MATEMATIKA 1 1.7.2020. α

1. **(20 bodova)** Odredite točku N simetričnu točki $M(-1,0,4)$ obzirom na pravac $p ... \frac{x+1}{2} = \frac{y+5}{3} = \frac{z-3}{-1}$. Skicirajte te odredite udaljenost točke M od točke N.

2. a) **(10 bodova)** Odredite svojstvene vrijednosti matrice $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$ te odredite svojstveni vektor najveće svojstvene vrijednosti.

   b) **(10 bodova)** Odredite opći član te ispitajte konvergenciju reda

$$\frac{0}{3^1 \cdot 1} + \frac{1}{3^2 \cdot 2} + \frac{2}{3^3 \cdot 3} + \frac{3}{3^4 \cdot 4} + \cdots .$$

3. **(20 bodova)** Odredite prirodno područje definicije, nultočke, intervale rasta i pada, ekstreme, asimptote te skicirajte graf funkcije $f(x) = \frac{x}{x^2 - 10x + 16}$.

4. **(20 bodova)** Odredite

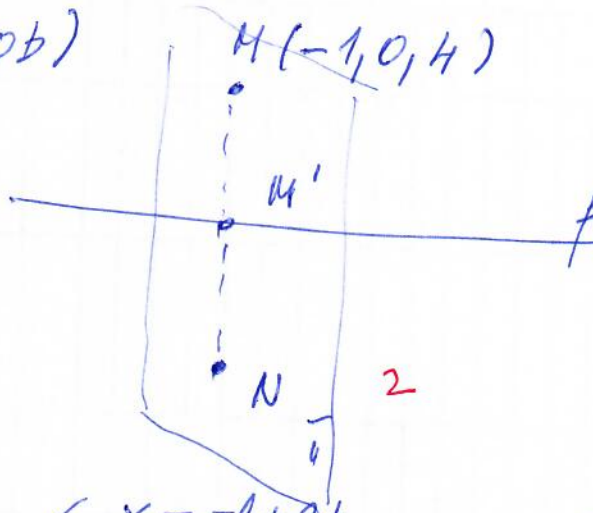
$$\int \frac{e^{3x} + e^x}{e^{4x} - e^{3x}}\, dx.$$

5. (a) **(10 bodova)** Izračunajte nepravi integral $\int_0^3 \frac{1}{(x-1)^2}\, dx$.

   (b) **(10 bodova)** Izračunajte površinu dijela ravnine omeđenog sa $y = x^2$, $y = (x-3)^2$ i $x = 3$. Skicirajte.

MAT1, 1.7.2020. (α)

1. (20b)



$p \ldots \frac{x+1}{2} = \frac{y+5}{3} = \frac{z-3}{-1}$

2

$\pi \perp p \,,\; M \in \pi$

$$p \ldots \begin{cases} x = -1+2t \\ y = -5+3t \\ z = 3-t \end{cases}$$ 3

$\pi \ldots 2(x+1)+3y-(z-4)=0$

$\underline{\pi \ldots 2x+3y-z+6=0}$, 4

$M' = p \cap \pi \ldots \quad 2(-1+2t)+3(-5+3t)-3+t+6=0$ 3

$4t+9t+t-2-15-3+6=0 \Rightarrow 14t=14 \Rightarrow t=1$ 1

$M'(1,-2,2)$ polovište dužine $\overline{MN}$

$N(x,y,z)$

$$\left.\begin{aligned} \frac{x+1}{2} &= 1 \Rightarrow x = 1 \\ \frac{y+0}{2} &= -2 \Rightarrow y = -4 \\ \frac{z+4}{2} &= 2 \Rightarrow z = 0 \end{aligned}\right\} \quad N(1,-4,0)$$ 4

$d(M,N) = \sqrt{(1+1)^2+(-4-0)^2+(0-4)^2} = \sqrt{4+16+16} = \boxed{6}$

3

2. a) (10b)

$$\begin{vmatrix} 2-\lambda & 3 \\ 1 & 4-\lambda \end{vmatrix} = (2-\lambda)(4-\lambda) - 3 = \lambda^2 - 6\lambda + 5 = 0$$

2

$$\lambda_{1,2} = \frac{6 \pm \sqrt{36-20}}{2} \rightarrow \boxed{\begin{matrix}\lambda_1 = 1 \\ \lambda_2 = 5\end{matrix}} \Big\} 3$$ svojstvene vrijednosti

$\underline{\lambda_2 = 5}$

$$\begin{bmatrix} -3 & 3 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow x - y = 0 \Rightarrow x = y = t$$

2 2

$$\boxed{v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} t \ , \ t \neq 0}$$

1

b) (10b)

$$a_n = \frac{n-1}{3^n \cdot n}$$

3

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{n}{3^{\cancel{n+1}}(n+1)}}{\frac{n-1}{\cancel{3^n} \cdot n}} = \lim_{n\to\infty} \frac{n^2}{3(n+1)(n-1)}$$

$$= \frac{1}{3} < 1$$

5

$\Rightarrow$ prema D'Alembertovom testu red konvergira

1 1

3. (20b) $f(x) = \frac{x}{x^2-10x+16}$

DOMENA: $x^2-10x+16=0 \Rightarrow x_{1,2} = \frac{10 \pm \sqrt{100-64}}{2}$

$\Rightarrow x_1 = 2, \ x_2 = 8$ 2

$\boxed{D_f = \mathbb{R} \setminus \{2,8\}}$ 1

NULTOČKA: $\boxed{x=0}$ 1

EKSTREMI:

$$f'(x) = \frac{x^2-10x+16-x(2x-10)}{(x^2-10x+16)^2} = \frac{-x^2+16}{(x^2-10x+16)^2} = 0$$ 3

$-x^2+16=0 \Rightarrow x^2=16 \Rightarrow \boxed{x_{1,2} = \pm 4 \quad S.T.}$ 2

$-\infty \quad -4 \quad 2 \quad 4 \quad 8 \quad +\infty$

f'(x)	−	+	+	−	−
f(x)	↘	↗	↗	↘	↘
	min		max		

3 Intervali rasta:
$\langle -4, 2\rangle, \langle 2, 4\rangle$
Intervali pada:
$\langle -\infty, -4\rangle, \langle 4, 8\rangle, \langle 8, +\infty\rangle$

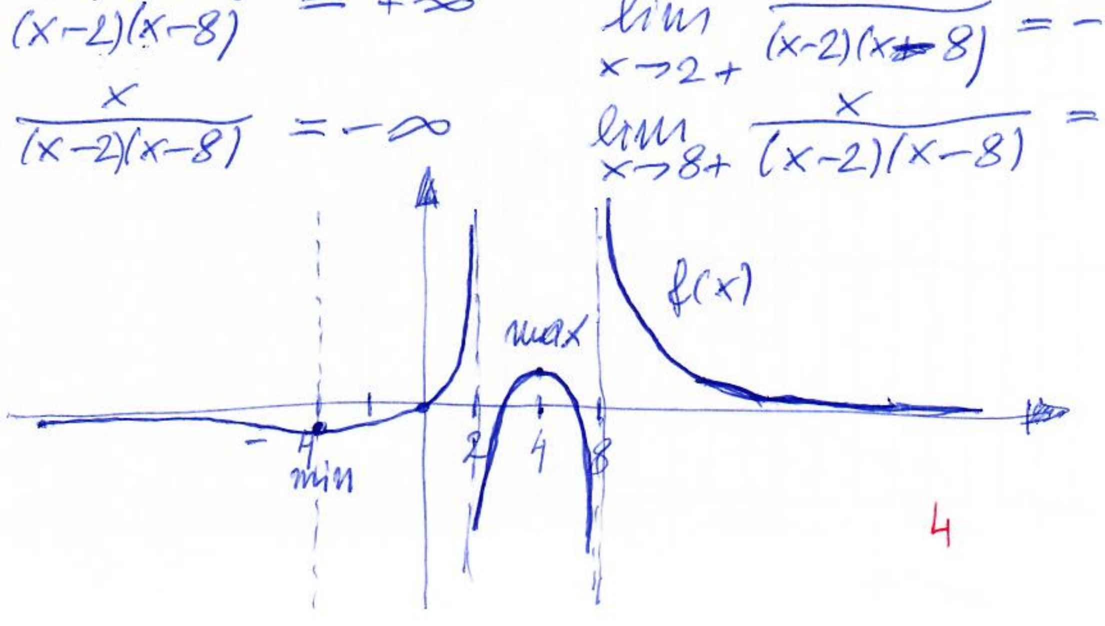
ASIMPTOTE:

$\lim_{x \to \pm\infty} \frac{x}{x^2-10x+16} = 0 \Rightarrow$ y=0 je obostrana H.A. 2

$\lim_{x \to 2-} \frac{x}{(x-2)(x-8)} = +\infty \qquad \lim_{x \to 2+} \frac{x}{(x-2)(x-8)} = -\infty$

$\lim_{x \to 8-} \frac{x}{(x-2)(x-8)} = -\infty \qquad \lim_{x \to 8+} \frac{x}{(x-2)(x-8)} = +\infty$ 2

GRAF:



4

4. (20b)

$$I=\int\frac{e^{3x}+e^{x}}{e^{4x}-e^{3x}}\,dx = \left|\begin{aligned} t&=e^{x}\\ dt&=\underbrace{e^{x}}_{t}dx\\ dx&=\frac{dt}{t}\end{aligned}\right|_{3} = \int\frac{t^3+t}{t^4-t^3}\cdot\frac{dt}{t}$$

$$=\int\frac{t^2+1}{t^3(t-1)}\,dt = \underline{I}$$

1

$$\frac{t^2+1}{t^3(t-1)}=\frac{A}{t}+\frac{B}{t^2}+\frac{C}{t^3}+\frac{D}{t-1}\quad\Big/\cdot t^3(t-1)$$

5

$$t^2+1=A(t^3-t^2)+B(t^2-t)+C(t-1)+Dt^3$$

$$t^2+1=(A+D)t^3+(B-A)t^2+(C-B)t-C$$

$A+D=0$ $\quad B-A=1$ $\quad C-B=0$ $\quad -C=1$

$\boxed{D=2}$ $\quad \boxed{A=-2}$ $\quad \boxed{B=-1}$ $\quad \boxed{C=-1}$

5

$$I=-2\int\frac{dt}{t}-\int\frac{dt}{t^2}-\int\frac{dt}{t^3}+2\int\frac{dt}{t-1}$$

1

$$=\boxed{-2\ln|t|_{1}+\frac{1}{t}_{1}+\frac{1}{2t^2}_{1}+2\ln|t-1|_{1}+C_{1}}$$

1 vraćanje

5. a) (10b)

$$\int_0^3\frac{dx}{(x-1)^2}=\lim_{b\to1-}\int_0^b\frac{dx}{(x-1)^2}+\lim_{a\to1+}\int_a^3\frac{dx}{(x-1)^2}$$

4

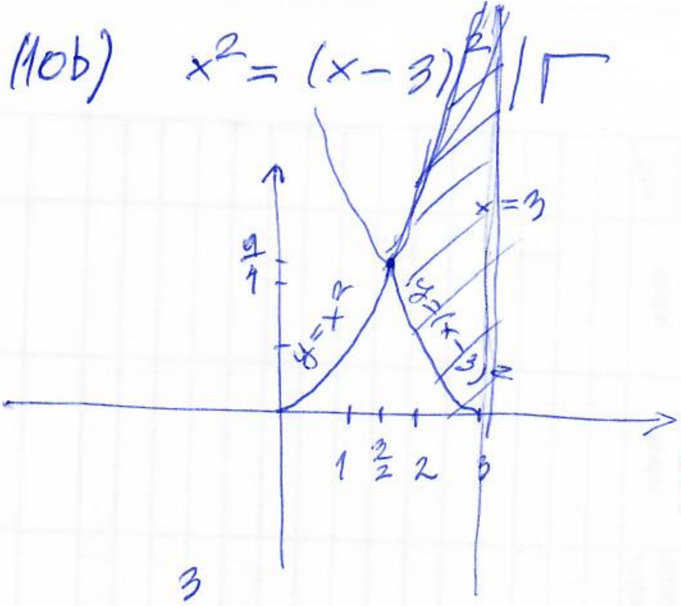
$$=\lim_{b\to1-}-\frac{1}{x-1}\Big|_0^b{}_{1}+\lim_{a\to1+}-\frac{1}{x-1}\Big|_a^3{}_{1}$$

$$=\lim_{b\to1-}\Big(-\underset{\substack{\downarrow\\+\infty}}{\frac{1}{b-1}}-1\Big)_{1}+\lim_{a\to1+}\Big(+\underset{\substack{\downarrow\\+\infty}}{\frac{1}{a-1}}-\frac{1}{2}\Big)_{1}$$

$$=\boxed{+\infty}_{2}$$

5b) (10b) $x^2 = (x-3)^2 \;\Big|\sqrt{}$ $\Rightarrow$ $x = -x + 3 \Rightarrow \underbrace{x = \frac{3}{2}}$

2



2

$$P = \int_{\frac{3}{2}}^{3} \left(x^2 - (x-3)^2\right) dx = \int_{\frac{3}{2}}^{3} \left(\cancel{x^2} - \cancel{x^2} + 6x - 9\right) dx$$

2

$$= \int_{\frac{3}{2}}^{3} (6x - 9)\, dx = \left(\cancel{6} \cdot \frac{x^2}{\cancel{2}} - 9x\right)\Bigg|_{\frac{3}{2}}^{3} = \cancel{27} - \cancel{27} - \frac{27}{4} + \frac{27}{2}$$

1 1 1

$$= \boxed{\frac{27}{4}}$$

1