OTPORNOST MATERIJALA 1

grupa A

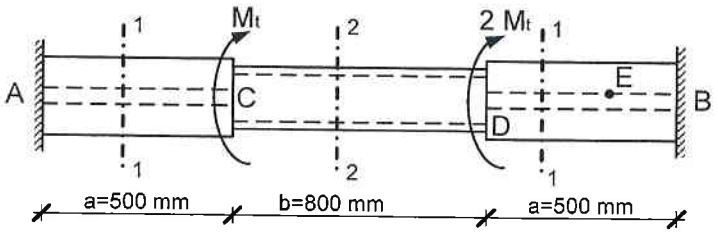
2. KOLOKVIJ 10. siječnja 2011. godine

Prezime i ime:_____

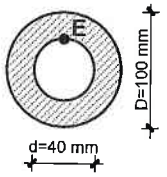
1. Odredite maksimalna naprezanja u presjecima **1-1** i **2-2** te kut zaokreta u točki **C** ako je zadano naprezanje u točki E, τ_E = 10 MPa.

Zadano:

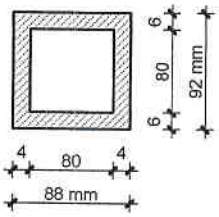
$G = 0{,}7 \cdot 10^5$ MPa



Presjek 1 - 1



Presjek 2 - 2



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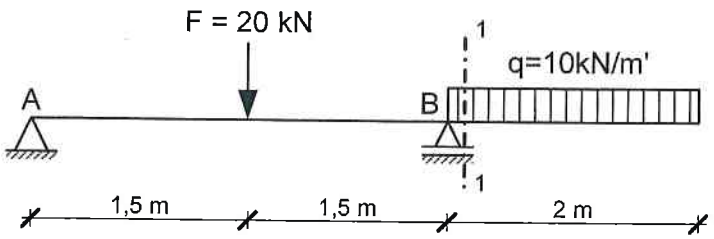
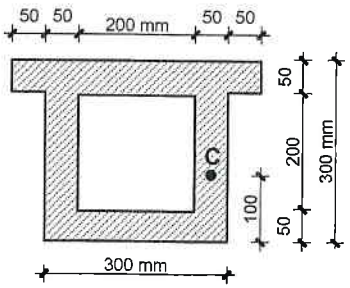
2. Za nosač prikazan na slici treba odrediti:

a) Maksimalno normalno i posmično naprezanje u kritičnim presjecima i nacrtati odgovarajuće dijagrame naprezanja.

b) Smjerove i iznose glavnih naprezanja u točki **C** presjeka **1 – 1**.



Presjek 1 - 1



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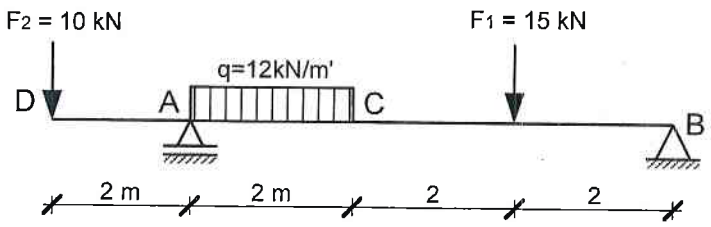
3. Analitičkim postupkom treba odrediti progib u točki **C** i kut nagiba na elastičnu liniju u točki **D**. Skicirajte progibnu liniju.

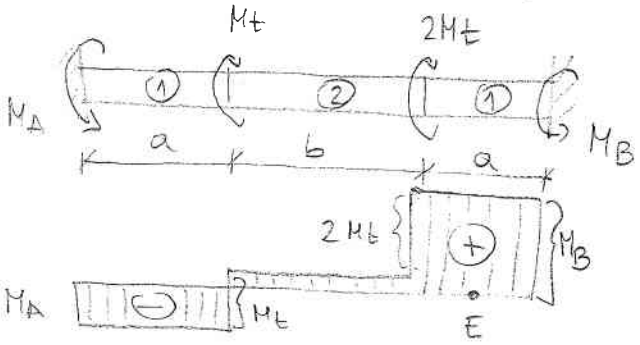
Zadano:

$E = 2 \cdot 10^5$ MPa
$I_y = 5 \cdot 10^7$ mm^4
EI = const.

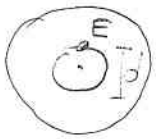


(1) $I_{p_1} = \frac{\pi(D^4 - d^4)}{32} = \frac{\pi(100^4 - 40^4)}{32} = 9{,}566 \cdot 10^6 \text{ mm}^4$

$I_{t_1} = \frac{4A_0^2}{\sum \frac{\Delta i}{t_i}} = \frac{4(84 \cdot 86)^2}{2\left(\frac{84}{6} + \frac{86}{4}\right)} = 2{,}94 \cdot 10^6 \text{ mm}^4$

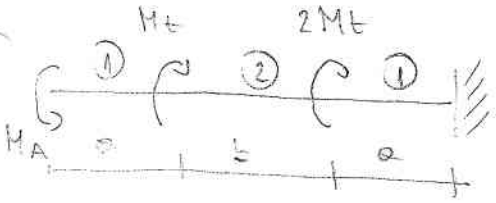


$M_A + M_B = 3M_t$



$\tau_E = \frac{M_B}{I_{p_1}} \cdot \frac{d}{2}$

$M_B = \frac{\tau_E \cdot I_{p_1} \cdot 2}{d} = \frac{10 \cdot 9{,}566 \cdot 10^6 \cdot 2}{40}$



$M_B = 4783000 \text{ Nmm} = 4{,}783 \text{ kNm}$

$\varphi_A = 0 = -\frac{M_A \cdot 2a}{G \cdot I_{p_1}} - \frac{M_A \cdot b}{G \cdot I_{t_2}} + \frac{M_t \cdot b}{G \cdot I_{t_2}} + \frac{M_t \cdot a}{G \cdot I_{p_1}} + \frac{2M_t \cdot a}{G \cdot I_{p_1}} = 0 \;/\; \cdot \frac{G \cdot I_{p_1}}{a}$

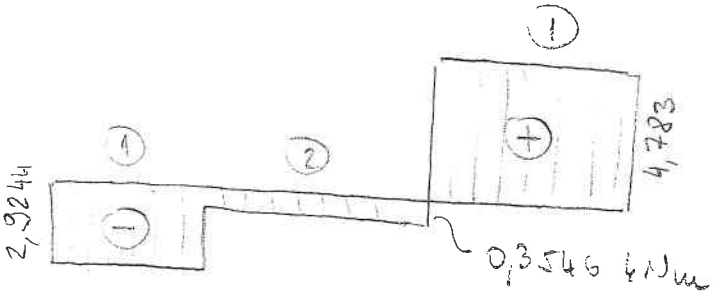
$-2M_A - M_A \frac{I_{p_1} \cdot b}{I_{t_2} \cdot a} + M_t \frac{I_{p_1} b}{I_{t_2} \cdot a} + 3M_t = 0$

$M_t \left[3 + \frac{I_{p_1} b}{I_{t_2} a}\right] = M_A \left[2 + \frac{I_{p_1} b}{I_{t_2} a}\right] \Rightarrow M_A = M_t \cdot \left[\frac{3 + \frac{I_{p_1} b}{I_{t_2} a}}{2 + \frac{I_{p_1} b}{I_{t_2} a}}\right]$

$M_A = 1{,}1388 \; M_t$

$1{,}1388 \; M_t + M_B = 3M_t \Rightarrow M_t = \frac{M_B}{1{,}861} = 2{,}5698 \text{ kNm}$

$M_A = 2{,}9244 \text{ kNm}$

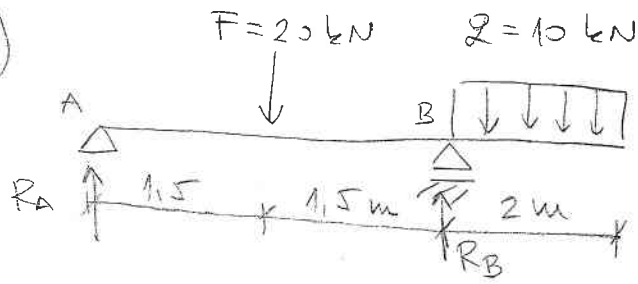


$\tau_{max}^1 = \frac{M_B}{I_{p_1}} \cdot \frac{D}{2} = \frac{4{,}783 \cdot 10^6}{9{,}566 \cdot 10^6} \cdot \frac{100}{2} = 25 \text{ MPa}$

$\tau_{max}^2 = \frac{M_2}{2A_0 \cdot t_{min}} = \frac{0{,}3546 \cdot 10^6}{2 \cdot (86 \cdot 84) \cdot 4} = 6{,}14 \text{ MPa}$

$\varphi_C = -\frac{M_A \cdot a}{G \cdot J_{t1}} = -2{,}185 \cdot 10^{-3} \text{ rad} = -0°\,7'\,30.69''$

(2.)



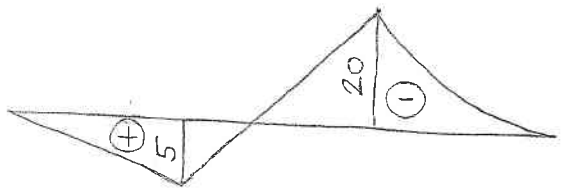
$\Sigma M_A = 0 \quad 20 \cdot 1,5 + 10 \cdot 2 \cdot 1 - R_B \cdot 3 = 0$

$R_B = 36,67$ kN

$\Sigma M_B = 0 \quad 20 \cdot 1,5 - R_A \cdot 3 - 10 \cdot 2 \cdot 1 = 0$

$R_A = 3,33$ kN

M



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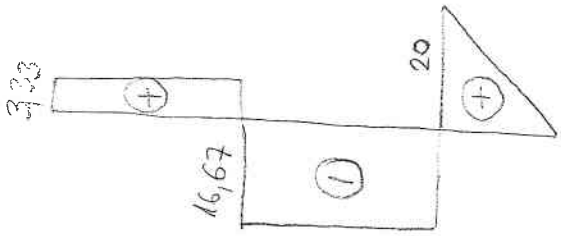
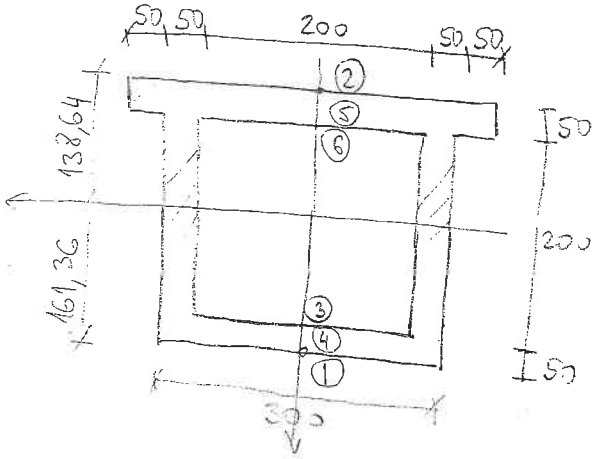




$A = 400 \cdot 300 - 200 \cdot 200 - 2 \cdot 50 \cdot 250 = 55\,000$ mm

$z_T = \dfrac{400 \cdot 300 \cdot 150 - 200 \cdot 200 \cdot 150 - 2 \cdot 50 \cdot 250 \cdot 125}{55\,000} = 161,36$ mm

$I_y = \dfrac{400 \cdot 300^3}{12} + 400 \cdot 300 \cdot 11,36^2 - \left[\dfrac{200 \cdot 200^3}{12} + 200 \cdot 200 \cdot 11,36^2\right] - 2\left[\dfrac{50 \cdot 250^3}{12} + 50 \cdot 250 \cdot 36,36^2\right] = 613,73 \cdot 10^6$ mm^4

$\sigma_{max} = \dfrac{M_B}{I_y} \cdot z_1 = \dfrac{-20 \cdot 10^6}{613,73 \cdot 10^6} \cdot 161,36 = -5,26$ MPa (tah)

$\sigma_{min} = \dfrac{M_B}{I_y} \cdot z_2 = \dfrac{-20 \cdot 10^6}{613,73 \cdot 10^6} \cdot (-138,64) = +4,52$ MPa (tlak)

$S_\Delta = 300 \cdot 50 (161,36 - 25) + 2\left[111,36 \cdot 50 \cdot \dfrac{111,36}{2}\right] = 2,665 \cdot 10^6$ mm^3

$\tau_{max} = \dfrac{T_{max} \cdot S_y}{I_y \cdot b} = \dfrac{20 \cdot 10^3 \cdot 2,665 \cdot 10^6}{613,73 \cdot 10^6 \cdot 100} = 0,868$ MPa

$S_{3,4} = 50 \cdot 300 (161,36 - 25) = 2,045 \cdot 10^6$ mm^3

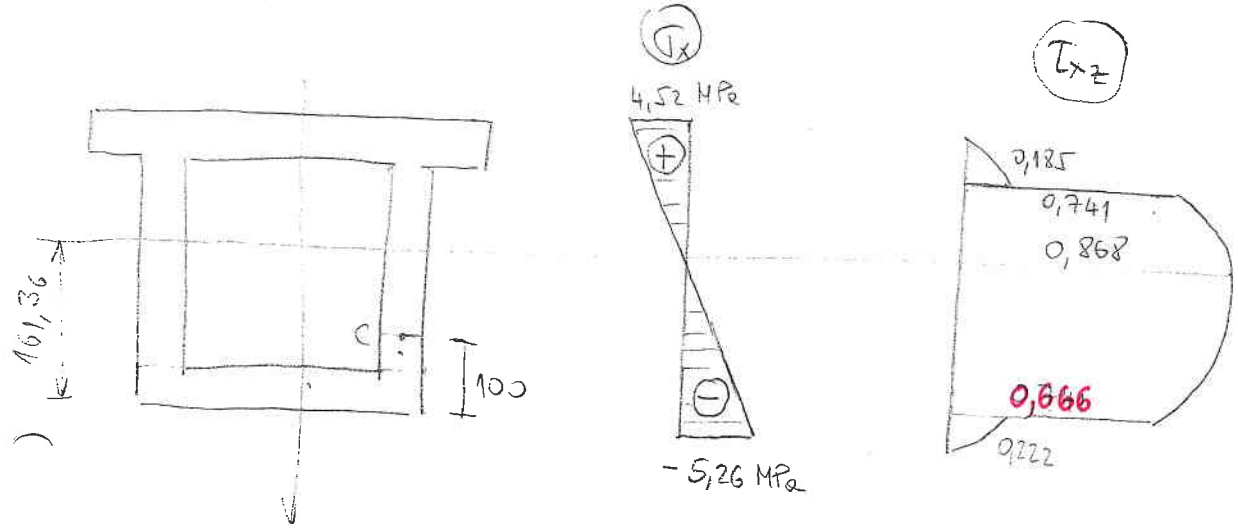
$\tau_3 = \dfrac{20 \cdot 10^3 \cdot 2,045 \cdot 10^6}{613,73 \cdot 10^6 \cdot 100} = 0,666$ MPa

$\tau_4 = \dfrac{20 \cdot 10^3 \cdot 2,045 \cdot 10^6}{613,73 \cdot 10^6 \cdot 300} = 0,222$ MPa

$$S_{5,6} = 50 \cdot 400 (138,64 - 25) = 2,273 \cdot 10^6 \text{ mm}^3$$

$$\tau_5 = \frac{20 \cdot 10^3 \cdot 2,273 \cdot 10^6}{613,73 \cdot 10^6 \cdot 400} = 0,185 \text{ MPa}$$

$$\tau_6 = \frac{20 \cdot 10^6 \cdot 2,273 \cdot 10^6}{613,73 \cdot 10^6 \cdot 100} = 0,741 \text{ MPa}$$



b) $M_c = -20 \text{ kNm}$

$T_c = 20 \text{ kN}$

$$\sigma_c = \frac{M_c}{I_y} \cdot z_c = \frac{-20 \cdot 10^6}{613,73 \cdot 10^6} \cdot 61,36 = -2 \text{ MPa}$$

$$\tau_c = \frac{T_c \cdot S_y^c}{I_y \cdot b_c} = \frac{20 \cdot 10^3 \cdot 2,477 \cdot 10^6}{613,73 \cdot 10^6 \cdot 100} = 0,807 \text{ MPa}$$

$$S_y^c = 300 \cdot 50 (161,36 - 25) + 2 \cdot 50 \cdot 50 (61,36 + 25) = 2,477 \cdot 10^6 \text{ mm}^3$$

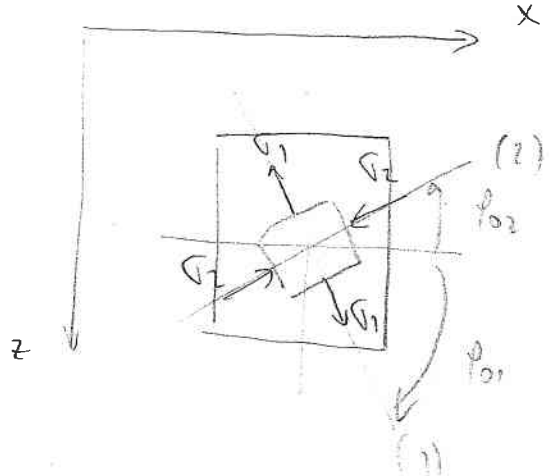
$$\sigma_{1,2} = \frac{\sigma_x}{2} \pm \frac{1}{2}\sqrt{\sigma_x^2 + 4\tau_{xz}^2} = -\frac{2}{2} \pm \frac{1}{2}\sqrt{(-2)^2 + 4 \cdot 0,807^2} = -1 \pm 1,285$$

$\sigma_1 = 0,285 \text{ MPa}$

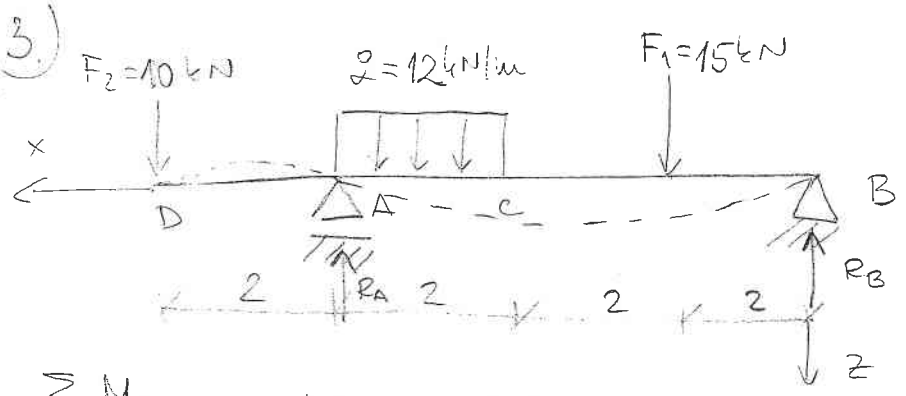
$\sigma_2 = -2,285 \text{ MPa}$

$$\operatorname{tg} \varphi_{01} = \frac{0,807}{0,285} \qquad \varphi_{01} = 70,55°$$

$$\operatorname{tg} \varphi_{02} = \frac{0,807}{-2,285} \qquad \varphi_{02} = -19,45°$$



3)



$E = 2 \cdot 10^5$ MPa

$I = 5 \cdot 10^7$ mm^4

$EI = 10 \cdot 10^3$ kNm^2

$w_C = ?$

$\varphi_D = ?$

$\sum M_A = 0 \quad 10 \cdot 2 - 12 \cdot 2 \cdot 1 - 15 \cdot 4 + R_B \cdot 6 = 0$

$\sum M_B = 0 \quad 10 \cdot 8 - R_A \cdot 6 + 12 \cdot 2 \cdot 5 + 15 \cdot 2 = 0$

$R_B = 10{,}67$ kN

$R_A = 38{,}33$ kN

$$M(x) = R_B \cdot x - F_1 \cdot (x-2) - q\frac{(x-4)^2}{2} + q\frac{(x-6)^2}{2} + R_A(x-6)$$

$$\frac{d^2w}{dx^2} EI = -R_B x + F_1(x-2) + q\frac{(x-4)^2}{2} - q\frac{(x-6)^2}{2} - R_A(x-6) \quad / \int$$

$$\frac{dw}{dx} EI = -R_B\frac{x^2}{2} + F_1\frac{(x-2)^2}{2} + q\frac{(x-4)^3}{6} - q\frac{(x-6)^3}{6} - R_A\frac{(x-6)^2}{2} + C \quad / \int$$

$$w \cdot EI = -R_B\frac{x^3}{6} + F_1\frac{(x-2)^3}{6} + q\frac{(x-4)^4}{24} - q\frac{(x-6)^4}{24} - R_A\frac{(x-6)^3}{6} + Cx + D$$

P.U.I. $w(0) = 0 \Rightarrow D = 0$

$w(6) = 0$

$$0 \cdot EI = -10{,}67\frac{6^3}{6} + 15\frac{4^3}{6} + 12\frac{2^4}{24} + C \cdot 6 = 0 \qquad C = 36{,}02 \text{ kNm}^2$$

$$w_C = \left[-10{,}67\frac{4^3}{6} + 15\frac{2^3}{6} + 36{,}02 \cdot 4\right]\frac{1}{EI} = \frac{50{,}266}{10 \cdot 10^3} = 5{,}026 \cdot 10^{-3} \text{ m}$$

$x=4$

$= 5{,}026$ mm

$$\varphi_D = \left[-10{,}67\frac{8^2}{2} + 15\frac{6^2}{2} + 12\frac{4^3}{6} - 12\frac{2^3}{6} - 38{,}33\frac{2^2}{2} + 36{,}02\right]\frac{1}{EI} =$$

$x=8$

$$= \frac{-0{,}08}{10 \cdot 10^3} = -8 \cdot 10^{-6} \text{ rad}$$