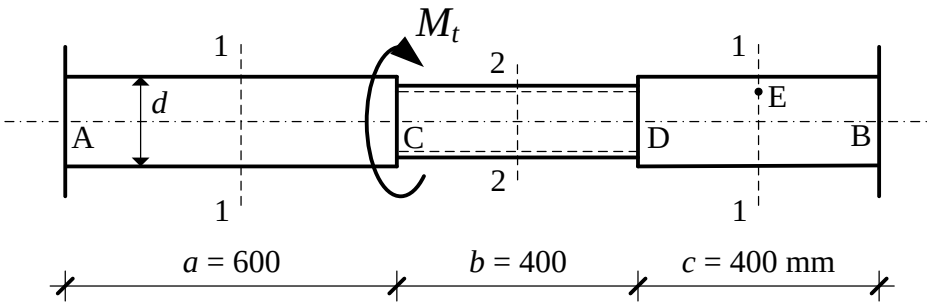
# OTPORNOST MATERIJALA 1 — grupa A

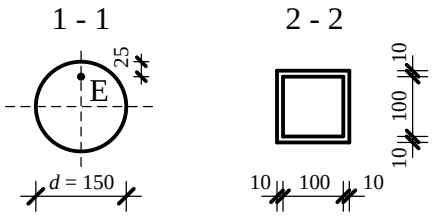
**2. KOLOKVIJ** 08. siječnja 2009. godine

Prezime i ime:_____

**1.** Na štap prikazan na slici djeluje moment torzije $M_t$.

Treba nacrtati dijagram momenata torzije i odrediti kutove zaokreta presjeka **C** i **D**, ako je zadano naprezanje u točki **E** $\tau_E = 30$ MPa i modul posmika $G = 0{,}8 \cdot 10^5$ MPa.

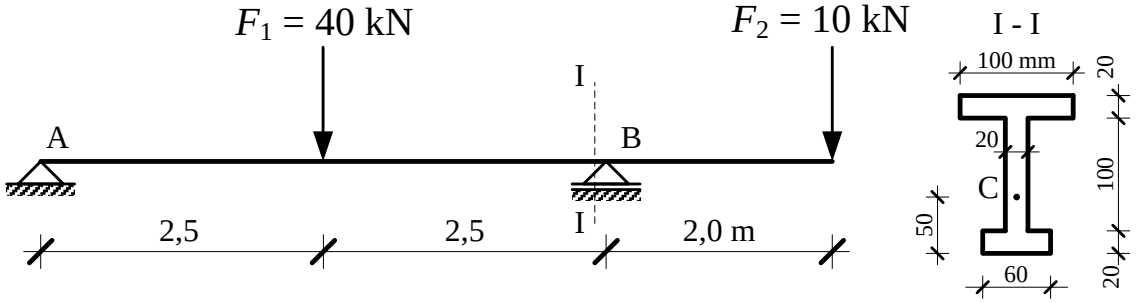



**2.** Za nosač prikazan na slici treba odrediti:

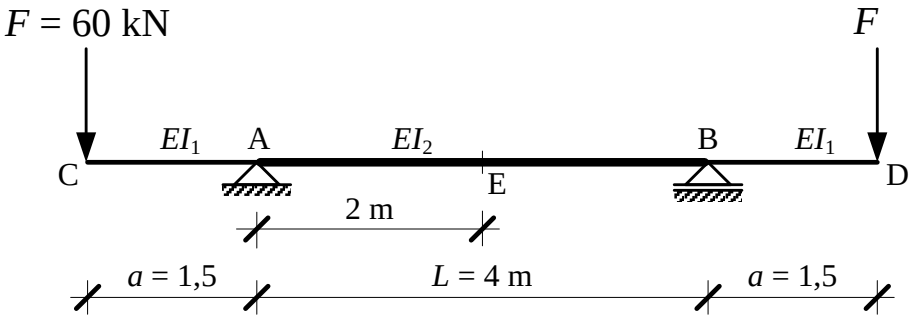
a) Maksimalno normalno i posmično naprezanje, te nacrtati odgovarajuće dijagrame naprezanja u kritičnim presjecima.

b) Veličinu i smjer glavnih naprezanja u točki **C** presjeka **I-I**, te skicirati trajektorije naprezanja u području točke **C**.



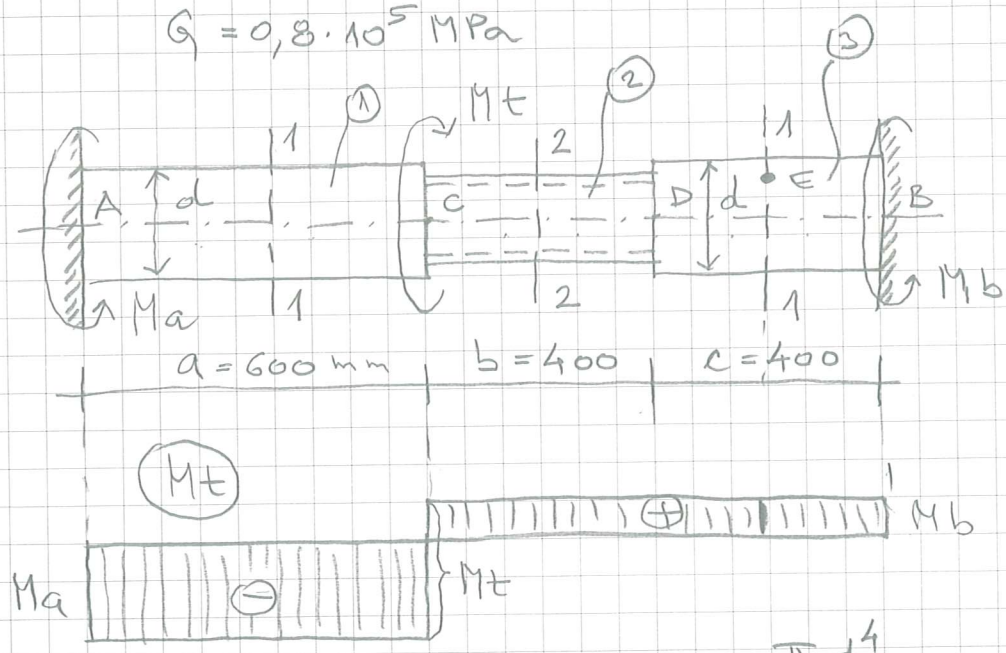

**3.** Grafoanalitičkim postupkom treba odrediti progib točke **E** i kut nagiba na elastičnu liniju presjeka **C**.

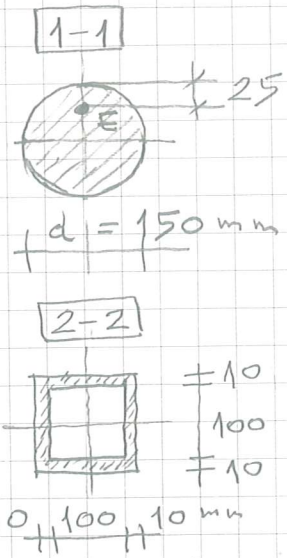
Zadano je: $E = 2{,}0 \cdot 10^5$ MPa
$I_1 = 1{,}5 \cdot 10^8$ mm$^4$
$I_2 = 2{,}0 \cdot 10^8$ mm$^4$.



1. A) $\tau_E = 30$ MPa

$G = 0{,}8 \cdot 10^5$ MPa




U.R. $M_a + M_b = M_t$ (1)

$$\tau_E = \frac{M_b}{J_{p1}} \cdot \rho_E$$

$$\rho_E = 75 - 25 = 50 \text{ mm}$$

$$J_{p1} = \frac{\pi \cdot d^4}{32} = 49{,}701 \cdot 10^6 \text{ mm}^4$$

$$J_{t2} = \frac{4 \cdot A_0^2}{\sum \frac{s_i}{t_i}} = \frac{4 \cdot (110^2)^2}{4 \cdot \frac{110}{10}} = 13{,}31 \cdot 10^6 \text{ mm}^4$$

$$A_0 = 110^2 = 12100 \text{ mm}^2$$

$$M_b = \frac{\tau_E \cdot J_{p1}}{\rho_E} = \frac{30 \cdot 49{,}701 \cdot 10^6}{50} = \underline{29{,}82 \text{ kNm}}$$

U.D. $$\varphi'_B = \frac{M_b \cdot c}{G J_{p1}} + \frac{M_b \cdot b}{G J_{t2}} + \frac{M_b \cdot a}{G J_{p1}} - \frac{M_t \cdot a}{G J_{p1}} = \phi \;\Big|\cdot \frac{G J_{p1}}{a}$$ (2)

$$M_t = M_b\left(\frac{c}{a} + \frac{J_{p1}}{J_{t2}} \cdot \frac{b}{a} + 1\right) \rightarrow M_t = 4{,}156 \cdot M_b$$

$\underline{M_t = 123{,}93 \text{ kNm}}$ $\quad M_a = M_t - M_b = \underline{94{,}11 \text{ kNm}}$

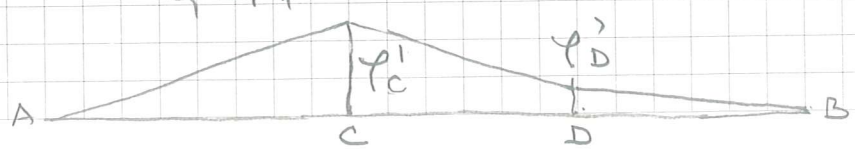
$$\varphi'_C = \frac{M_a \cdot a}{G J_{p1}} = \underline{0{,}0142 \text{ rad} = 0° 48' 49''}$$

$$\varphi'_D = \frac{M_b \cdot c}{G J_{p1}} = \underline{0{,}0030 \text{ rad} = 0° 10' 19''}$$

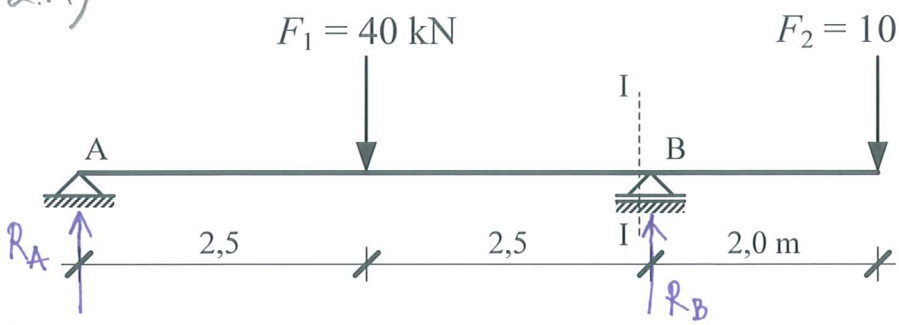
$\tau^{(1)}_{max} = \underline{142{,}01 \text{ MPa}}$

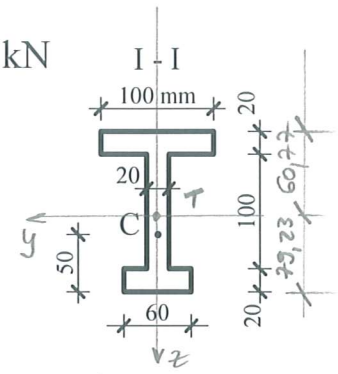
$\tau^{(2)}_{max} = \underline{123{,}22 \text{ MPa}}$

$\tau^{(3)}_{max} = \underline{45{,}0 \text{ MPa}}$

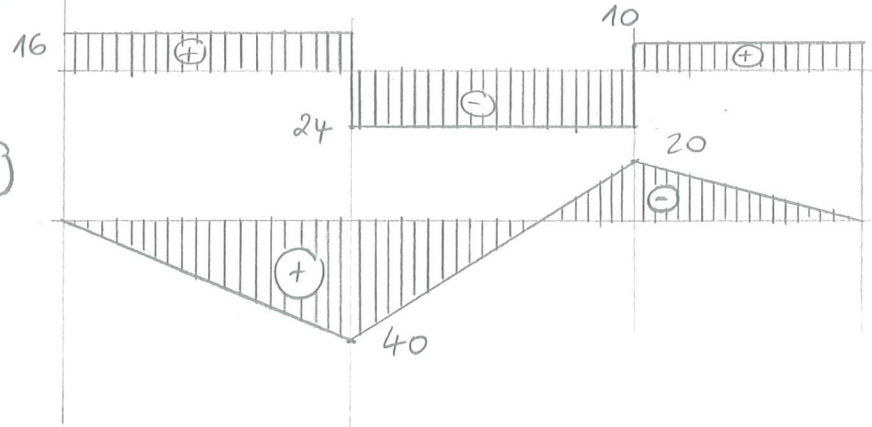


2.A)




(T)




(M)

$\Sigma M_A = 0$

$40 \cdot 2{,}5 - R_B \cdot 5 + 10 \cdot 7 = 0$

$R_B = 34$ kN

$R_A = 16$ kN

$$z_T = \frac{\Sigma A_i \cdot z_i}{\Sigma A_i} = \frac{60 \cdot 20 \cdot 10 + 100 \cdot 20 \cdot 70 + 100 \cdot 20 \cdot 130}{60 \cdot 20 + 100 \cdot 20 + 100 \cdot 20} = 79{,}23 \text{ mm}$$

$$J_y = \frac{60 \cdot 20^3}{12} + 60 \cdot 20 \cdot 69{,}23^2 + \frac{20 \cdot 100^3}{12} + 20 \cdot 100 \cdot 9{,}23^2 + \frac{100 \cdot 20^3}{12} + 100 \cdot 20 \cdot 50{,}77^2$$

$$= 1{,}285 \cdot 10^7 \text{ mm}^4$$

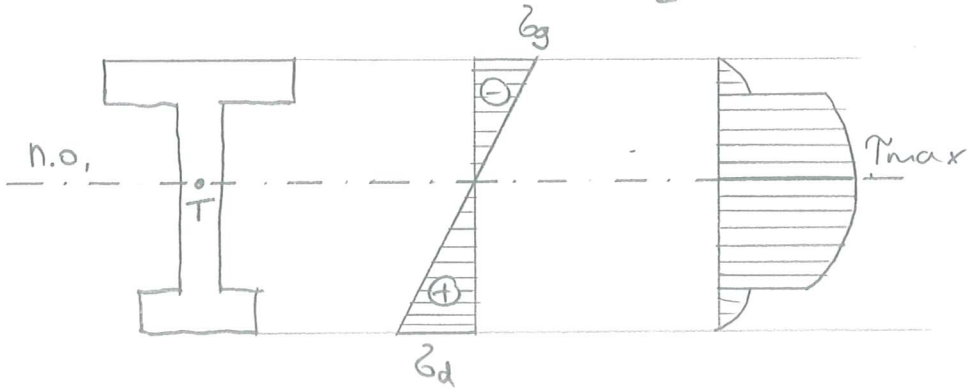
a) $\sigma_{max} = \frac{M_{max}}{J_y} z_{max}$

$$\sigma_g = \frac{40 \cdot 10^6}{1{,}285 \cdot 10^7} \cdot (-60{,}77) = -189{,}17 \text{ MPa}$$

$$\sigma_d = \frac{40 \cdot 10^6}{1{,}285 \cdot 10^7} \cdot 79{,}23 = 246{,}63 \text{ MPa}$$

$$\tau_{max} = \frac{T_{max} \cdot S_{ymax}}{J_y \cdot b} = \frac{-24 \cdot 10^3 \cdot 1{,}18 \cdot 10^5}{1{,}285 \cdot 10^7 \cdot 20} = -11{,}01 \text{ MPa}$$

$$S_{ymax} = 60 \cdot 20 \cdot 69{,}23 + 20 \cdot \frac{59{,}23^2}{2} = 1{,}18 \cdot 10^5 \text{ mm}^3$$



b) $\sigma_x^c = \frac{M_{I-I}}{J_y} \cdot z_c = \frac{-20 \cdot 10^6}{1{,}285 \cdot 10^7} \cdot 29{,}23 = -45{,}49 \text{ MPa}$

$\tau_{xz}^c = \frac{T_{I-I} \cdot S_{yc}}{J_y \cdot b} = \frac{-24 \cdot 10^3 \cdot 1{,}096 \cdot 10^5}{1{,}285 \cdot 10^7 \cdot 20} = -10{,}24 \text{ MPa}$

$S_{yc} = 60 \cdot 20 \cdot 69{,}23 + 30 \cdot 20 \cdot (15 + 29{,}23) = 1{,}096 \cdot 10^5 \text{ mm}^3$

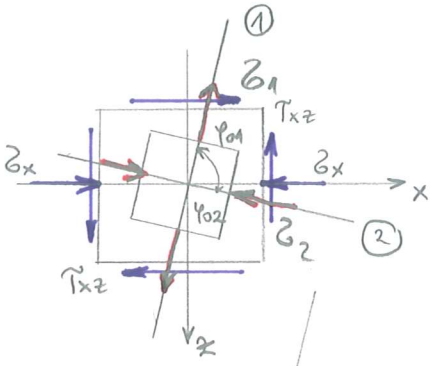
$\sigma_{1,2} = \frac{\sigma_x + \sigma_z}{2} \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_z)^2 + 4\tau_{xz}^2} =$

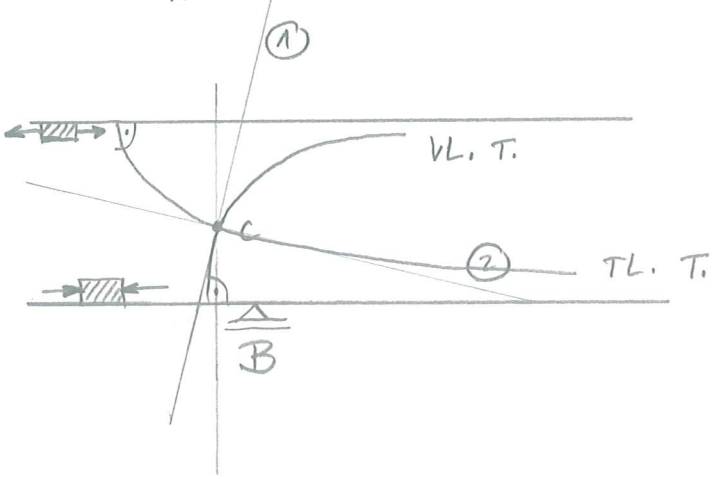
$= \frac{-45{,}49}{2} \pm \frac{1}{2}\sqrt{45{,}49^2 + 4 \cdot 10{,}24^2} = -22{,}75 \pm 24{,}94$

$\sigma_1 = 2{,}19 \text{ MPa} \qquad \sigma_2 = -47{,}69 \text{ MPa}$

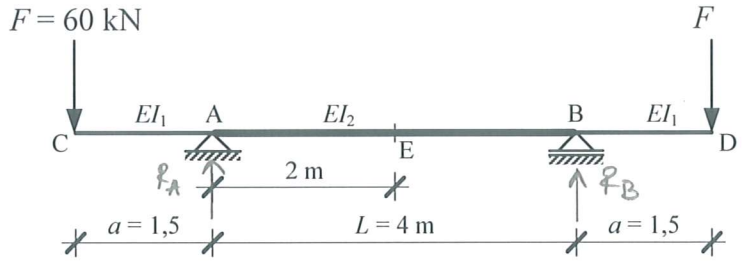
$tg\,\varphi_{01} = \frac{\tau_{xz}}{\sigma_1} = \frac{-10{,}24}{2{,}19} = -4{,}6758 \Rightarrow \varphi_{01} = -77{,}93°$

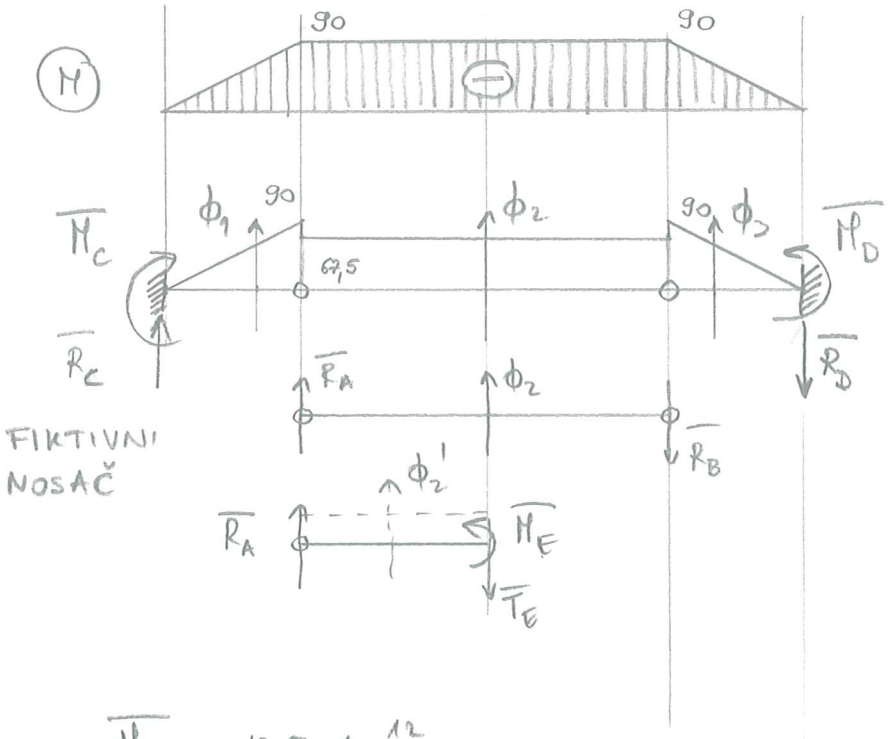
$tg\,\varphi_{02} = \frac{\tau_{xz}}{\sigma_2} = \frac{-10{,}24}{-47{,}69} = 0{,}2147 \Rightarrow \varphi_{02} = 12{,}12°$



3.A)




$$E = 2 \cdot 10^5 \text{ MPa}$$

$$J_1 = 1{,}5 \cdot 10^8 \text{ mm}^4$$

$$J_2 = 2 \cdot 10^8 \text{ mm}^4$$

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$$\Sigma M_A = 0$$

$$R_B \cdot 4 - F \cdot 5{,}5 + F \cdot 1{,}5 = 0$$

$$R_B = R_A = 60 \text{ kN} \qquad J_0 = J_1$$

$$M^* = M \frac{J_0}{J} = 90 \cdot \frac{1{,}5}{2} = 67{,}5 \text{ kNm}$$

$$\phi_1 = \phi_3 = \frac{90 \cdot 1{,}5}{2} = 67{,}5 \text{ kNm}^2$$

$$\phi_2 = 67{,}5 \cdot 4 = 270 \text{ kNm}^2$$

$$\overline{R}_D = \frac{\phi_1 + \phi_2 + \phi_3}{2} = 202{,}5 \text{ kNm}^2$$

$$\overline{R}_C = -202{,}5 \text{ kNm}^2$$

$$\Sigma \overline{M}_B = 0$$

$$\overline{R}_A \cdot 4 + \phi_2 \cdot 2 = 0$$

$$\overline{R}_A = -135 \text{ kNm}^2$$

$$\phi_2' = 135 \text{ kNm}^2$$

$$\Sigma \overline{M}_E = 0$$

$$\overline{M}_E - \overline{R}_A \cdot 2 - \phi_2' \cdot 1 = 0$$

$$\overline{M}_E = -135 \text{ kNm}^3$$

$$w_E = \frac{\overline{M}_E}{E I_0} = \frac{-135 \cdot 10^{12}}{2 \cdot 10^5 \cdot 1{,}5 \cdot 10^8} = -4{,}5 \text{ mm}$$

$$\varphi_C = \frac{\overline{R}_C}{E I_0} = \frac{-202{,}5 \cdot 10^9}{2 \cdot 10^5 \cdot 1{,}5 \cdot 10^8} = -6{,}75 \cdot 10^{-3} \text{ rad}$$