MATEMATIKA 1 8.2.2021.

1. a) (8 bodova) Izračunajte površinu trokuta čiji su vrhovi $A(-4,-3,0), B(1,-2,1)$ i $C(4,-3,3)$.

   b) (17 bodova) Ispitajte u kojem su odnosu pravac $p \ldots \frac{x-7}{-5}=\frac{y-4}{-1}=\frac{z-7}{-4}$ i ravnina $\pi \ldots 3x-2y+4z-12=0$ te odredite ortogonalnu projekciju pravca p na ravninu π. Skicirajte!

2. a) (10 bodova) Odredite opći član reda

$$\frac{1}{3\cdot 1}+\frac{1\cdot 2}{9\cdot 4}+\frac{1\cdot 2\cdot 3}{27\cdot 9}+\frac{1\cdot 2\cdot 3\cdot 4}{81\cdot 16}+\cdots$$

   i ispitajte njegovu konvergenciju.

   b) (5 bodova) Izračunajte $\lim\limits_{x\to 0}\frac{\sin(2x)}{\sin(3x)}$.

3. (20 bodova) Odredite prirodnu domenu, intervale rasta i pada, ekstreme, asimptote, te skicirajte graf funkcije

$$f(x)=\frac{x}{\sqrt{1-x^2}}$$

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4. (15 bodova) Odredite

$$\int \frac{\cos x\, dx}{\sqrt[3]{\sin^2 x}\left(\sqrt[3]{\sin x}+\sin x\right)}.$$

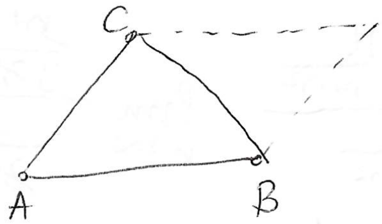
5. (a) (13 bodova) Izračunajte površinu lika koji zatvaraju krivulje

$$y=-x^2-5x+6 \quad \text{i} \quad y=-x^2+x$$

   s osi x. Skicirajte lik.

   (b) (12 bodova) Izračunajte volumen tijela koje nastaje rotacijom krivulje $y=\frac{1}{\sqrt{x}\cdot \ln x}$ oko osi x na segmentu $[e,e^3]$.

1a)



$P_{▱} = |\vec{AB} \times \vec{AC}|$

$P_{\Delta} = \frac{1}{2} |\vec{AB} \times \vec{AC}|$

$\vec{AB} = 5\vec{i} + \vec{j} + \vec{k}$

$\vec{AC} = 8\vec{i} + 3\vec{k}$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 5 & 1 & 1 \\ 8 & 0 & 3 \end{vmatrix} =$$

$$= i(3-0) - \vec{j}(15-8) + \vec{k}(0-8)$$

$$= 3\vec{i} - 7\vec{j} - 8\vec{k}$$

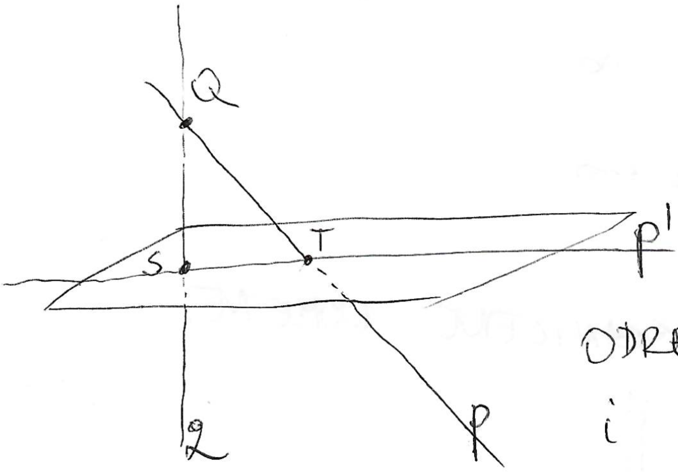
$$|\vec{AB} \times \vec{AC}| = \sqrt{3^2 + (-7)^2 + (-8)^2} = \sqrt{122} \Rightarrow P_{\Delta} = \frac{\sqrt{122}}{2}$$

b) $$p \equiv \begin{cases} x = -5t + 7 \\ y = -t + 4 \\ z = -4t + 7 \end{cases}$$

ODNOS PR. I RAV:

$$3x - 2y + 4z - 12 = 0$$
$$3(-5t+7) - 2(-t+4) + 4(-4t+7) - 12 = 0$$
$$-15t + 21 + 2t - 8 - 16t + 28 - 12 = 0$$
$$-29t + 29 = 0$$

$t = 1 \Rightarrow$ PRAVAC I RAVNINA SE SIJEKU U $T(2,3,3)$



ODABERIMO JOŠ NEKU TOČKU NA p, npr za $t = 0 \Rightarrow Q(7,4,7)$

ODREDIMO q TAKO DA $q \perp \pi$ i $Q \in q$.

$$q \equiv \frac{x-7}{3} = \frac{y-4}{-2} = \frac{z-7}{4}$$

$$q \equiv \begin{cases} x = 3t + 7 \\ y = -2t + 4 \\ z = 4t + 7 \end{cases}$$

$q \cap \pi = ?$

$$3x - 2y + 4z - 12 = 0$$
$$3(3t+7) - 2(-2t+4) + 4(4t+7) - 12 = 0$$
$$9t + 21 + 4t - 8 + 16t + 28 - 12 = 0$$
$$29t = -29$$

$t = -1 \Rightarrow q \cap \pi = S(4,6,3)$

p'... pravac kroz S i T

$$\boxed{p' \equiv \frac{x-2}{2} = \frac{y-3}{3} = \frac{z-3}{0}}$$

2a) $a_n = \frac{n!}{3^n \cdot n^2}$

D'ALAM. KRITERIJ $\lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty} \frac{\frac{(n+1)!}{3^{n+1}\cdot(n+1)^2}}{\frac{n!}{3^n \cdot n^2}} = \lim\limits_{n\to\infty} \frac{\frac{\cancel{n!}\cdot\cancel{(n+1)}}{3\cdot\cancel{3^n}\cdot(n+1)^{\cancel{2}}}}{\frac{\cancel{n!}}{\cancel{3^n}\cdot n^2}} =$

$= \lim\limits_{n\to\infty} \frac{n^2}{3(n+1)} = \infty$ ⇒ RED DIVERGIRA

b) $\lim\limits_{x\to 0} \frac{\sin(2x)}{\sin(3x)} = \lim\limits_{x\to 0} \frac{\overset{\to 1}{\frac{\sin 2x}{2x}}}{\underset{\to 1}{\frac{\sin 3x}{3x}}} \cdot \frac{2x}{3x} = \frac{2}{3}$

ILI L'HOSP. PRAVILO

$\lim\limits_{x\to 0} \frac{\sin(2x)}{\sin(3x)} \overset{L'H}{=} \lim\limits_{x\to 0} \frac{\overset{\to 1}{\cos(2x)}}{\underset{\to 1}{\cos(3x)}} \cdot \frac{2}{3} = \frac{2}{3}$

3. $f(x) = \frac{x}{\sqrt{1-x^2}}$ $D_f = \langle -1, 1 \rangle$

NULTOČKA $x=0$

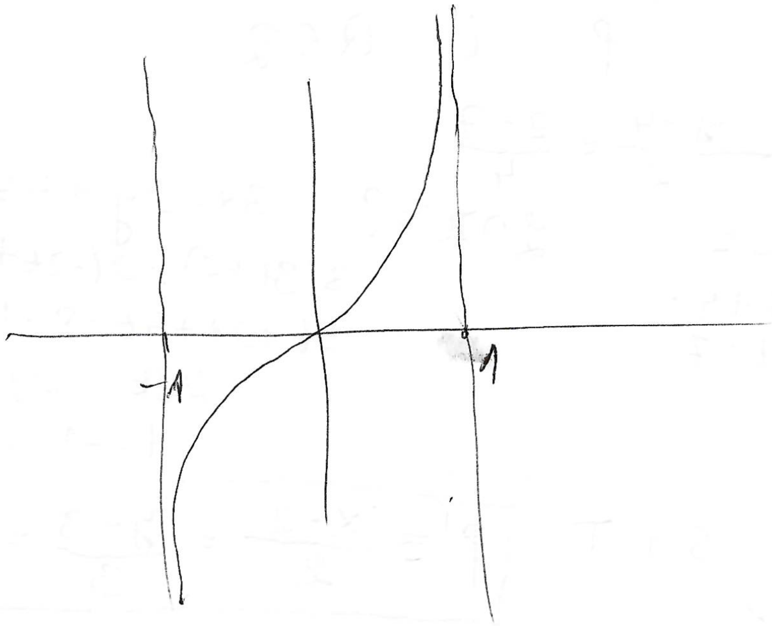
$f'(x) = \frac{\sqrt{1-x^2} - \frac{x}{2\sqrt{1-x^2}}\cdot(-2x)}{1-x^2} = \frac{\frac{2-2x^2+2x^2}{2\sqrt{1-x^2}}}{1-x^2} = \frac{\cancel{2}^{\,1}}{{}_1\cancel{2}(1-x^2)\sqrt{1-x^2}}$

⇒ NEMA EKSTREMA, RASTE NA CIJELOJ DOMENI

V.A.: $\lim\limits_{x\to -1^+} \frac{x}{\sqrt{1-x^2}} = \frac{-1}{0^+} = -\infty$

$\lim\limits_{x\to 1^-} \frac{x}{\sqrt{1-x^2}} = \frac{1}{0^+} = +\infty$

H.A. i K.A. NEMA ZBOG OGRANIČENE DOMENE.



4.) $\int \frac{\cos x\, dx}{\sqrt[3]{\sin^2 x}\left(\sqrt[3]{\sin x} + \sin x\right)} = \begin{cases} t^3 = \sin x \\ 3t^2 dt = \cos x\, dx \end{cases}$

$= \int \frac{3\cancel{t^2} dt}{\cancel{t^2}(t+t^3)} = \int \frac{3\, dt}{t(1+t^2)} = (*)$

$\frac{3}{t(1+t^2)} = \frac{A}{t} + \frac{Bt+C}{1+t^2} \quad / \; t(1+t^2)$

$3 = A(1+t^2) + Bt^2 + Ct$

$3 = t^2(A+B) + t \cdot C + A$

$\Rightarrow A = 3 \Rightarrow B = -3 \Rightarrow C = 0$

$(*) = \int \frac{3\, dt}{t} - \underbrace{\int \frac{3t\, dt}{1+t^2}}_{\square} = (*)$

$\square = \int \frac{3t\, dt}{1+t^2} = \begin{cases} w = 1+t^2 \\ dw = 2t\, dt \end{cases}$

$= \frac{3}{2} \int \frac{dw}{w} = \frac{3}{2} \ln|w| = \frac{3}{2} \ln|1+t^2|$

$(*) = 3 \ln|t| - \frac{3}{2} \ln|1+t^2| = 3 \ln\left|\sqrt[3]{\sin x}\right| - \frac{3}{2} \ln\left|\sqrt[3]{\sin^2 x} + 1\right| + C$

5a) NULTOČKE :

$-x^2 - 5x + 6 = 0$

$x_{1,2} = \frac{5 \pm \sqrt{25+24}}{-2} = 1, -6$

$-x^2 + x = 0$

$x(-x+1) = 0$

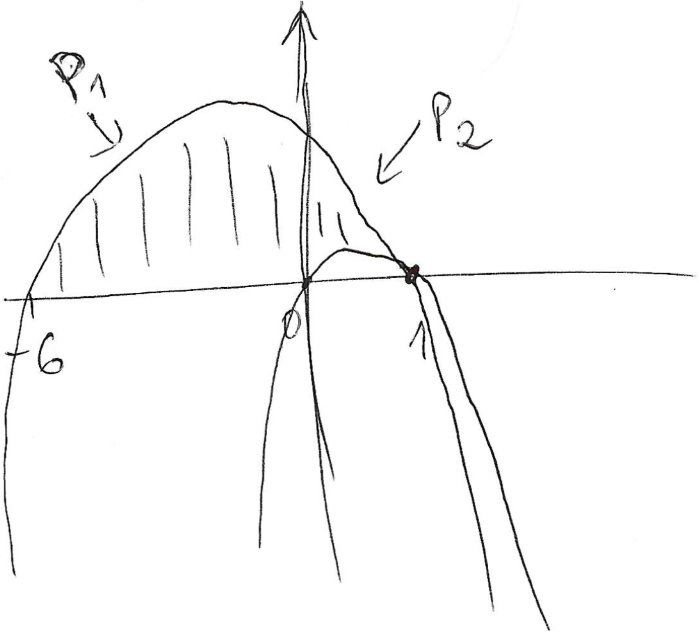
$x_{1,2} = 0, 1$

SJECIŠTA

$-x^2 - 5x + 6 = x^2 + x$

$6x = 6$

$x = 1$



$$P_1 = \int_{-6}^{0} (-x^2 - 5x + 6)\,dx = \left(-\frac{1}{3}x^3 - \frac{5}{2}x^2 + 6x\right)\Big|_{-6}^{0} = 0 - \left(-\frac{(-216)}{3} - \frac{180}{2} - 36\right)$$

$$= 0 - (\ 72 - 90 - 36) = 54$$ //

$$P_2 = \int_0^1 (-x^2 - 5x + 6 - (-x^2 + x))\,dx = \int_0^1 (-6x + 6)\,dx = (-3x^2 + 6x)\Big|_0^1 = 3$$ //

$$P = 54 + 3 = 57$$ //

b) $$V_x = \pi \int_a^b (f(x))^2\,dx = \pi \int_e^{e^3} \frac{dx}{x \ln^2 x} = \begin{cases} t = \ln x \\ dt = \frac{dx}{x} \\ x = e \Rightarrow t = 1 \\ x = e^3 \Rightarrow t = 3 \end{cases}$$

$$= \pi \int_1^3 \frac{dt}{t^2} = -\pi \left(\frac{1}{t}\right)\Big|_1^3 = \frac{-\pi}{3} + \pi = \frac{2}{3}\pi$$ //