

OTPORNOST MATERIJALA 2

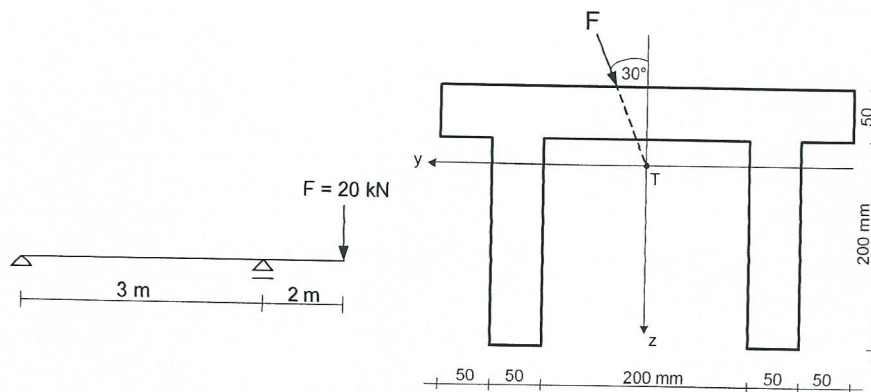
1. kolokvij, 4. travnja 2016.

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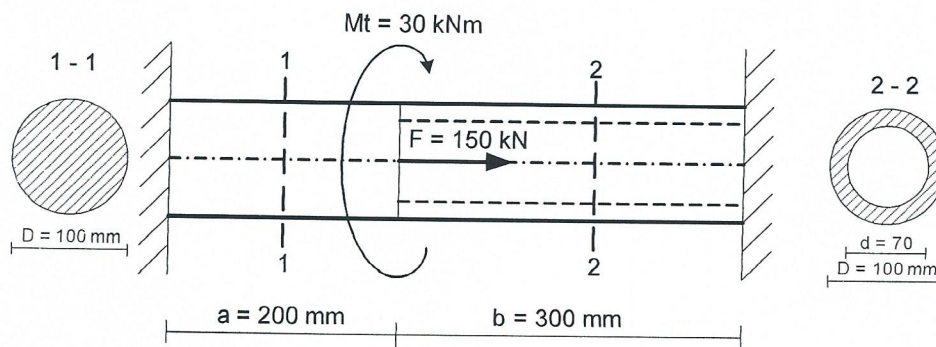
Ime i prezime:

1. Za zadani nosač opterećen prema slici treba primjenom jezgre poprečnog presjeka odrediti najveća normalna naprezanja te nacrtati dijagram. 35



2. Čelična osovina opterećena je momentom torzije i uzdužnom silom. Pomoću 3. teorije čvrstoće treba odrediti koeficijent sigurnosti. 30

$$\sigma_T = 260 \text{ MPa}$$



3. Za sistem prikazan na slici treba odrediti i nacrtati dijagrame unutarnjih sila ako je: 35

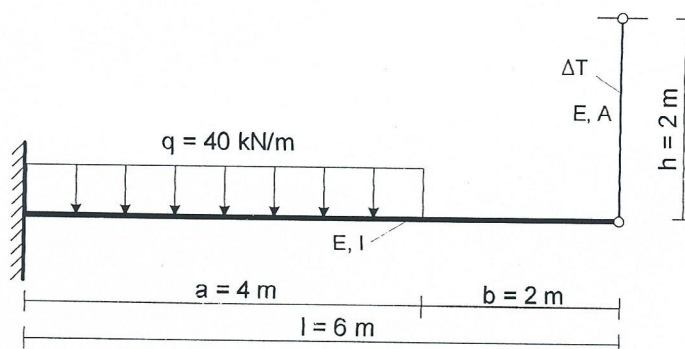
$$E = 2 \cdot 10^5 \text{ MPa}$$

$$I = 1,5 \cdot 10^8 \text{ mm}^4$$

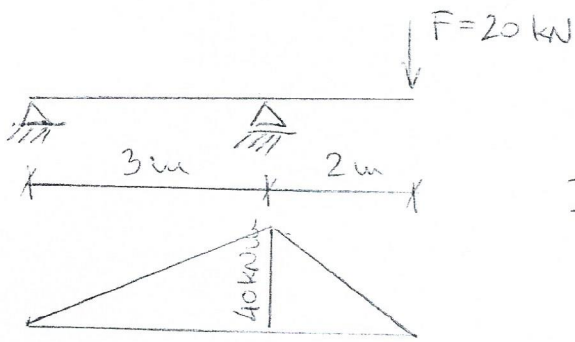
$$A = 200 \text{ mm}^2$$

$$\alpha_T = 1,2 \cdot 10^{-5} \text{ K}^{-1}$$

$$\Delta T = +20 \text{ K}$$



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$$A = 40000 \text{ mm}^2$$

$$z_T = 87,5 \text{ mm}$$

$$I_y = \frac{400 \cdot 50^3}{12} + 400 \cdot 50 \cdot 62,5^2 + 2 \left(\frac{50 \cdot 200^3}{12} + 50 \cdot 200 \cdot 62,5^2 \right)$$

$$= 227,083 \cdot 10^6 \text{ mm}^4$$

$$I_z = \frac{50 \cdot 400^3}{12} + 2 \left(\frac{200 \cdot 50^3}{12} + 200 \cdot 50 \cdot 125^2 \right)$$

$$= 583,333 \cdot 10^6 \text{ mm}^4$$

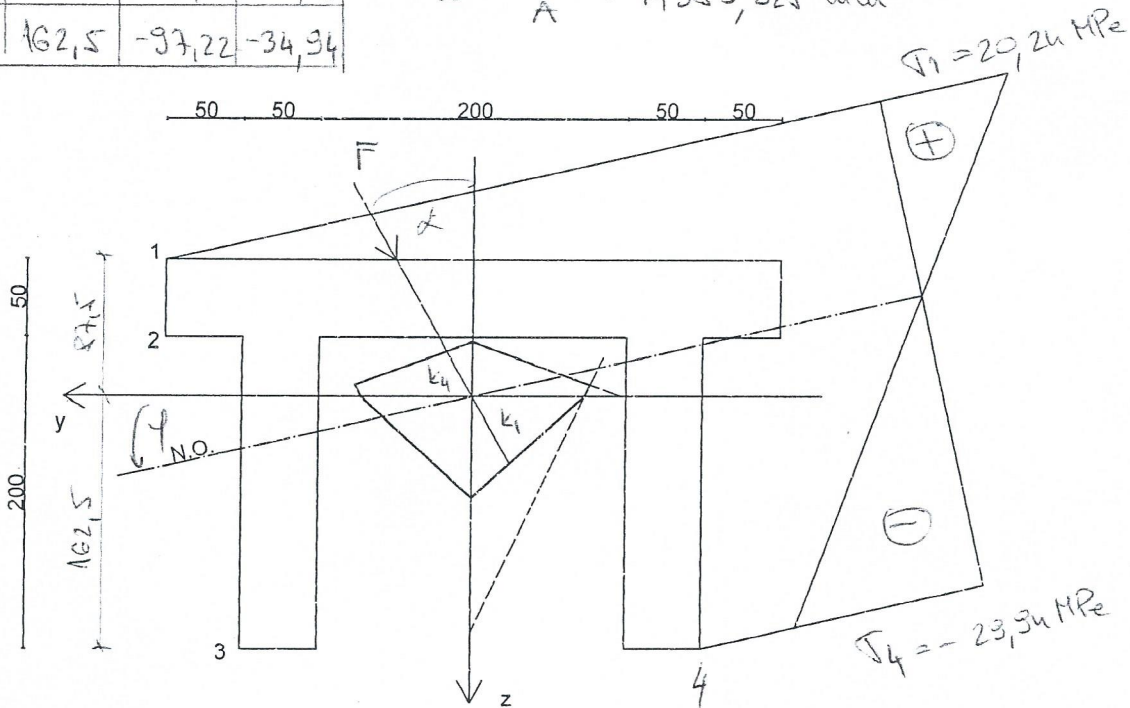
	y	z	α_y	α_z
1	200	-87,5	-72,92	64,88
2	200	-37,5	-72,92	151,33
3	150	162,5	-97,22	-34,94

$$i_y^2 = \frac{I_y}{A} = 5677,075 \text{ mm}^2$$

$$i_z^2 = \frac{I_z}{A} = 14583,325 \text{ mm}^2$$

$$\alpha_y = -\frac{i_z^2}{y}$$

$$\alpha_z = -\frac{i_y^2}{z}$$



$$\alpha = -30^\circ$$

$$\tan \varphi = -\frac{I_y}{I_z} \tan \alpha$$

$$\varphi = 12,67^\circ$$

MJ 1:5

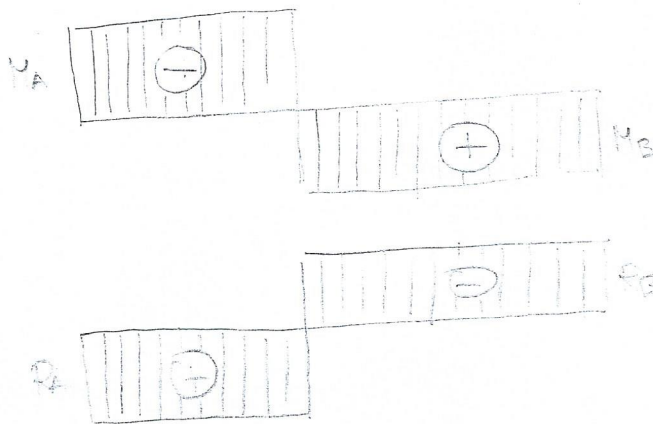
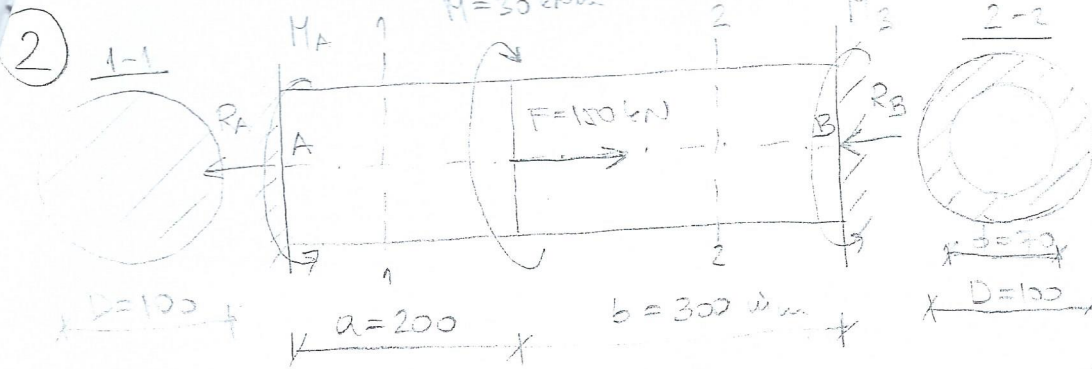
$$k_1 = 49,4 \text{ mm}$$

$$k_4 = 33,4 \text{ mm}$$

$$\sigma_1 = \sigma_{\max} = \frac{M}{k_1 \cdot A} = 20,24 \text{ MPa}$$

$$\sigma_4 = \sigma_{\min} = \frac{M}{k_4 \cdot A} = -23,94 \text{ MPa}$$

$$\tau_T = 260 \text{ MPa}$$



$$I_{p1} = \frac{D^4 T}{32} = 98175 \cdot 10^5 \text{ mm}^4$$

$$I_{p2} = \frac{(D^4 - d^4) \pi}{32} = 7,4603 \cdot 10^6 \text{ mm}^4$$

$$A_1 = 7853,98 \text{ mm}^2$$

$$A_2 = 4005,53 \text{ mm}^2$$

$$W_{p1} = \frac{I_{p1}}{50} = 196350 \text{ mm}^3$$

$$W_{p2} = \frac{I_{p2}}{50} = 149206 \text{ mm}^3$$

$$M_A + M_B = M_T$$

$$\frac{M_B \cdot 300}{G \cdot I_{p2}} + \frac{M_B \cdot 200}{G \cdot I_{p1}} - \frac{M_T \cdot 200}{G \cdot I_{p1}} = 0 \quad / \cdot \frac{G \cdot I_{p1}}{200}$$

$$M_B \left(\frac{3}{2} \frac{I_{p1}}{I_{p2}} + 1 \right) = M_T \Rightarrow M_B = 10,09 \text{ kNm}$$

$$M_A = 19,91 \text{ kNm}$$

$$R_A + R_B = F$$

$$\Delta_B = 0 \Rightarrow \frac{R_B \cdot 200}{E \cdot A_1} + \frac{R_B \cdot 300}{E \cdot A_2} - \frac{F \cdot 200}{E \cdot A_1} = 0 \quad / \cdot \frac{E \cdot A_1}{200}$$

$$R_B \left(1 + \frac{3}{2} \frac{A_1}{A_2} \right) = F \Rightarrow R_B = 38,06 \text{ kN}$$

$$R_A = 11,94 \text{ kN}$$

$$\sigma = \frac{R_A}{A_1} = 14,25 \text{ MPa}$$

$$\sigma_{1/2} = 7,125 \pm 101,65$$

$$\tau = \frac{V_A}{W_{p1}} = 101,4 \text{ MPa}$$

$$\sigma_1 = 102,775$$

$$\sigma_2 = -94,525$$

3. T. č.

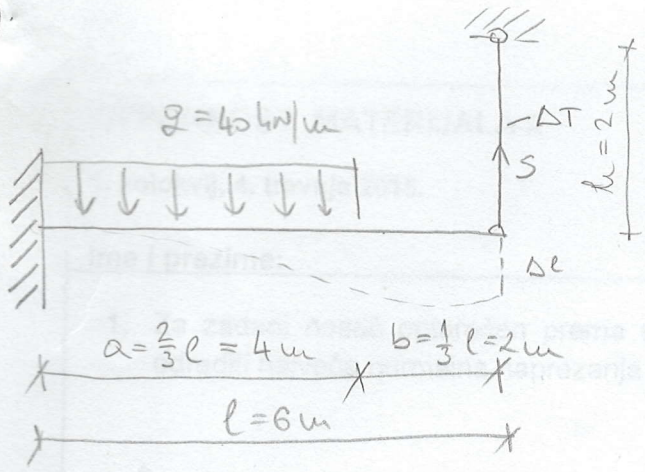
$$\sigma_{\text{velu}} = \sigma_1 - \sigma_2 = 203,3 \text{ MPa}$$

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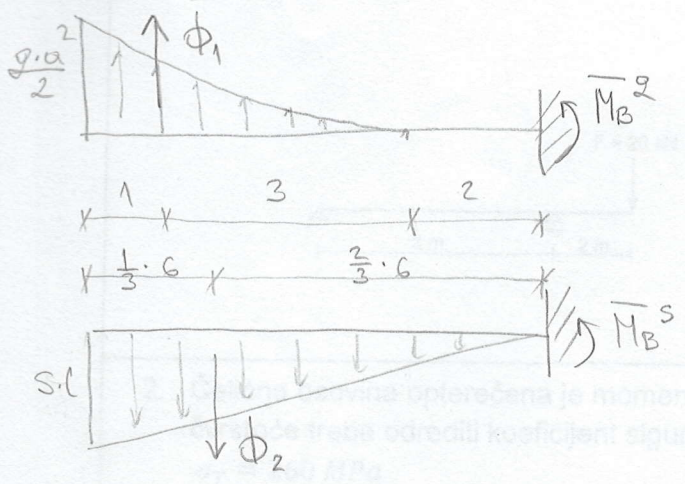
$$\sigma_{\text{velu}} = \sqrt{9^2 + 40^2} = 203,3 \checkmark$$

$$k = \frac{\sigma_T}{\sigma_{\text{velu}}} = 1,28$$

$E = 2 \cdot 10^5 \text{ MPa}$
 $I = 1,5 \cdot 10^8 \text{ mm}^4$
 $A = 200 \text{ mm}^2$
 $\Delta T = +20 \text{ K}$
 $\alpha_T = 1,2 \cdot 10^{-5} \text{ K}^{-1}$



$\Delta l = \frac{S \cdot h}{EA} + \alpha \cdot \Delta T \cdot h$
 $\Delta l = |W_B^q| - |W_B^S| = \frac{M_B^q}{EI} - \frac{M_B^S}{EI}$



$\phi_1 = \frac{q \cdot a^2}{2} \cdot \frac{1}{3} a = \frac{q a^3}{6} = 426,67 \text{ [kNm}^2\text{]}$

$\phi_2 = S \cdot l \cdot l \cdot \frac{1}{2} = \frac{S l^2}{2} = 18 \cdot S \text{ [kNm}^2\text{]}$

$M_B^q = \frac{5}{6} l \cdot \phi_1 = \frac{5}{36} a l q = 2133,35 \text{ kNm}^3$

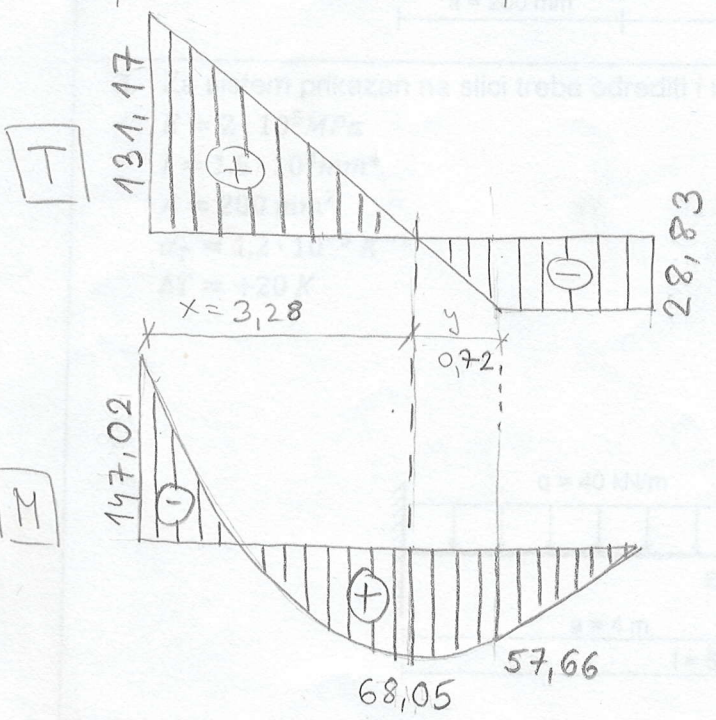
$M_B^S = -\phi_2 \cdot \frac{2}{3} l = -\frac{S l^3}{3} = -72 \cdot S \text{ kNm}^3$

$w = \frac{M_i}{EI}$

$\frac{5 a^3 \cdot l \cdot q}{36 EI} - \frac{S \cdot l^3}{3 EI} = \frac{S \cdot h}{EA} + \alpha \cdot \Delta T \cdot h \cdot EI$

$\frac{5 \cdot 4000 \cdot 6000 \cdot 40}{36} - 1,2 \cdot 10^{-5} \cdot 20 \cdot 2000 \cdot 2 \cdot 10^5 \cdot 1,5 \cdot 10^8 = S \left(\frac{6000^3}{3} + \frac{2000 \cdot 1,5 \cdot 10^8}{200} \right)$

$2,11893 \cdot 10^{15} = S \cdot 7,05 \cdot 10^{10} \rightarrow \boxed{S = 28,83 \text{ kN}}$



$131,17 - q \cdot x = 0 \rightarrow x = 3,28 \text{ m}$

$M_{max} + \frac{q \cdot y^2}{2} - S \cdot 2,72 = 0$

$M_{max} = 68,05 \text{ kNm}$

