1.). $y'' - y' - 12y = 7e^{4x}$ α

homogena: $y'' - y' - 12y = 0$ karakteristična: $\lambda^2 - \lambda - 12 = 0$

$\Rightarrow \lambda_{1,2} = \frac{1 \pm \sqrt{49}}{2} = \frac{1 \pm 7}{2}$ $\lambda_1 = -3,\ \lambda_2 = 4$ $\Rightarrow y_H = C_1 e^{-3x} + C_2 e^{4x}$

partikularno: $f(x) = e^{4x}$ $\alpha = 4 = \lambda_2 \neq \lambda_1$ (jedno poklapanje)

$\Rightarrow y_p = x \cdot A e^{4x}$ $y_p' = Ae^{4x} + 4Axe^{4x} = A(4x+1) \cdot e^{4x}$

$y_p'' = 4Ae^{4x} + 4A \cdot (4x+1)e^{4x} = (16Ax + 8A)e^{4x}$

Uvrstimo u početnu:

$(\cancel{16Ax} + 8A)e^{4x} - \cancel{4Axe^{4x}} - Ae^{4x} - \cancel{12Axe^{4x}} = 7e^{4x}$

$7Ae^{4x} = 7e^{4x}$ $\rightarrow$ $A = 1$ $\Rightarrow y_p = xe^{4x}$

$y = y_H + y_p = C_1 e^{-3x} + C_2 e^{4x} + xe^{4x}$

2. a) (7 bodova) Odredite i skicirajte prirodnu domenu funkcije

$$f(x,y) = \ln\left(\frac{x}{y}\right) + \arccos\frac{x-2}{2}.$$

b) (5 bodova) Zadana je $g(x,y) = \frac{2xy}{\sqrt{4x^2+5y^2}}$. Izračunajte $\frac{\partial g}{\partial y}(2,-2)$.

a) $\frac{x}{y} > 0 \Rightarrow$ točke u I. i III. kvadrantu

$-1 \leq \frac{x-2}{2} \leq 1 \quad /\cdot 2 > 0 \Rightarrow -2 \leq x-2 \leq 2$

$0 \leq x \leq 4$

$D(f) = \{(x,y) \in \mathbb{R}^2 : 0 < x \leq 4,\ 0 < y\}$

osi nisu uključene

b) $$\frac{\partial g}{\partial y} = \frac{2x\sqrt{4x^2+5y^2} - 2xy\cdot\frac{1}{2\sqrt{4x^2+5y^2}}\cdot 10y}{4x^2+5y^2}$$

$$\sqrt{4x^2+5y^2}\Big|_{(2,-2)} = \sqrt{4\cdot 4+5\cdot 4} = \sqrt{36} = 6$$

$$\frac{\partial g}{\partial y}(2,-2) = \frac{4\cdot 6 - 2\cdot \cancel{2}\cdot(-2)\cdot\frac{10\cdot(-2)}{\cancel{2}\cdot 6}}{36}$$

$$= \frac{24 - \frac{80}{6}}{36} = \frac{32}{108} = \frac{8}{27}$$

3. (12 bodova) Nađite lokalne ekstreme funkcije $f(x,y) = xy(6-x-y)$.

$$\frac{\partial f}{\partial x} = y(6-x-y) + xy(-1) = y(6-2x-y) = 0$$

$$\frac{\partial f}{\partial y} = x(6-x-y) + xy(-1) = x(6-x-2y) = 0$$

$$\left.\begin{aligned} y(6-2x-y) &= 0 \\ x(6-x-2y) &= 0 \end{aligned}\right\}$$ jedan od faktora u svakom redu mora biti 0 pa imamo četiri mogućnosti:

1° $y=0,\ x=0$ $\quad T_1(0,0)$

2° $y=0,\ 6-x-2y=0 \Rightarrow x=6,\ T_2(6,0)$

3° $6-2x-y=0,\ x=0 \Rightarrow y=6 \quad T_3(0,6)$

4° $6-2x-y=0,\ 6-x-2y=0 \Rightarrow x=y=2 \quad T_4(2,2)$

$$\frac{\partial^2 f}{\partial x^2} = -2y$$

$$\frac{\partial^2 f}{\partial y \partial x} = 6-2x-y+y(-1) = 6-2x-2y$$

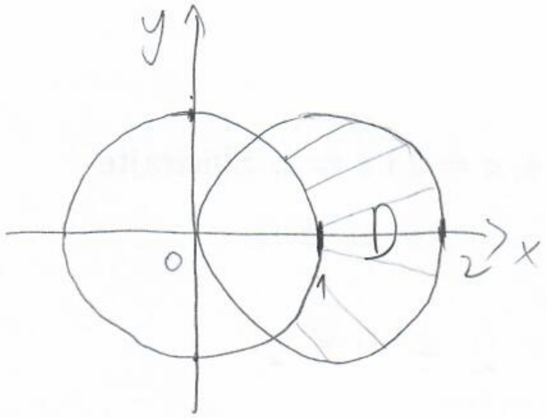
$$\frac{\partial^2 f}{\partial y^2} = -2x$$

	$T_1(0,0)$	$T_2(6,0)$	$T_3(0,6)$	$T_4(2,2)$
$A=-2y$	0	0	-12	-4
$B=6-2x-2y$	6	-6	-6	0
$C=-2x$	0	-12	0	-4
$AC-B^2$	-36	-36	-36	$16>0$
	sedlasta	sedlasta	sedlasta	$-4<0$ $\Downarrow$ max.

Funkcija f u $T_4(2,2)$ postiže

lokalni maksimum $f(2,2) = 4\cdot 2 = 8$

4. (12 bodova) Izračunajte površinu područja D, dijela ravnine koji se nalazi izvan kružnice $x^2+y^2=1$, a unutar kružnice $x^2+y^2=2x$. Skicirajte D.



$$x^2-2x+y^2=0$$

$$(x-1)^2+y^2=1$$

$$r=1 \qquad r^2=2r\cos\varphi$$

$$r=2\cos\varphi$$

$$2\cos\varphi=1 \Rightarrow \cos\varphi=\frac{1}{2} \Rightarrow \varphi=\pm\frac{\pi}{3}$$

$$D\ldots \quad -\frac{\pi}{3}\le\varphi\le\frac{\pi}{3}$$

$$1\le r\le 2\cos\varphi$$

$$P(D)=\iint_D dx\,dy=\iint_D r\,dr\,d\varphi=\int_{-\pi/3}^{\pi/3}d\varphi\int_1^{2\cos\varphi}r\,dr$$

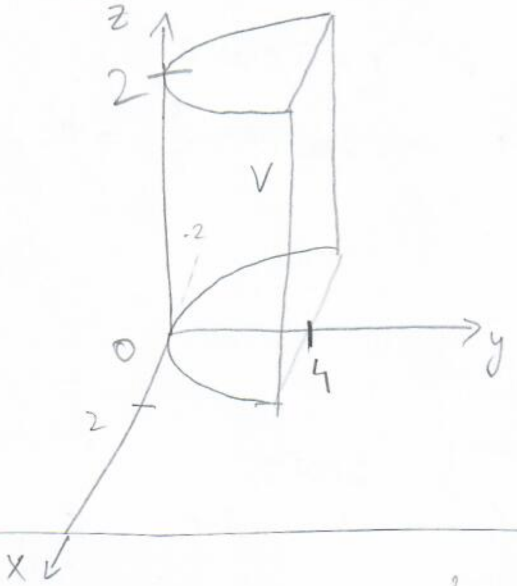
$$=\int_{-\pi/3}^{\pi/3}\frac{r^2}{2}\Big|_1^{2\cos\varphi}d\varphi=\int_{-\pi/3}^{\pi/3}\frac{4\cos^2\varphi-1}{2}\,d\varphi$$

$$=\int_{-\pi/3}^{\pi/3}\left(2\cos^2\varphi-\frac{1}{2}\right)d\varphi=\int_{-\pi/3}^{\pi/3}(1+\cos2\varphi)-\frac{\varphi}{2}\Big|_{-\pi/3}^{\pi/3}$$

$$=\left(\varphi+\frac{\sin2\varphi}{2}\right)\Big|_{-\pi/3}^{\pi/3}-\frac{1}{\cancel{2}}\cdot\frac{\cancel{2}\pi}{3}$$

$$=\frac{2\pi}{3}+\frac{1}{2}\cdot\left(\sin\frac{2\pi}{3}\right)\cdot2-\frac{\pi}{3}=\frac{\pi}{3}+\frac{\sqrt3}{2}$$

5. (12 bodova) Izračunajte
$$\iiint\limits_V \cos x \mathrm{d}x\mathrm{d}y\mathrm{d}z$$
ako je V tijelo omeđeno plohama $y = x^2$, $y = 4$, $z = 0$ i $z = 2$. Skicirajte tijelo.



$$y = 4 = x^2 \Rightarrow x = \pm 2$$

$$-2 \leq x \leq 2$$

$$x^2 \leq y \leq 4$$

$$0 \leq z \leq 2$$

$$\iiint\limits_V y \, dx\, dy\, dz = \int_{-2}^{2} dx \int_{x^2}^{4} y \, dy \int_{0}^{2} dz$$

$$= \int_{-2}^{2} dx \int_{x^2}^{4} y \cdot z \Big|_0^2 \, dy = \int_{-2}^{2} dx \int_{x^2}^{4} 2y \, dy$$

$$= \int_{-2}^{2} y^2 \Big|_{x^2}^{4} dx = \int_{-2}^{2} (16 - x^4)\, dx = \text{(parna funkcija na simetričnoj domeni)}$$

$$= 2 \int_0^2 (16 - x^4) dx = 2 \cdot \left(16x - \frac{x^5}{5}\right)\Big|_0^2$$

$$= 2 \cdot \left(32 - \frac{32}{5}\right) = 2 \cdot \frac{160 - 32}{5} = \frac{256}{5}$$