

1.) Riješite dif. jednačinu  $y' = 4 + y^2$ .

Pj.1 Separirano varijable:

$$\frac{dy}{dx} = 4 + y^2$$

$$\frac{dy}{4 + y^2} = dx$$

$$\int \frac{dy}{4 + y^2} = \int dx$$

$$\frac{1}{2} \arctan \frac{y}{2} = x + C$$

$$y = 2 \tan(2x + 2C)$$

↳ može se staviti  $C_1 = 2C$ .

2. a) Domena funkcije  $f(x,y) = \arcsin\left(1 - 2\left(\frac{x-y}{x+y}\right)^2\right)$

Domena od  $\arcsin x$  je  $(-1, 1)$  pa zaključujemo da je nužno i dovoljno.

$$\left. \begin{aligned} -1 &\leq 1 - 2\left(\frac{x-y}{x+y}\right)^2 \leq 1 \\ \text{i nužno } x+y &\neq 0. \end{aligned} \right\}$$

Njednakost rastavimo: a)  $1 - 2\left(\frac{x-y}{x+y}\right)^2 \leq 1$

$$-\left(\frac{x-y}{x+y}\right)^2 \leq 0 \rightarrow \text{Vrijedi uvijek}$$

$$b) -1 \leq 1 - 2\left(\frac{x-y}{x+y}\right)^2$$

$$-2 \leq -2\left(\frac{x-y}{x+y}\right)^2$$

$$1 \geq \left(\frac{x-y}{x+y}\right)^2$$

→ množenje s pozitivnim  $(x+y)^2$

$$(x+y)^2 \geq (x-y)^2$$

$$4xy \geq 0$$

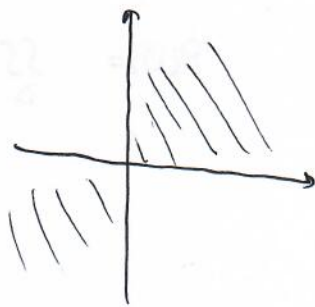
$$\boxed{xy \geq 0}$$

Dakle, domena funkcije je  $xy \geq 0$  uz ujet  $x+y \neq 0$ ,

to je I. i II. kvadrant bez pravca  $y = -x$ .

ovaj pravac ne prolazi kroz I. i II. kvadrant

pa je rješenje ustvari  $\boxed{xy \geq 0}$  i  $\boxed{(x,y) \neq (0,0)}$



$$b) \quad g(x,y) = \ln(x \ln(y-x))$$

$$\frac{\partial g}{\partial y} = \frac{1}{x \ln(y-x)} \cdot x \cdot \frac{1}{y-x} = \frac{y-x}{\ln(y-x)}$$

$$\frac{\partial g}{\partial y}(e, 2e) = \frac{e}{\ln(e)} = e$$

3.) Lokalni ekstremi funkcije  $f(x,y) = 5x^2 + 10x - 4y^2 + 48y + 7$

$$\frac{\partial f}{\partial x} = 10x + 10 = 0 \Rightarrow x = -1$$

$$\frac{\partial f}{\partial y} = -12y^2 + 48 = 0 \Rightarrow y = \pm 2$$

Stacionarne točke su  $(-1, 2)$  i  $(-1, -2)$ .

$$\frac{\partial^2 f}{\partial x^2} = 10$$

$$\frac{\partial^2 f}{\partial x \partial y} = 0$$

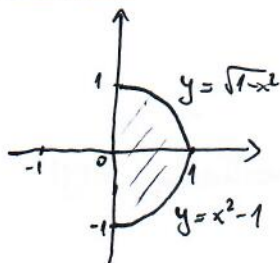
$$\frac{\partial^2 f}{\partial y^2} = -24y$$

Uz oznake kao na vježbama vrijedi:

a)  $(-1, 2) \rightarrow A = 10 > 0$   
 $Ac - B^2 = -480 < 0 \Rightarrow$  sedlasta točka

b)  $(-1, -2) \rightarrow A = 10 > 0$   
 $Ac - B^2 = 480 > 0 \} \Rightarrow (-1, -2)$  je točka minimuma.

4.) skica



$\rightarrow$  površina četvrtine kruga

$$P(D) = \iint_D dx dy = \iint_{D_1} dx dy + \iint_{D_2} dx dy \quad \text{gdje je } D_1 \text{ gornji dio } (y \geq 0)$$

$D_2$  donji dio

$$= \frac{\pi}{4} + \int_0^1 \int_{x^2-1}^0 dx dy = \frac{\pi}{4} + \int_0^1 (1-x^2) dx$$

$$= \frac{\pi}{4} + \left(x - \frac{x^3}{3}\right) \Big|_0^1 = \frac{\pi}{4} + \frac{2}{3}$$

4. Alternativno rješenje:

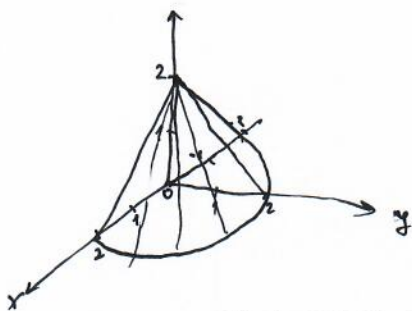
$$\begin{aligned} \iint_D dx dy &= \int_0^1 \int_{x^2-1}^{\sqrt{1-x^2}} dy dx = \int_0^1 y \Big|_{x^2-1}^{\sqrt{1-x^2}} dx \\ &= \int_0^1 (\sqrt{1-x^2} + 1-x^2) dx \\ &= \int_0^1 \sqrt{1-x^2} dx + \int_0^1 (1-x^2) dx \end{aligned}$$

Možemo izračunati direktno sa substitucijom  $x = \sin t$   
 $dx = \cos t$

$$\begin{aligned} &= \frac{2}{3} + \int_0^{\frac{\pi}{2}} \cos^2 t dt \\ &= \frac{2}{3} + \int_0^{\frac{\pi}{2}} \frac{1+\cos 2t}{2} dt = \frac{2}{3} + \left[ \frac{t}{2} + \frac{1}{4} \sin 2t \right]_0^{\frac{\pi}{2}} \\ &= \frac{2}{3} + \frac{\pi}{4}. \end{aligned}$$

5.] Skica: funkcija  $f(x,y) = \sqrt{x^2+y^2}$  predstavlja stožac prema gore s vrhom u  $(0,0)$

$\Rightarrow z = 2 - \sqrt{x^2+y^2}$  je stožac prema dolje s visine 2



Sjecač u  $xy$  ravnini kad je  $z=0$ , dakle

$$\begin{aligned} z = 2 - \sqrt{x^2+y^2} &= 0 \\ x^2+y^2 &= 4 \end{aligned}$$

Tijelo  $V$  je opisano s

$$\left. \begin{aligned} 0 \leq z \leq 2 - \sqrt{x^2+y^2} \\ x^2+y^2 \leq 4 \\ y \geq 0 \end{aligned} \right\}$$

pojednostak za cilindrične koordinate

$$\Rightarrow \left. \begin{aligned} x &= \rho \cos \varphi \\ y &= \rho \sin \varphi \\ z &= z \end{aligned} \right\}$$

$\Rightarrow$  Tijelo  $V$  u cil. coord. je

$$\begin{aligned} 0 \leq z \leq 2 - \rho \\ 0 \leq \rho \leq 2 \\ 0 \leq \varphi \leq \pi \end{aligned}$$

$$\iiint_V y^3 dx dy dz$$

$$\begin{aligned} &= \int_0^{\pi} \int_0^2 \int_0^{2-\rho} \rho^3 \sin^3 \varphi \cdot \rho d\rho d\varphi dz \\ &= \int_0^{\pi} \int_0^2 (\rho^4 \sin^3 \varphi \cdot z \Big|_0^{2-\rho}) d\rho d\varphi \end{aligned}$$

$$= \int_0^{\pi} \int_0^2 (2\rho^4 - \rho^5) \sin^3 \varphi d\rho d\varphi = \int_0^{\pi} \sin^3 \varphi \left[ \frac{2\rho^5}{5} - \frac{\rho^6}{6} \right]_0^2 d\varphi$$

$$= \int_0^{\pi} \sin^3 \varphi \frac{32}{15} d\varphi = \frac{32}{15} \int_0^{\pi} (1 - \cos^2 \varphi) \sin \varphi d\varphi$$

Substituiramo  $u = \cos \varphi$   
 $du = -\sin \varphi d\varphi$

$$= \frac{32}{15} \int_{-1}^1 (u^2 - 1) du = \frac{32}{15} \left[ \frac{u^3}{3} - u \right]_{-1}^1 = \frac{128}{45}$$

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