MATEMATIKA 1 1.2.2021.

1. a) (8 bodova) Dane su točke $A(1,1,1)$, $B(1,3,5)$, $C(2,2,2)$ i $D(-1,-3,5)$. Izračunajte volumen paralelepipeda razapetog vektorima $\overrightarrow{AB}$, $\overrightarrow{AC}$ i $\overrightarrow{AD}$.

   b) (17 bodova) Izračunajte udaljenost između pravaca $p_1 \ldots \frac{x+2}{3} = \frac{y-1}{-2} = \frac{z+1}{4}$ i $p_2 \ldots \frac{x-6}{3} = \frac{y+4}{-2} = \frac{z-5}{4}$.

2. (15 bodova) Riješite sustav:

$$\begin{aligned} 2x - y - 4z &= 1 \\ x + y - 5z &= -1 \\ 4x - 6y &= 6 \end{aligned}$$

3. (20 bodova) Odredite prirodnu domenu, intervale rasta i pada, ekstreme, asimptote, te skicirajte graf funkcije $f(x) = \frac{x^2}{x^2-1}$.

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4. (15 bodova) Odredite

$$\int \frac{\ln(\ln(\ln x))\,dx}{x \ln x}.$$

5. (a) (10 bodova) Izračunajte površinu lika koji zatvaraju krivulje

$$y = x^2 + 1 \quad \text{i} \quad y = -x^2 - 2x + 5.$$

   Skicirajte lik.

   (b) (15 bodova) Izračunajte volumen tijela koje nastaje rotacijom lika omeđenog krivuljom $y = \cos(2x)$ nad segmentom $[0, \pi]$ oko osi x. Skicirajte tijelo.

$$P = \begin{vmatrix} 0 & 2 & 4 \\ 1 & 1 & 1 \\ -2 & -4 & 4 \end{vmatrix} = -2(4+2) + 4(-4+2)$$

$$= -12 - 8 = -20$$

$$P = |-20| = 20$$

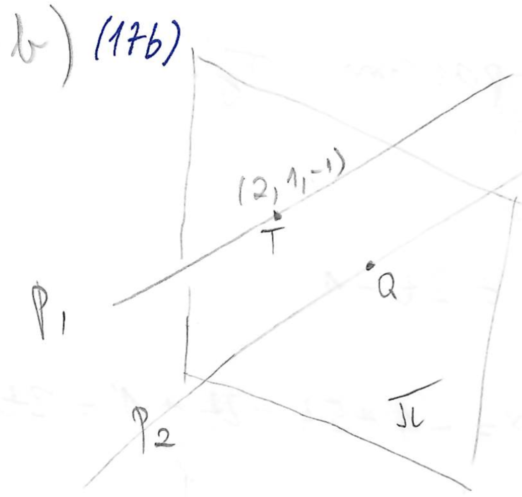
$\vec{AB} = 2\vec{j} + 4\vec{k}$

$\vec{AC} = \vec{i} + \vec{j} + \vec{k}$

$\vec{AD} = -2\vec{i} - 4\vec{j} + 4\vec{k}$

$$\boxed{P = \left| \left(\vec{AB} \times \vec{AC} \right) \cdot \vec{AD} \right|}$$

b) (17b)



$\vec{c} = \vec{n} = (3, -2, 4)$

$T(-2, 1, -1) \in p_1$

π SADRŽI T, $\pi \perp p_1$

$3(x+2) - 2(y-1) + 4(z+1) = 0$

$\pi \ldots \ 3x - 2y + 4z + 12 = 0$

$$p_2 \equiv \begin{cases} x = 3t + 6 \\ y = -2t - 4 \\ z = 4t + 5 \end{cases}$$

$3(3t+6) - 2(-2t-4) + 4(4t+5) + 12 = 0$

$9t + 18 + 4t + 8 + 16t + 20 + 12 = 0$

$29t = -58$

$t = -2$

$\rightarrow Q(0, 0, -3) = \pi \cap p_2$

$$d(p_1, p_2) = d(T, Q) = \sqrt{(-2)^2 + 1^2 + (-1+3)^2} =$$

$$= \sqrt{9} = 3$$

2) (15b) $\left[\begin{array}{ccc|c} 2 & -1 & -4 & 1 \\ 1 & 1 & -5 & -1 \\ 4 & -6 & 0 & 6 \end{array}\right] \overset{\mathrm{II} \leftrightarrow \mathrm{I}}{=} \left[\begin{array}{ccc|c} 1 & 1 & -5 & -1 \\ 2 & -1 & -4 & 1 \\ 4 & -6 & 0 & 6 \end{array}\right] \begin{array}{l} \mathrm{II}-2\mathrm{I} \\ \mathrm{III}-4\mathrm{I} \end{array} \sim$

$\sim \left[\begin{array}{ccc|c} 1 & 1 & -5 & -1 \\ 0 & -3 & 6 & 3 \\ 0 & -10 & 20 & 10 \end{array}\right] \begin{array}{l} \\ /:3 \\ /:10 \end{array} \sim \left[\begin{array}{ccc|c} 1 & 1 & -5 & -1 \\ 0 & -1 & 2 & 1 \\ 0 & -1 & 2 & 1 \end{array}\right] \mathrm{III}-\mathrm{II} \sim$

$\sim \left[\begin{array}{ccc|c} 1 & 1 & -5 & -1 \\ 0 & -1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$ 1-param. rj.

$z = t$

$-y + 2z = 1 \Rightarrow y = 2t - 1$

$x + y - 5z = -1 \Rightarrow x = -1 + 5t - 2t + 1 = 3t$

Rj: $\begin{bmatrix} 3t \\ 2t-1 \\ t \end{bmatrix}$ //

3.) (20b) $f(x) = \dfrac{x^2}{x^2-1}$ $D_f = \mathbb{R} \setminus \{-1, 1\}$

NULTOČKE $x = 0$

DERIVACIJA

$$f'(x) = \frac{2x(x^2-1) - x^2(2x)}{(x^2-1)^2} = \frac{2x^3 - 2x - 2x^3}{(x^2-1)^2} = \frac{-2x}{(x^2-1)^2}$$

STAC. TOČKA $x = 0$

	$-\infty$	-1	0	1	$+\infty$
$f'(x)$	$+$	$+$	$-$	$-$	
$f(x)$	$\nearrow$	$\nearrow$	$\searrow$	$\searrow$	

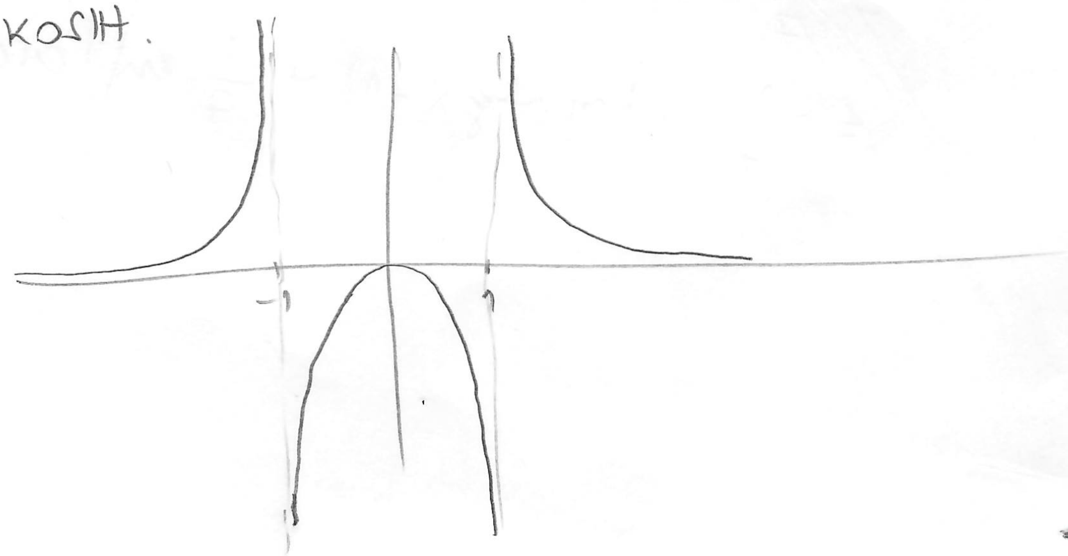
MAX

$\Rightarrow (0,0)$ je MAX

V.A.
$$\lim_{x \to 1^-} \frac{x^2}{x^2-1} = \frac{1}{0^-} = -\infty$$
$$\lim_{x \to 1^+} \frac{x^2}{x^2-1} = \frac{1}{0^+} = +\infty$$
$$\lim_{x \to -1^-} \frac{x^2}{x^2-1} = \frac{1}{0^+} = +\infty$$
$$\lim_{x \to -1^+} \frac{x^2}{x^2-1} = \frac{1}{0^-} = -\infty$$

H.A. $\displaystyle\lim_{x \to \pm\infty} \frac{x^2}{x^2-1} = 1$ $y = 1$ je H.A.

NEMA KOSIH.



4) (15b) $\int \frac{\ln(\ln(\ln x))}{x \ln x} = \left\{ \begin{array}{l} t = \ln x \\ dt = \frac{1}{x} dx \end{array} \right.$

$= \int \frac{\ln(\ln t)}{t} dt = \left\{ \begin{array}{l} w = \ln t \\ dw = \frac{1}{t} dt \end{array} \right.$

$= \int \ln w \, dw = \left\{ \begin{array}{ll} u = \ln w & dv = dw \\ du = \frac{1}{w} dw & v = w \end{array} \right\}$

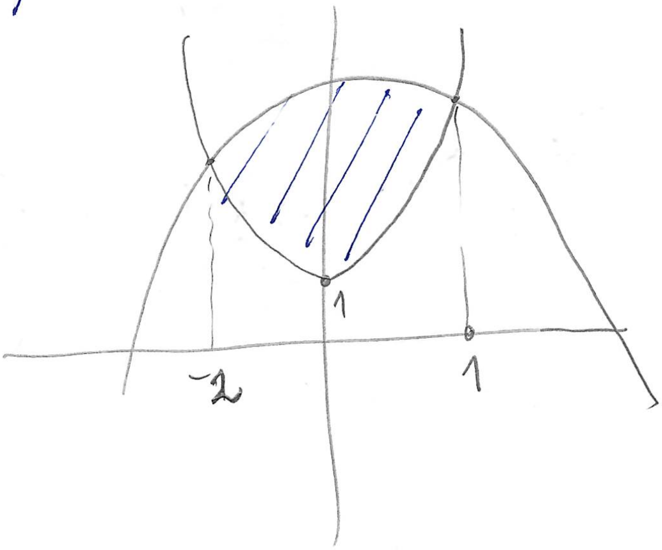
$= w \ln w - \int w \frac{1}{w} dw$

$= w \ln w - w + C$

$= \ln t \ln(\ln t) - \ln t + C$

$= \ln(\ln x) \cdot \ln(\ln(\ln x)) - \ln(\ln x) + C$

5a) (10b)



GRANICE

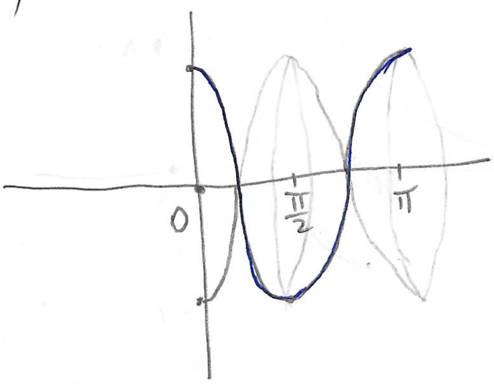
$$x^2+1=-x^2-2x+5$$
$$2x^2+2x-4=0$$
$$x^2+x-2=0$$
$$x_{1,2}=\frac{-1\pm\sqrt{1+8}}{2}=-2,1$$

$$\int_{-2}^{1}\left(-x^2-2x+5-x^2-1\right)dx=\int_{-1}^{1}(-2x^2-2x+4)\,dx$$

$$=\left(\frac{-2}{3}x^3-x^2+4x\right)\Big|_{-2}^{1}=\left(\frac{-2}{3}-1+4\right)-\left(\frac{16}{3}-4-8\right)$$

$$=-6+3+12=9$$

b) (15b)



$$V_x=\pi\int_0^{\pi}\cos^2(2x)\,dx$$

$$=\pi\int_0^{\pi}\frac{1+\cos 4x}{2}\,dx=$$

$$=\pi\int_0^{\pi}\frac{dx}{2}+\frac{\pi}{2}\int_0^{\pi}\cos 4x\,dx=\begin{cases}t=4x\\ dt=4dx\\ x=0\Rightarrow t=0\\ x=\pi\Rightarrow t=4\pi\end{cases}$$

$$=\pi\left(\frac{1}{2}x\right)\Big|_0^{\pi}+\frac{\pi}{8}\int_0^{4\pi}\cos t\,dt=$$

$$=\frac{1}{2}\pi^2+\sin t\Big|_0^{4\pi}\cdot\frac{\pi}{8}=\frac{1}{2}\pi^2$$