \lambda_1, \lambda_2 &= \tfrac{3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot 2}}{2} \\ \lambda_1, \lambda_2 &= \tfrac{3 \pm 1}{2} \end{aligned}$$

$$\lambda_1 = 1 \quad , \quad \lambda_2 = 2$$

Sada tražimo pripadne svojstvene vektore:

$\lambda = 1$

$(\lambda I - A)v = 0$

$$\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}\right) \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 & -2 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{aligned} -2x_1 - 2x_2 &= 0 \\ x_1 + x_2 &= 0 \end{aligned}$$

$x_2 = -x_1$, $x_1 = t \in \mathbb{R}$

$$v = \begin{bmatrix} t \\ -t \end{bmatrix}, t \in \mathbb{R} \setminus \{0\}$$

$\lambda = 2$

$(\lambda I - A)v = 0$

$$\left(\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}\right) \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{aligned} -x_1 - 2x_2 &= 0 \\ x_1 + 2x_2 &= 0 \end{aligned}$$

$x_1 = -2x_2,\ x_2 = t \in \mathbb{R}$

$$v = \begin{bmatrix} -2t \\ t \end{bmatrix}, t \in \mathbb{R} \setminus \{0\}$$

Zadatak (3.30.)

Odredite svojstvene vrijednosti i svojstvene vektore matrice

$$A = \begin{bmatrix} 2 & 0 & 3 \\ 2 & 2 & 2 \\ 3 & 0 & 2 \end{bmatrix}.$$

Zadatak (3.30.)

Odredite svojstvene vrijednosti i svojstvene vektore matrice
$A = \begin{bmatrix} 2 & 0 & 3 \\ 2 & 2 & 2 \\ 3 & 0 & 2 \end{bmatrix}$.

$$
\begin{aligned}
\lambda I - A &= \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 2 & 0 & 3 \\ 2 & 2 & 2 \\ 3 & 0 & 2 \end{bmatrix} \\
&= \begin{bmatrix} \lambda - 2 & 0 & -3 \\ -2 & \lambda - 2 & -2 \\ -3 & 0 & \lambda - 2 \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
k(\lambda) &= \det(\lambda I - A) \\
&= \begin{vmatrix} \lambda-2 & 0 & -3 \\ -2 & \lambda-2 & -2 \\ -3 & 0 & \lambda-2 \end{vmatrix} \\
&= (-1)^{2+2}(\lambda-2)\begin{vmatrix} \lambda-2 & -3 \\ -3 & \lambda-2 \end{vmatrix} \\
&= (\lambda-2)\left[(\lambda-2)^2-9\right] \\
&= (\lambda-2)\left(\lambda^2-4\lambda-5\right) \\
&= (\lambda-2)(\lambda-5)(\lambda+1)
\end{aligned}
$$

$\lambda_1 = 2,\ \lambda_2 = 5,\ \lambda_3 = -1$

Sada tražimo pripadne svojstvene vektore:

$\lambda_1 = 2$

$(\lambda_1 I - A)v = 0$

$$\begin{bmatrix} 0 & 0 & -3 \\ -2 & 0 & -2 \\ -3 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left.\begin{aligned} -3x_3 &= 0 \\ -2x_1 - 2x_3 &= 0 \\ -3x_1 &= 0 \end{aligned}\right\} \Longrightarrow \begin{aligned} x_1 &= 0 \\ x_2 &= t \\ x_3 &= 0 \end{aligned}$$

$$v = \begin{bmatrix} 0 \\ t \\ 0 \end{bmatrix}, t \in \mathbb{R} \setminus \{0\}$$

$\lambda_2 = 5$

$(\lambda_2 I - A)v = 0$

$$\begin{bmatrix} 3 & 0 & -3 \\ -2 & 3 & -2 \\ -3 & 0 & 3 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left.\begin{aligned} 3x_1 - 3x_3 &= 0 \\ -2x_1 + 2x_2 - 2x_3 &= 0 \\ -3x_1 - 3x_3 = 0 &= 0 \end{aligned}\right\} \Longrightarrow \begin{aligned} x_3 &= t \\ x_1 &= x_3 \\ x_2 &= \frac{4}{3}t \end{aligned}$$

$$v = \begin{bmatrix} t \\ \frac{4}{3}t \\ t \end{bmatrix}, t \in \mathbb{R} \setminus \{0\}$$

$\lambda_3 = -1$

$(\lambda_3 I - A)v = 0$

$$\begin{bmatrix} -3 & 0 & -3 \\ -2 & -3 & -2 \\ -3 & 0 & -3 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left.\begin{aligned} -3x_1 - 3x_3 &= 0 \\ -2x_1 - 3x_2 - 2x_3 &= 0 \\ -3x_1 - 3x_3 &= 0 \end{aligned}\right\} \implies \begin{aligned} x_1 &= -x_3 \\ x_2 &= 0 \\ x_3 &= t \end{aligned}$$

$$v = \begin{bmatrix} -t \\ 0 \\ t \end{bmatrix}, t \in \mathbb{R} \setminus \{0\}$$

Zadatak (3.31.)

Odredite svojstvene vrijednosti matrice $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$.

Zadatak (3.31.)

Odredite svojstvene vrijednosti matrice $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$.

$$\begin{aligned} \lambda I - A &= \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix} \\ &= \begin{bmatrix} \lambda - 1 & 1 & -1 \\ -1 & \lambda - 1 & 1 \\ -1 & 1 & \lambda - 1 \end{bmatrix} \end{aligned}$$

$$
\begin{aligned}
k(\lambda) &= \det(\lambda I - A) \\
&= \begin{vmatrix} \lambda - 1 & 1 & -1 \\ -1 & \lambda - 1 & 1 \\ -1 & 1 & \lambda - 1 \end{vmatrix} \\
&= (-1)^{1+1}(\lambda - 1)\begin{vmatrix} \lambda - 1 & 1 \\ 1 & \lambda - 1 \end{vmatrix} + (-1)^{1+2}\begin{vmatrix} -1 & 1 \\ -1 & \lambda - 1 \end{vmatrix} + (-1)^{1+3}(-1)\begin{vmatrix} -1 & \lambda - 1 \\ -1 & 1 \end{vmatrix} \\
&= (\lambda - 1)\left[(\lambda - 1)^2 - 1\right] - \left[-(\lambda - 1) + 1\right] - (-1 + \lambda - 1) \\
&= (\lambda - 1)\left(\lambda^2 - 2\lambda + 1 - 1\right) - (-\lambda + 2) - \lambda + 2 \\
&= (\lambda - 1)\left(\lambda^2 - 2\lambda\right) + \lambda - 2 - \lambda + 2 \\
&= (\lambda - 1)\lambda\,(\lambda - 2)
\end{aligned}
$$

$\lambda_1 = 1,\ \lambda_2 = 0,\ \lambda_3 = 2$

Zadatak (3.32.)

Odredite barem jedan svojstveni vektor matrice A koji pripada svojstvenoj vrijednosti λ ako je:

a) $A = \begin{bmatrix} 1 & 3 & 1 \\ 1 & 4 & -4 \\ 1 & -2 & 4 \end{bmatrix}$ i $\lambda = -1$

b) $A = \begin{bmatrix} 5 & 4 & -4 \\ 1 & 3 & 3 \\ 2 & 0 & -9 \end{bmatrix}$ i $\lambda = 1$

c) $A = \begin{bmatrix} 4 & 4 & 6 \\ 3 & 3 & -1 \\ 1 & -3 & -5 \end{bmatrix}$ i $\lambda = 2$

Rješenje:

$\lambda = -1$

$$
\begin{aligned}
(\lambda I - A)v &= 0 \\
(-I - A)v &= 0 \\
(I + A)v &= 0 \\
\left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 3 & 1 \\ 1 & 4 & -4 \\ 1 & -2 & 4 \end{bmatrix} \right) \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} &= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \\
\begin{bmatrix} 2 & 3 & 1 \\ 1 & 5 & -4 \\ 1 & -2 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} &= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}
\end{aligned}
$$

$$\left[\begin{array}{rrr|r} 2 & 3 & 1 & 0 \\ 1 & 5 & -4 & 0 \\ 1 & -2 & 5 & 0 \end{array}\right] \begin{array}{l} \\ \mathrm{II} \leftrightarrow \mathrm{I} \\ \\ \end{array} \sim \left[\begin{array}{rrr|r} 1 & 5 & -4 & 0 \\ 2 & 3 & 1 & 0 \\ 1 & -2 & 5 & 0 \end{array}\right] \begin{array}{l} \\ \mathrm{II}-2\mathrm{I} \\ \mathrm{III}-\mathrm{I} \end{array}$$

$$\sim \left[\begin{array}{rrr|r} 1 & 5 & -4 & 0 \\ 0 & -7 & 9 & 0 \\ 0 & -7 & 9 & 0 \end{array}\right] \begin{array}{l} \\ \\ \mathrm{III}-\mathrm{II} \end{array} \sim \left[\begin{array}{rrr|r} 1 & 5 & -4 & 0 \\ 0 & -7 & 9 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{aligned} x_1 + 5x_2 - 4x_3 &= 0 \\ -7x_2 + 9x_3 &= 0 \end{aligned} \qquad x_3 = t,\ t \in \mathbb{R}$$

$$\begin{aligned} -7x_2 + 9x_3 &= 0 \\ x_2 &= \tfrac{9}{7}x_3 \\ x_2 &= \tfrac{9}{7}t \end{aligned} \qquad \begin{aligned} x_1 + 5x_2 - 4x_3 &= 0 \\ x_1 &= -5x_2 + 4x_3 \\ x_1 &= -5\tfrac{9}{7}t + 4t \\ x_1 &= -\tfrac{17}{7}t \end{aligned}$$

$$x = \begin{bmatrix} -\frac{17}{7}t \\ \frac{9}{7}t \\ t \end{bmatrix} = t \begin{bmatrix} -\frac{17}{7} \\ \frac{9}{7} \\ 1 \end{bmatrix}, t \in \mathbb{R} \setminus \{0\}, \text{ npr. za } t = 7 \text{ imamo } v_1 = \begin{bmatrix} -17 \\ 9 \\ 7 \end{bmatrix}$$

Zadatak (3.33.)

Odredite svojstvene vrijednosti i svojstvene vektore matrice
$$A = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 3 & -1 \\ -2 & 2 & 0 \end{bmatrix}.$$

Zadatak (3.33.)

Odredite svojstvene vrijednosti i svojstvene vektore matrice
$A = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 3 & -1 \\ -2 & 2 & 0 \end{bmatrix}$.

Rješenje:

$$\begin{aligned} \lambda I - A &= \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 1 & 1 & -1 \\ -1 & 3 & -1 \\ -2 & 2 & 0 \end{bmatrix} \\ &= \begin{bmatrix} \lambda - 1 & -1 & 1 \\ 1 & \lambda - 3 & 1 \\ 2 & -2 & \lambda \end{bmatrix} \end{aligned}$$

$$
\begin{aligned}
k(\lambda) &= \det(\lambda I - A) \\
&= \begin{bmatrix} \lambda - 1 & -1 & 1 \\ 1 & \lambda - 3 & 1 \\ 2 & -2 & \lambda \end{bmatrix} \\
&= (\lambda - 1) \begin{vmatrix} \lambda - 3 & 1 \\ -2 & \lambda \end{vmatrix} + \begin{vmatrix} 1 & 1 \\ 2 & \lambda \end{vmatrix} + \begin{vmatrix} 1 & \lambda - 3 \\ 2 & -2 \end{vmatrix} \\
&= (\lambda - 1)\, [\lambda(\lambda - 3) + 2] + \lambda - 2 - 2 - 2\,(\lambda - 3) \\
&= (\lambda - 1) \left[\lambda^2 - 3\lambda + 2\right] + \lambda - 2 - 2 - 2\lambda + 6 \\
&= \lambda^3 - 3\lambda^2 + 2\lambda - \lambda^2 + 3\lambda - 2 - \lambda + 2 \\
&= \lambda^3 - 4\lambda^2 + 4\lambda \\
&= \lambda \left(\lambda^2 - 4\lambda + 4\right) \\
&= \lambda\, (\lambda - 2)^2
\end{aligned}
$$

$\lambda_1 = 0,\ \lambda_{2,3} = 2 \leftarrow$ dvostruka svojstvena vrijednost

Sada tražimo pripadne svojstvene vektore:

$\lambda_1 = 0$

$(\lambda_1 I - A)v = 0$

$Av = 0$

$$\left[\begin{array}{rrr|r} 1 & 1 & -1 & 0 \\ -1 & 3 & -1 & 0 \\ -2 & 2 & 0 & 0 \end{array}\right] \begin{array}{l} \\ \text{II+I} \\ \text{III+2I} \end{array} \sim \left[\begin{array}{rrr|r} 1 & 1 & -1 & 0 \\ 0 & 4 & -2 & 0 \\ 0 & 4 & -2 & 0 \end{array}\right] \begin{array}{l} \\ \\ \text{III}-\text{II} \end{array}$$

$$\sim \left[\begin{array}{rrr|r} 1 & 1 & -1 & 0 \\ 0 & 4 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \qquad \begin{aligned} x_1 + x_2 - x_3 &= 0 \\ 4x_2 - 2x_3 &= 0 \end{aligned}$$

$$\begin{aligned} x_1 &= \tfrac{1}{2}t \\ x_2 &= \tfrac{1}{2}t \\ x_3 &= t \end{aligned} \implies v = \begin{bmatrix} \frac{1}{2}t \\ \frac{1}{2}t \\ t \end{bmatrix}, t \in \mathbb{R} \setminus \{0\}, \text{ npr. za } t = 2,\ v_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$\lambda_{2,3} = 2$

$(\lambda_{2,3} I - A)v = 0$

$(2I - A)v = 0$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 1 & -1 & 1 & 0 \\ 2 & -2 & 2 & 0 \end{array}\right] \begin{array}{l} \\ \text{II}-\text{I} \\ \text{III}-2\text{I} \end{array} \quad \sim \quad \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$x_1 - x_2 + x_3 = 0 \Longrightarrow \begin{aligned} x_1 &= t_1 - t_2 \\ x_2 &= t_1 \\ x_3 &= t_2 \end{aligned}$$

$$v = \begin{bmatrix} t_1 - t_2 \\ t_1 \\ t_2 \end{bmatrix} = t_1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + t_2 \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, t_1, t_2 \in \mathbb{R}, (t_1, t_2) \neq (0,0)$$

Npr. za $t_1 = 0, t_2 = 1,\ v_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ i za $t_1 = 1, t_2 = 0,\ v_3 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

Teorem

Svaka matrica poništava svoj karakteristični polinom, tj. $k(A) = 0$.

Zadatak (3.34.)

Koji od vektora $x = \begin{bmatrix} 11 \\ 0 \\ 0 \end{bmatrix}$ i $y = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ je svojstveni vektor matrice $A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & -1 \end{bmatrix}$ za svojstvenu vrijednost $\lambda = 1$? Pokažite da matrica poništava svoj karakteristični polinom.

Rješenje:

$Ax \overset{?}{=} \lambda x$, za $\lambda = 1$

$$Ax = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 11 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 11 \\ 0 \\ 0 \end{bmatrix}$$

x je svojstveni vektor za $\lambda = 1$

$$Ay = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix} \neq y$$

y nije svojstveni vektor za $\lambda = 1$

$$
\begin{aligned}
k(\lambda) &= \det(\lambda I - A) \\
&= \begin{bmatrix} \lambda - 1 & 2 & -1 \\ 0 & \lambda - 2 & -3 \\ 0 & 0 & \lambda + 1 \end{bmatrix} \\
&= (\lambda - 1) \begin{vmatrix} \lambda - 2 & -3 \\ 0 & \lambda + 1 \end{vmatrix} \\
&= (\lambda - 1)(\lambda - 2)(\lambda + 1)
\end{aligned}
$$

$$
\begin{aligned}
k(A) &= (A-I)(A-2I)(A+I) \\
&= \begin{bmatrix} 0 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} -1 & -2 & 1 \\ 0 & 0 & 3 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} 2 & -2 & 1 \\ 0 & 3 & 3 \\ 0 & 0 & 0 \end{bmatrix} \\
&= \begin{bmatrix} 0 & 0 & -9 \\ 0 & 0 & -6 \\ 0 & 0 & 6 \end{bmatrix} \begin{bmatrix} 2 & -2 & 1 \\ 0 & 3 & 3 \\ 0 & 0 & 0 \end{bmatrix} \\
&= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
\end{aligned}
$$

Zadatak (3.35.)

Pronađite svojstvene vrijednosti matrice A i provjerite je li vektor v svojstveni vektor koji odgovara svojstvenoj vrijednosti λ ako je:

a) $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & 1 \\ 2 & -1 & 3 \end{bmatrix}, \; v = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \; \lambda = 1$

b) $A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 0 & 3 \\ 3 & 1 & 2 \end{bmatrix}, \; v = \begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix}, \; \lambda = -1$

Zadatak (3.36.)

Zadana je matrica $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ i $B = A^2 - \lambda A$. Odredite $\lambda \in \mathbb{R}$ tako da matrica B bude singularna (nije regularna).

Rješenje:

$$\begin{aligned} B &= A^2 - \lambda A \\ &= \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & \lambda & 0 \\ 0 & \lambda & 0 \\ \lambda & 0 & \lambda \end{bmatrix} \\ &= \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & \lambda & 0 \\ 0 & \lambda & 0 \\ \lambda & 0 & \lambda \end{bmatrix} \\ &= \begin{bmatrix} 1-\lambda & 2-\lambda & 0 \\ 0 & 1-\lambda & 0 \\ 2-\lambda & 1 & 1-\lambda \end{bmatrix} \end{aligned}$$

$$
\begin{aligned}
0 &= \det B \\
&= \begin{vmatrix} 1-\lambda & 2-\lambda & 0 \\ 0 & 1-\lambda & 0 \\ 2-\lambda & 1 & 1-\lambda \end{vmatrix} \\
&= (-1)^{3+3}(1-\lambda)\begin{vmatrix} 1-\lambda & 2-\lambda \\ 0 & 1-\lambda \end{vmatrix} \\
&= (1-\lambda)(1-\lambda)^2 \\
&= (1-\lambda)^3
\end{aligned}
$$

$$\lambda = 1$$

Zadatak (3.37.)

Zadana je matrica $A = \begin{bmatrix} 1 & 3 & 2 \\ 4 & 0 & 5 \\ 2 & \lambda & 1 \end{bmatrix}$. Odredite $\lambda \in \mathbb{R}$ tako da matrica A bude regularna.

Zadatak (3.38.)

Gaussovom metodom riješite sustav:

$$\begin{cases} x + y + z + w = 2 \\ -2x - 4y + z + w = 1 \\ x - y - z + w = 5 \end{cases}$$