MATEMATIKA 1 15.2.2021.

1. a) (10 bodova) Odredite konstantu λ tako da vektori $\vec{a} = -\vec{i} - \vec{j} + \vec{k}$, $\vec{b} = \lambda\vec{i} + 3\vec{j} - 2\vec{k}$ i $\vec{c} = -8\vec{i} - 11\vec{j} + 8\vec{k}$ budu komplanarni. Prikažite vektor $\vec{c}$ kao linarnu kombinaciju vektora $\vec{a}$ i $\vec{b}$.

   b) (15 bodova) Odredite kanonsku i parametarsku jednadžbu pravca zadanog kao presjek ravnina $\begin{cases} \pi_1 & \equiv & -y + 5z - 8 = 0 \\ \pi_2 & \equiv & 8x - 3y - z - 16 = 0 \end{cases}$

2. (15 bodova) Gauss - Jordanovom metodom riješite sustav: $\begin{array}{rcl} 3x - 2y & = & -3 \\ x + y - 5z & = & -1 \\ 4x - y - 5z & = & -4 \end{array}$

3. (20 bodova) Odredite prirodnu domenu, intervale rasta i pada, ekstreme, asimptote, te skicirajte graf funkcije
$$f(x) = \operatorname{arctg} \sqrt{x}.$$

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4. (a) (10 bodova) Odredite
$$\int \frac{dx}{x^2 - 3x + 2}.$$

   (b) (10 bodova) Izračunajte
$$\int_0^{+\infty} \frac{dx}{x\left(\ln^2 x + 1\right)}.$$

5. (a) (10 bodova) Izračunajte površinu lika omeđenog parabolom $y = x^2 - 4x - 5$ i pravcem $y = -x - 5$. Skicirajte.

   (b) (10 bodova) Primjenom integralnog računa izračunajte volumen kugle koja nastaje rotacijom kružnice $x^2 + y^2 = R^2$ oko osi x. Skicirajte tijelo.

1. a) 10b $\vec{c} = \alpha \vec{a} + \beta \vec{b}$

$$-8\vec{i} - 11\vec{j} + 8\vec{k} = \alpha(-\vec{i} - \vec{j} + \vec{k}) + \beta(\lambda\vec{i} + 3\vec{j} - 2\vec{k})$$

$$-8\vec{i} - 11\vec{j} + 8\vec{k} = \vec{i}(-\alpha + \lambda\beta) + \vec{j}(-\alpha + 3\beta) + \vec{k}(\alpha - 2\beta)$$

$$\begin{aligned} -\alpha + \lambda\beta &= -8 \\ -\alpha + 3\beta &= -11 \\ \alpha - 2\beta &= 8 \end{aligned} \Big/ +$$

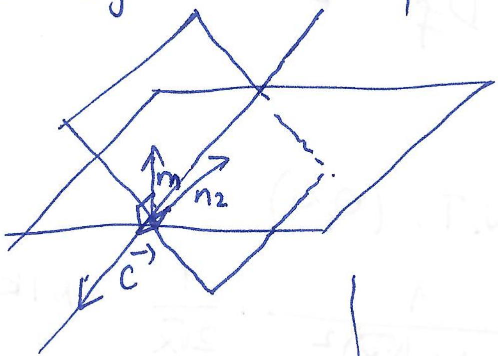
$$\beta = -3 \Rightarrow \alpha = 8 + 2\cdot(-3) = 2$$

$$\Rightarrow -2 + \lambda\cdot(-3) = -8$$

$$\lambda = 2 /\!/$$

$$\vec{b} = 2\vec{i} + 3\vec{j} - 2\vec{k} \quad , \quad \vec{c} = 2\vec{a} - 3\vec{b}$$

b) 15b



$\vec{c} \perp n_1, \ \vec{c} \perp n_2$

$$\vec{c} = \vec{n_1} \times \vec{n_2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & -1 & 5 \\ 8 & -3 & -1 \end{vmatrix} =$$

$\vec{n_1} = (0, -1, 5)$

$\vec{n_2} = (8, -3, -1)$

$$= \vec{i}(1 + 15) - \vec{j}(0 - 40) + \vec{k}(0 + 8)$$

$$= 16\vec{i} + 40\vec{j} + 8\vec{k}$$

MOŽEMO UZETI KOLINEARNI VEKTOR $\frac{1}{8}\vec{c} = 2\vec{i} + 5\vec{j} + 2\vec{k}$ I NJEGA UZETI ZA $\vec{c}$. ODABERIMO NEKU TOČKU NA $\pi_1 \cap \pi_2$ npr. za $z = 0$. Imamo

$$\begin{aligned} -y - 8 &= 0 \\ 8x - 3y &= 16 \\ \hline y &= -8 \\ x &= -1 \end{aligned}$$

$T(-1, -8, 0)$

TRAŽENA JEDNADŽBA:

$$p \equiv \frac{x+1}{2} = \frac{y+8}{5} = \frac{z}{1} \quad i \quad p \equiv \begin{cases} x = 2t - 1 \\ y = 5t - 8 \\ z = t \end{cases}$$

2. 15b

$$\left[\begin{array}{ccc|c} 3 & -2 & 0 & -3 \\ 1 & 1 & -5 & -1 \\ 4 & -1 & -5 & -4 \end{array}\right] \text{I} \leftrightarrow \text{II} \sim \left[\begin{array}{ccc|c} 1 & 1 & -5 & -1 \\ 3 & -2 & 0 & -3 \\ 4 & -1 & -5 & -4 \end{array}\right] \begin{array}{l} \\ \text{II}-3\text{I} \\ \text{III}-4\text{I} \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & -5 & -1 \\ 0 & -5 & 15 & 0 \\ 0 & -5 & 15 & 0 \end{array}\right] \begin{array}{l} \\ /:-5 \\ \text{III}-\text{II} \end{array} \sim \left[\begin{array}{ccc|c} 1 & 1 & -5 & -1 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$z = t \Rightarrow y = 3t \Rightarrow x + 3t - 5t = -1$

$x = 2t - 1$

Rj: $\begin{bmatrix} 2t-1 \\ 3t \\ t \end{bmatrix}$

3. 20b $D(\text{arctg}) = \mathbb{R}$

ZBOG $\sqrt{x}$, $\Rightarrow x \geq 0 \Rightarrow D_f = [0, +\infty\rangle$

NULTOČKA $\text{arctg}\sqrt{x} = 0$

$\sqrt{x} = 0$

$x = 0$ N.T. $(0,0)$

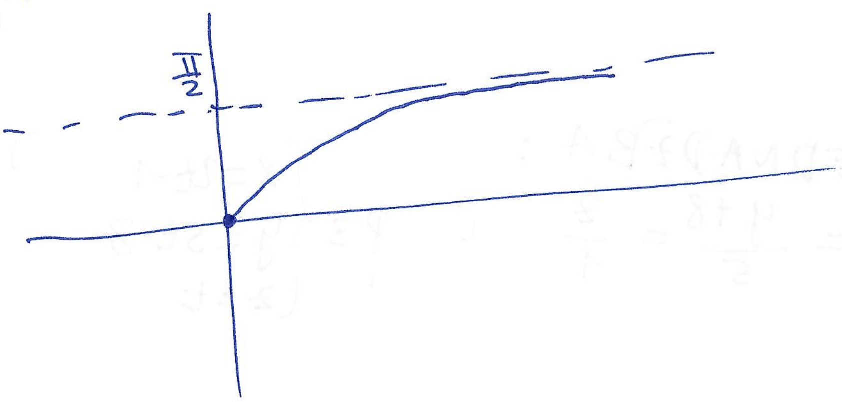
DERIVACIJA: $f'(x) = \frac{1}{1+(\sqrt{x})^2} \cdot \frac{1}{2\sqrt{x}}$ NIKAD NIJE 0

NEMA EKSTREMA, RASTE NA CIJELOJ DOMENI.

V.A. $\lim\limits_{x \to 0^+} \text{arctg}\sqrt{x} = \text{arctg}\,0 = 0$ (NIJE ASIMPTOTA)

H.A. $\lim\limits_{x \to +\infty} \text{arctg}\sqrt{x} = \text{arctg}(+\infty) = \frac{\pi}{2}$ (H.A.)

NEMA K.A.



4a) 10b $\displaystyle\int \frac{dx}{x^2-3x+2} = \int \frac{dx}{x^2-2x-x+2} = \int \frac{dx}{x(x-2)-(x-2)} = \int \frac{dx}{(x-1)(x-2)} = (*)$

$$\frac{1}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2} \quad / \cdot (x-1)(x-2)$$

$$1 = Ax - 2A + Bx - B$$

$$A + B = 0 \Rightarrow A = -B$$

$$-2A - B = 1$$

$$-2A + A = 1 \Rightarrow A = -1, \quad B = 1$$

$$(*) = -\int \frac{dx}{x-1} + \int \frac{dx}{x-2} = \ln|x-1| + \ln|x-2| + C$$

b) 10b $\displaystyle\int_0^{+\infty} \frac{dx}{x(\ln^2 x+1)} = \left\{ \begin{array}{ll} t = \ln x & x=0 \quad t=-\infty \\ dt = \frac{dx}{x} & x=+\infty \quad t=+\infty \end{array} \right.$

$$= \int_{-\infty}^{+\infty} \frac{dt}{t^2+1} = \lim_{a\to-\infty} \int_a^0 \frac{dt}{t^2+1} + \lim_{b\to+\infty} \int_0^b \frac{dt}{t^2+1} =$$

$$= \lim_{a\to+\infty} (\underbrace{\operatorname{arctg} 0}_{0} - \underbrace{\operatorname{arctg} a}_{-\frac{\pi}{2}}) + \lim_{b\to+\infty} (\underbrace{\operatorname{arctg} b}_{\frac{\pi}{2}} - \underbrace{\operatorname{arctg} 0}_{0}) = \pi$$

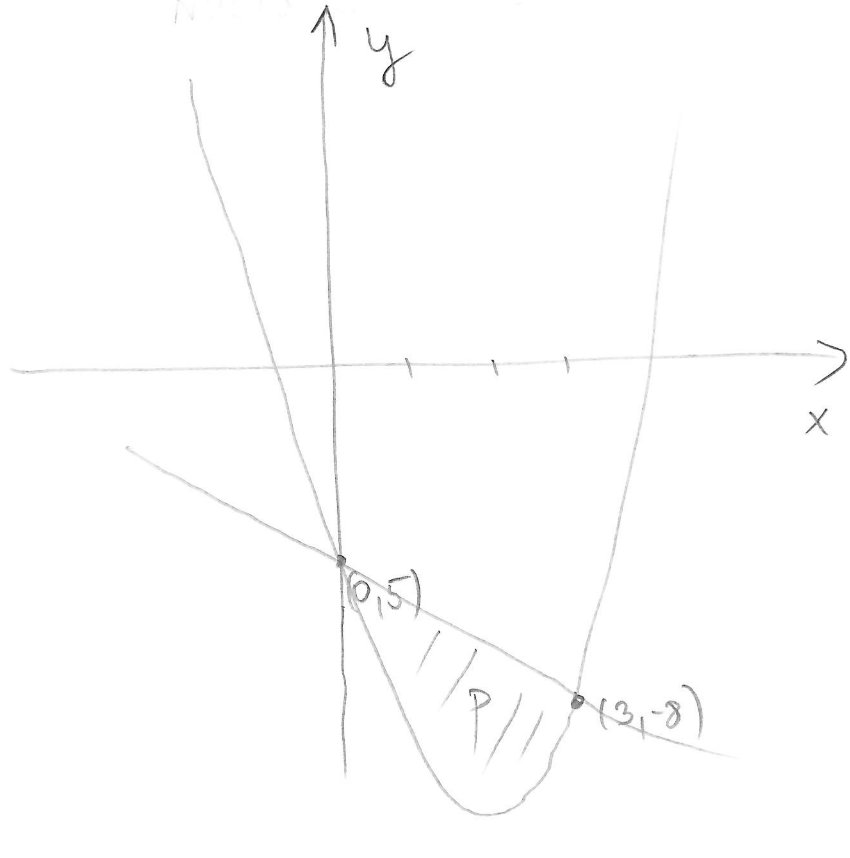
5a) 10b

SJECIŠTA:

$$x^2 - 4x - 5 = -x - 5$$

$$x^2 - 3x = 0$$

$$x_{1,2} = 0, 3$$



$$P = \int_0^3 (-x-5-(x^2-4x-5))\,dx$$

$$= \int_0^3 (-x^2+3x)\,dx$$

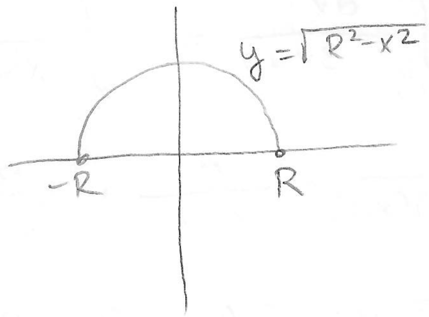
$$= \left(-\frac{1}{3}x^3 + \frac{3}{2}x^2\right)\Big|_0^3$$

$$= \left(-9 + \frac{27}{2}\right) - 0 = \frac{9}{2}$$

5 b) 10 b

$$x^2 + y^2 = R^2$$

$$y = \sqrt{R^2 - x^2}$$



$$V_x = \pi \int_{-R}^{R} \left(\sqrt{R^2 - x^2}\right)^2 dx$$

$$= \pi \int_{-R}^{R} (R^2 - x^2)\, dx$$

$$= \pi \left(R^2 x - \frac{1}{3} x^3\right) \Big|_{-R}^{R}$$

$$= \pi \Big(\underbrace{R^3 - \frac{1}{3} R^3}_{\frac{2}{3}R^3} - \big(\underbrace{-R^3 - \frac{1}{3}(-R)^3}_{-\frac{2}{3}R^3} \big) \Big)$$

$$= \frac{4}{3} R^3 \pi$$