Kolokvij iz Matematike 2 **5. svibnja 2021.** β

Ime i prezime: ______________________ Grupa: ________

1.	2.	3.	4.	5.	Σ

1. (12 bodova) Riješite diferencijalnu jednadžbu $y'' - 4y' + 4y = 8x$.

$y'' - 4y' + 4y = 8x$

$\lambda^2 - 4\lambda + 4 = 0$

$(\lambda - 2)^2 = 0$

$\lambda_1 = \lambda_2 = 2$

$y_H = C_1 e^{2x} + C_2 x e^{2x}$

$f(x) = 8x = P_n(x) e^{ax} \Rightarrow a = 0, \; n = 1$

$y_p = Ax + B$

$y_p' = A$

$y_p'' = 0$

$\Rightarrow -4A + 4Ax + 4B = 8x$

$4A = 8 \qquad -4A + 4B = 0$

$A = 2 \qquad B = 2$

$\Rightarrow \quad y = y_H + y_p$

$$\boxed{y = C_1 e^{2x} + C_2 x e^{2x} + 2x + 2}$$

2. a) (7 bodova) Odredite i skicirajte prirodnu domenu funkcije

$$f(x,y) = \ln\left(x^2 + y^2 - 9\right) + \arcsin\frac{x-2}{2}.$$

b) (5 bodova) Izračunajte $\frac{\partial f}{\partial y}(4,2)$.

a) Uvjet Ln: $x^2+y^2-9>0$

$\Rightarrow x^2+y^2>9$

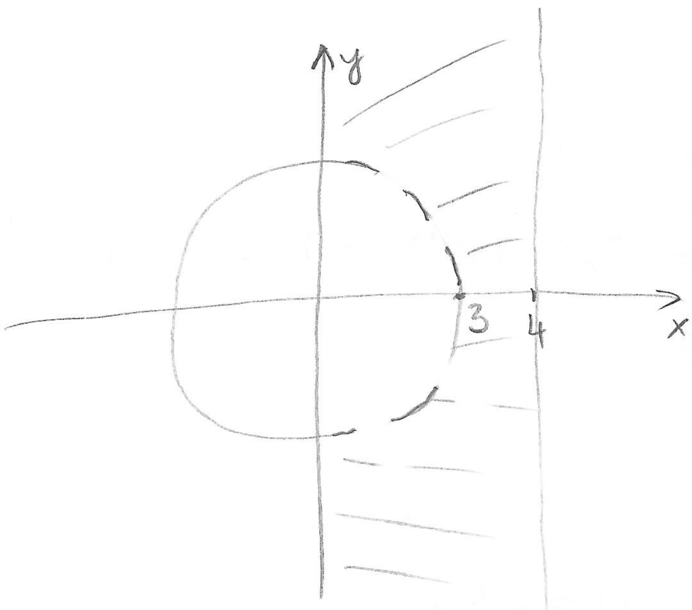
Uvjet arcsin: $-1 \le \frac{x-2}{2} \le 1$

$-1 \le \frac{x-2}{2}$ $\qquad$ $\frac{x-2}{2} \le 1$

$x-2 \geq -2$ $\qquad$ $x-2 \le 2$

$x \geq 0$ $\qquad$ $x \le 4$

SKICA



b) $\frac{\partial f}{\partial y} = \frac{2y}{x^2+y^2-9}$ $\qquad$ $\frac{\partial f}{\partial y}(4,2) = \frac{4}{11}$

3. (12 bodova) Nađite lokalne ekstreme funkcije $f(x, y) = xy(12 - x - y)$.

$f(x,y) = 12xy - x^2y - xy^2$

$\frac{\partial f}{\partial x} = 12y - 2xy - y^2$ $\quad A = \frac{\partial^2 f}{\partial x^2} = -2y$ $\quad C = \frac{\partial^2 f}{\partial y^2} = -2x$

$B = \frac{\partial^2 f}{\partial x \partial y} = 12 - 2x - 2y$

$\frac{\partial f}{\partial y} = 12x - x^2 - 2xy$

$y(12 - 2x - y) = 0$

$x(12 - x - 2y) = 0$

$y = 0$
$x = 0$
$T_1(0,0)$

$y = 0$
$12 - x - 2y = 0$
$T_2(12,0)$

$12 - 2x - y = 0$
$x = 0$
$T_3(0,12)$

$12 - 2x - y = 0 \ / \cdot(-2)$
$12 - x - 2y = 0 \ / +$
$-12 + 3x = 0$
$x = 4$
$y = 4$
$T_4(4,4)$

$\left.\begin{array}{l} T_1: AC - B^2 = 0 - 12^2 < 0 \\ T_2: AC - B^2 = 0 - (-12)^2 < 0 \\ T_3: AC - B^2 = 0 - (-12)^2 < 0 \end{array}\right\}$ NIJE EKSTREM

$T_4: AC - B^2 = (-2\cdot 4)\cdot(-2\cdot 4) - (12 - 2\cdot 4 - 2\cdot 4)^2 =$

$= 64 - 16 = 48 > 0$ i $A = -2\cdot 4 < 0$

$T_4(4,4)$ JE LOKALNI MAKSIMUM

4. (12 bodova) Izračunajte integral

$$\iint_D \frac{xy}{x^2+y^2+1}\,dx\,dy$$

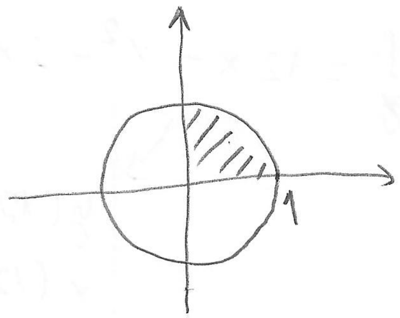
po području D, gdje je D dio ravnine zadan s $x^2+y^2 \le 1$ u prvom kvadrantu. Skicirajte D.

$x = r\cos\varphi$

$y = r\sin\varphi$

$0 \le \varphi \le \frac{\pi}{2}$

$0 \le r \le 1$



$$\iint_D \frac{xy}{x^2+y^2+1}\,dx\,dy = \int_0^{\pi/2} \underbrace{\int_0^1 \frac{r^3}{r^2+1}\,dr}_{(*)} \cdot \sin\varphi\cos\varphi\,d\varphi = (\Delta)$$

$$r^3 : (r^2+1) = r \quad \Rightarrow \quad \frac{r^3}{r^2+1} = r - \frac{r}{r^2+1}$$

$$\frac{-(r^3+r)}{-r}$$

$$(*) = \int_0^1 r\,dr - \int_0^1 \frac{r}{r^2+1}\,dr = \left\{ \begin{array}{l} r^2+1 = t \\ r\,dr = \frac{1}{2}dt \\ t=1 \quad t=2 \end{array} \right\} = \frac{1}{2} - \frac{1}{2}\int_1^2 \frac{dt}{t} = \frac{1}{2} - \frac{1}{2}\ln 2$$

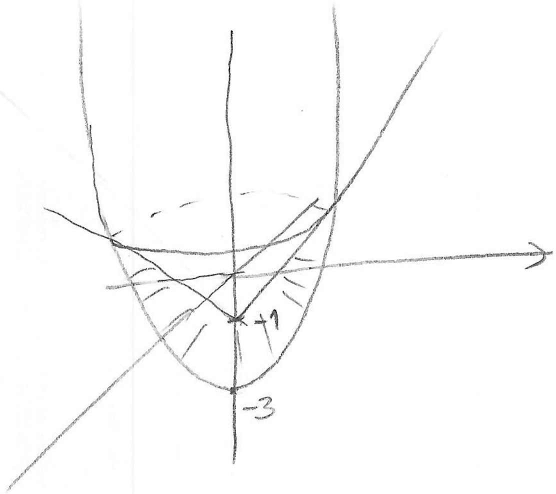
$$(\Delta) = \left(\frac{1}{2} - \frac{1}{2}\ln 2\right) \int_0^{\pi/2} \sin\varphi\cos\varphi\,d\varphi = \left\{ \begin{array}{l} t = \sin\varphi \\ \cos\varphi\,d\varphi = dt \\ t=0 \quad t=1 \end{array} \right.$$

$$= \left(\frac{1}{2} - \frac{1}{2}\ln 2\right) \underbrace{\int_0^1 t\,dt}_{\frac{1}{2}} = \frac{1}{4}(1-\ln 2) \quad //$$

5. (12 bodova) Odredite masu homogenog tijela V gustoće 1, ako je

$$V = \{(x,y,z) \in \mathbb{R}^3 : x \geq 0, y \geq 0,\ x^2 + y^2 - 3 \leq z \leq \sqrt{x^2+y^2} - 1\}.$$

Skicirajte tijelo.



$$\underbrace{x^2+y^2}_{t^2} - 3 = \underbrace{\sqrt{x^2+y^2}}_{t} - 1$$

$$t^2 - t - 2 = 0$$

$$t_{1,2} = \frac{+1 \pm \sqrt{1+8}}{2} = 2, -1$$

$$\sqrt{x^2+y^2} = 2 \ /^2 \qquad \cancel{\sqrt{x^2+y^2} = -1}$$

$$x^2+y^2 = 4 \Rightarrow \quad 0 \leq \rho \leq 2$$

$$0 \leq \varphi \leq \frac{\pi}{2}$$

$$x^2+y^2 = \rho^2 \Rightarrow \rho^2 - 3 \leq z \leq \rho - 1$$

$$\iiint_\Omega dx\,dy\,dz = \int_0^{\pi/2} \int_0^2 \int_{\rho^3-3}^{\rho-1} \rho\, dz\, d\rho\, d\varphi = \int_0^{\pi/2} \int_0^2 \rho \left(z \Big|_{\rho^2-3}^{\rho-1} \right) =$$

$$= \int_0^{\pi/2} \int_0^2 \rho(-\rho^2+\rho+2)\, d\rho\, d\varphi = \int_0^{\pi/2} \left(\frac{-1}{4}\rho^4 + \frac{1}{3}\rho^3 + 2\rho \right) \Big|_0^2 d\varphi$$

$$= \int_0^{\pi/2} \left(-4 + \frac{8}{3} + 4 \right) d\varphi = \frac{8}{3} \int_0^{\pi/2} d\varphi = \frac{8\pi}{6} = \frac{4\pi}{3} //$$