Matematika 2 - kolokvij

5. svibnja 2021.

IME I PREZIME: **Grupa na vježbama:** **Asistent:** δ

Bodovi:

1. Riješite diferencijalnu jednadžbu $y' + y = x$. *(12 bod.)*

$f(x) = 1$, $g(x) = x$

$\int f(x)dx = \int dx = x$

$\int e^{\int f(x)dx} g(x)dx = \int e^x x\,dx = \left| \begin{matrix} u = x & du = dx \\ dv = e^x dx & v = e^x \end{matrix} \right|$

$= xe^x - \int e^x dx = xe^x - e^x = (x-1)e^x$

$y(x) = e^{-\int f(x)dx} \left[\int e^{\int f(x)dx} g(x)dx + C \right]$

$= e^{-x} \left[(x-1)e^x + C \right] = x - 1 + Ce^{-x}$

2. (a) Odredite i skicirajte prirodnu domenu funkcije $f(x,y) = \sqrt{x+1-\sqrt{x+y}}$. *(8 bod.)*

(b) Izračunajte $\frac{\partial g}{\partial x}(1,0)$ ako je $g(x,y) = \operatorname{arctg} \frac{x+y}{x^2+y^2}$. *(5 bod.)*

a) $1^\circ\ x+y \geq 0 \Rightarrow y \geq -x$

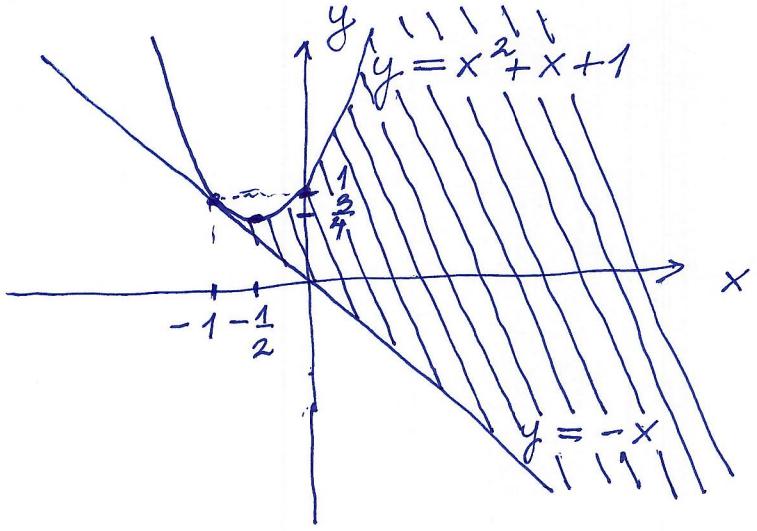
(i)

$2^\circ\ x+1-\sqrt{x+y} \geq 0 \Rightarrow x+1 \geq \sqrt{x+y} \geq 0$

$\Rightarrow x \geq -1$ (i) $(x+1)^2 \geq x+y \Rightarrow x^2+2x+1 \geq x+y$

$\Rightarrow x^2+x+1 \geq y$

$D_f = \{(x,y) \in \mathbb{R}^2 : x \geq -1,\ -x \leq y \leq x^2+x+1\}$



b) $\frac{\partial g}{\partial x} = \frac{1}{1+\left(\frac{x+y}{x^2+y^2}\right)^2} \cdot \frac{1\cdot(x^2+y^2)-2x(x+y)}{(x^2+y^2)^2}$

$\frac{\partial g}{\partial x}(1,0) = \frac{1}{1+\left(\frac{1+0}{1+0}\right)^2} \cdot \frac{1-2}{1} = \frac{1}{2}\cdot(-1) = \boxed{-\frac{1}{2}}$

3. Odredite lokalne ekstreme funkcije $f(x,y) = x^3 + 2y^2 - 3x - 8y - 5$. *(11 bod.)*

$\frac{\partial f}{\partial x} = 3x^2 - 3 = 0 \Rightarrow x^2 = 1 \Rightarrow x_{1,2} = \pm 1$

$\frac{\partial f}{\partial y} = 4y - 8 = 0 \Rightarrow y = 2$

} stacionarne točke: $T_1(-1,2)$, $T_2(1,2)$

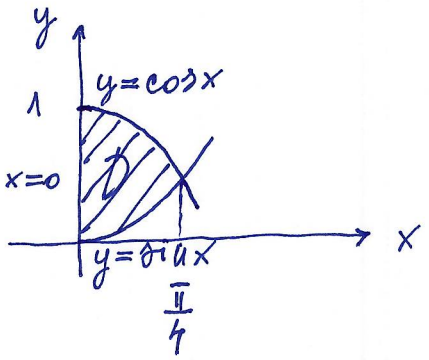
$\frac{\partial^2 f}{\partial x^2} = 6x \qquad \frac{\partial^2 f}{\partial x \partial y} = 0 \qquad \frac{\partial^2 f}{\partial y^2} = 4$

$T_1(-1,2) \Rightarrow A = -6,\ B = 0,\ C = 4 \Rightarrow AC - B^2 = -24 < 0$
$\Rightarrow T_1$ je sedlasta točka

$T_2(1,2) \Rightarrow A = 6,\ B = 0,\ C = 4 \Rightarrow AC - B^2 = 24 > 0,\ A > 0$
$\Rightarrow T_2$ je točka lok. minimuma

$f(T_2) = 1 + 8 - 3 - 16 - 5 = -15$

4. Neka je D lik u 1. kvadrantu omeđen krivuljama $y = \sin x$, $y = \cos x$ i $x = 0$. Izračunajte $\iint_D y^3\,dx\,dy$. Skicirajte D. *(12 bod.)*



Presjek krivulja!

$\sin x = \cos x \Rightarrow \operatorname{tg} x = 1 \Rightarrow x = \frac{\pi}{4}$

$D\ldots\ 0 \le x \le \frac{\pi}{4}$

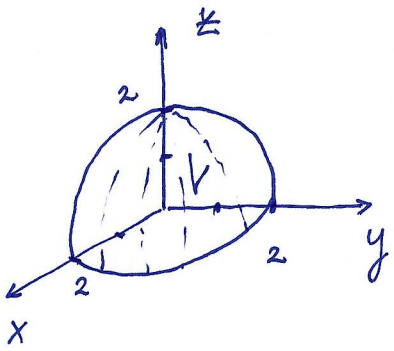
$\sin x \le y \le \cos x$

$$\iint_D y^3\,dx\,dy = \int_0^{\frac{\pi}{4}} dx \int_{\sin x}^{\cos x} y^3\,dy = \int_0^{\frac{\pi}{4}} \frac{y^4}{4}\Big|_{\sin x}^{\cos x}\,dx$$

$$= \frac{1}{4}\int_0^{\frac{\pi}{4}} (\cos^4 x - \sin^4 x)\,dx = \frac{1}{4}\int_0^{\frac{\pi}{4}} \underbrace{(\cos^2 x - \sin^2 x)}_{\cos 2x}\underbrace{(\cos^2 x + \sin^2 x)}_{=1}\,dx$$

$$= \frac{1}{4}\int_0^{\frac{\pi}{4}} \cos 2x\,dx = \frac{1}{8}\sin 2x\Big|_0^{\frac{\pi}{4}} = \frac{1}{8}\sin\frac{\pi}{2} = \boxed{\frac{1}{8}}$$

5. Izračunajte statički moment obzirom na xz-ravninu homogenog tijela V gustoće 1 omeđenog plohom $z = \sqrt{4 - x^2 - y^2}$ u 1. oktantu. Skicirajte tijelo. *(12 bod.)*



$V \dots 0 \le \varphi \le \frac{\pi}{2}$

$0 \le \vartheta \le \frac{\pi}{2}$

$0 \le r \le 2$

$$M_{xz} = \iiint_V y \, dx\,dy\,dz = \int_0^{\frac{\pi}{2}} d\varphi \int_0^{\frac{\pi}{2}} d\vartheta \int_0^2 r \sin\vartheta \sin\varphi \, r^2 \sin\vartheta \, dr$$

$$= \int_0^{\frac{\pi}{2}} \sin\varphi \int_0^{\frac{\pi}{2}} \underbrace{\sin^2\vartheta}_{\frac{1-\cos 2\vartheta}{2}} d\vartheta \int_0^2 r^3 dr$$

$$= -\cos\varphi \Big|_0^{\frac{\pi}{2}} \cdot \left(\frac{1}{2}\vartheta - \frac{1}{4}\sin 2\vartheta \right) \Big|_0^{\frac{\pi}{2}} \cdot \frac{r^4}{4} \Big|_0^2$$

$$= 1 \cdot \frac{\pi}{4} \cdot 4 = \boxed{\pi}$$