1. (20 bodova) U kojem su odnosu pravac

$$p \equiv \frac{x+3}{-7} = \frac{y-2}{2} = \frac{z-3}{3}$$

i ravnina

$$\pi \equiv 3x + 2y - 4z - 12 = 0?$$

Odredite ortogonalnu projekciju pravca p na ravninu π. Skicirajte.

2. (15 bodova) Riješite sustav:

$$\begin{aligned} x + y - 5z &= 1 \\ 2x - y - 4z &= -1 \\ x \quad\;\; - 3z &= 0 \end{aligned}$$

3. (25 bodova) Odredite prirodnu domenu, nultočke, intervale rasta i pada, ekstreme, područja konveksnosti i konkavnosti, asimptote, te skicirajte graf funkcije

$$f(x) = \frac{x^2}{x+2}.$$

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4. (a) (15 bodova) Izračunajte

$$\int_0^1 \ln\left(1 + x^2\right)\, dx.$$

5. (a) (12 bodova) Izračunajte površinu lika omeđenog krivuljama

$$y = \cos(\pi x),\ y = 6x^2 - 5x - 1,\ x = 0 \text{ i } x = \tfrac{1}{2}.$$

Skicirajte lik.

(b) (13 bodova) Izračunajte volumen tijela nastalog rotacijom krivulje

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

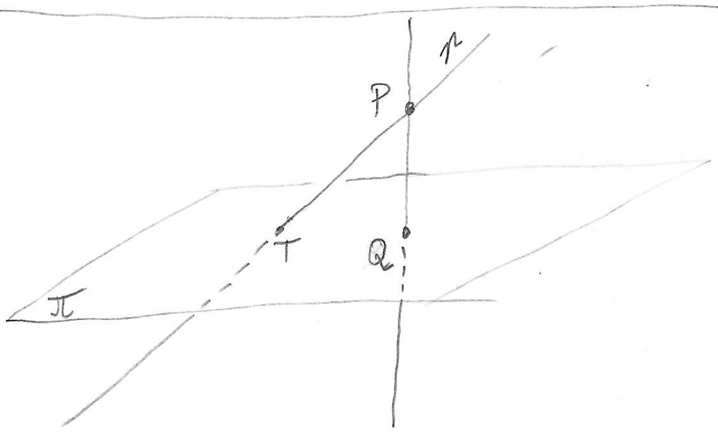
oko osi x. Skicirajte tijelo.

1. ODNOS PRAVCA I RAVNINE:

$$p \equiv \begin{cases} x = -7t - 3 \\ y = 2t + 2 \\ z = 3t + 3 \end{cases}$$

$$3(-7t-3) + 2(2t+2) + 3(3t+3) - 12 = 0$$
$$-21t + 4t + 9t = 12 + 9 - 4 - 9$$
$$-8t = 8$$
$$t = -1 \Rightarrow T(4,0,0) = p \cap \pi$$

$\boxed{\Rightarrow \text{PRAVAC I RAVNINA SE SIJEKU.}}$



ODABERIMO NEKU DRUGU TOČKU NA p. NPR. ZA $t=0$ IMAMO $P(-3,2,3)$. ODREDIMO JEDN. PRAVCA q T.D. JE $q \perp \pi$ I $P \in q$. $\Rightarrow \vec{c}_q = \vec{n}_\pi = (3,2,-4)$

$$\Rightarrow q \equiv \frac{x+3}{3} = \frac{y-2}{2} = \frac{z-3}{-4} \Rightarrow q \equiv \begin{cases} x = 3x - 3 \\ y = 2t + 2 \\ z = -4t + 3 \end{cases}$$

$\pi \cap q = ?$ $\quad 3(3t-3) + 2(2t+2) - 4(-4t+3) = 12$

$$29t = 29 \Rightarrow t = 1 \Rightarrow \pi \cap q = Q(0,4,-1)$$

p' JE PRAVAC KROZ T I Q

$$\Rightarrow p' \equiv \frac{x-0}{4-0} = \frac{y-4}{0-4} = \frac{z+1}{0+1}$$

$$\boxed{p' \equiv \frac{x}{4} = \frac{y-4}{-4} = \frac{z+1}{1}}$$

2.)

$$\left[\begin{array}{ccc|c} 1 & 1 & -5 & 1 \\ 2 & -1 & -4 & -1 \\ 1 & 0 & -3 & 0 \end{array}\right] \underset{\substack{II-2I \\ III-I}}{\sim} \left[\begin{array}{ccc|c} 1 & 1 & -5 & 1 \\ 0 & -3 & 6 & -3 \\ 0 & -1 & 2 & -1 \end{array}\right] /:-3$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & -5 & 1 \\ 0 & 1 & -2 & 1 \\ 0 & -1 & 2 & -1 \end{array}\right] \underset{III+II}{} \sim \left[\begin{array}{ccc|c} 1 & 1 & -5 & 1 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$z = t \quad \Rightarrow \quad y - 2t = 1$$

$$y = 2t + 1$$

$$x + 2t + 1 - 5t = 1$$

$$x = 3t$$

Rj: $\begin{bmatrix} 3t \\ 2t+1 \\ t \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} t + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

3. $D_f = \mathbb{R} \setminus \{-2\}$

NULTOČKE $x^2 = 0 \Rightarrow$ N.T. $(0,0)$

$$f'(x) = \frac{2x(x+2) - x^2}{(x+2)^2} = \frac{2x^2 + 4x - x^2}{(x+2)^2} = \frac{x(x+4)}{(x+2)^2} = \frac{x^2+4x}{(x+2)^2}$$

STAC. TOČKE $0, -4$

	$-\infty$		-4		-2		0		$+\infty$
f'		$+$		$-$		$-$		$+$	
f		↗		↘		↘		↗	
			M				m		

$\Rightarrow$ $(0,0)$ MINIMUM

$\Rightarrow$ $(-4,-8)$ MAKSIMUM

RASTE $\langle -\infty, -4 \rangle, \langle 0, +\infty \rangle$

PADA $\langle -4, -2 \rangle, \langle -2, 0 \rangle$

$$f''(x) = \frac{(2x+4)(x+2)^2 - (x^2+4x) \cdot 2(x+2)}{(x+2)^4} =$$

$$= \frac{(x+2)\left[(2x+4)(x+2) - 2(x^2+4x)\right]}{(x+2)^4}$$

$$= \frac{\cancel{2x^2} + 4x + 4x + 8 - \cancel{2x^2} - 8x}{(x+2)^3} = \frac{8}{(x+2)^3}$$

$\Rightarrow$ NEMA TOČAKA INF.

	$-\infty$		-2		$+\infty$
$f''(x)$		$-$		$+$	
$f(x)$		∩		∪	

V.A: $\lim\limits_{x\to -2^-} \frac{x^2}{x+2} = \frac{4}{0^-} = -\infty$

$\lim\limits_{x\to 2^+} \frac{x^2}{x+2} = +\infty$

H.A. $\lim\limits_{x\to\pm\infty}\frac{x^2}{x+2} = \pm\infty$ NEMA H.A.

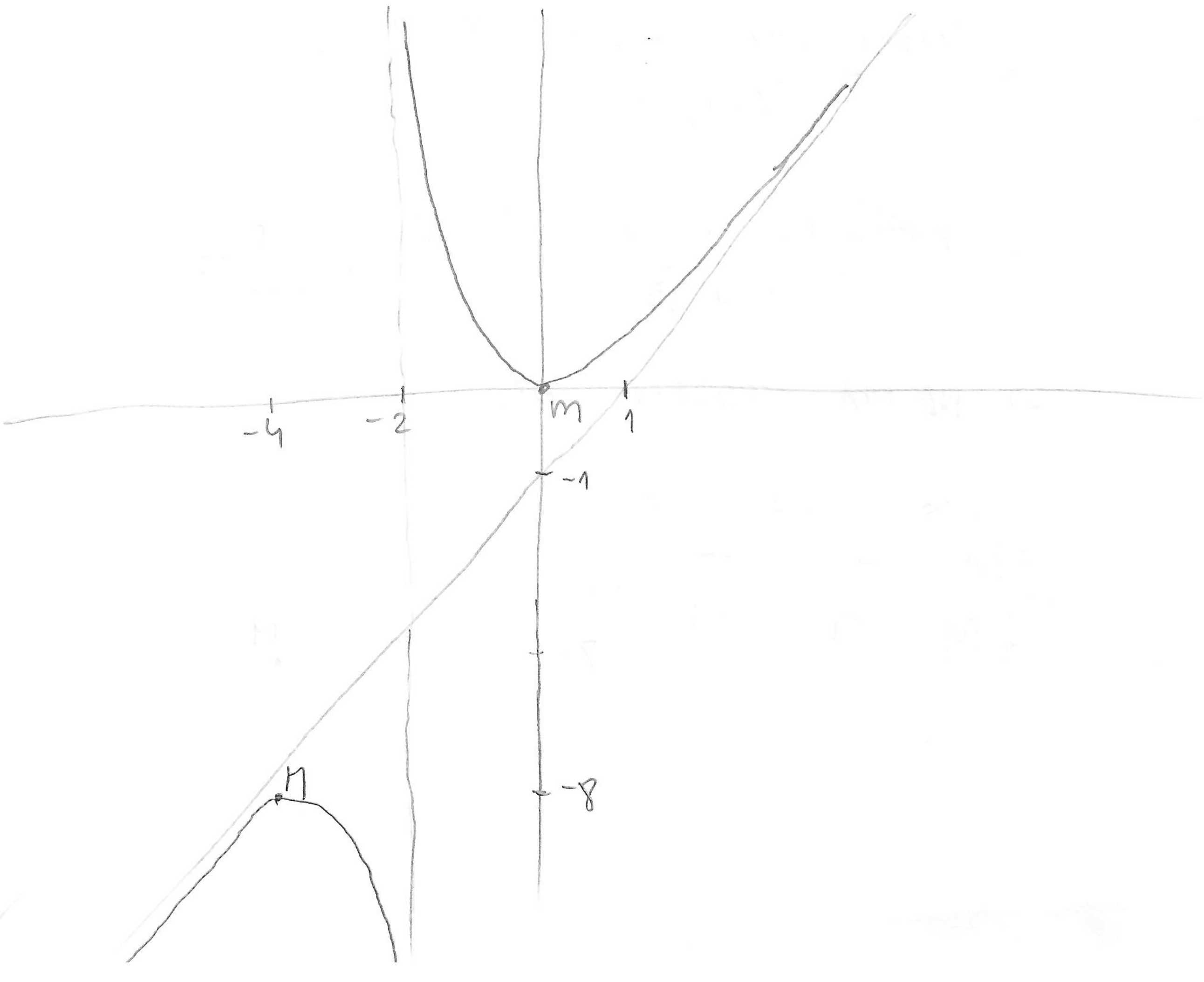
K.A. $\lim\limits_{x\to+\infty}\frac{f(x)}{x} = \lim\limits_{x\to+\infty}\frac{x^2}{x^2+2x}\begin{matrix}:x^2\\ :x^2\end{matrix} = \lim\limits_{x\to+\infty}\frac{1}{1+\frac{2}{x}} = 1$

$\lim\limits_{x\to-\infty}\frac{f(x)}{x} = 1$ $k = 1$

$l = \lim\limits_{x\to\pm\infty}(f(x)-kx) = \lim\limits_{x\to\pm\infty}\left(\frac{x^2}{x+2} - x\right) = \lim\limits_{x\to\pm\infty}\left(\frac{x^2-x^2-2x}{x+2}\right)$

$= -2$ //

$y = x-2$ JE K.A.



4.) $\displaystyle\int_0^1 \ln(1+x^2)\,dx = \left\{ \begin{array}{ll} u=\ln(1+x^2) & dv=dx \\ du=\frac{2x}{1+x^2} & v=x \end{array} \right.$

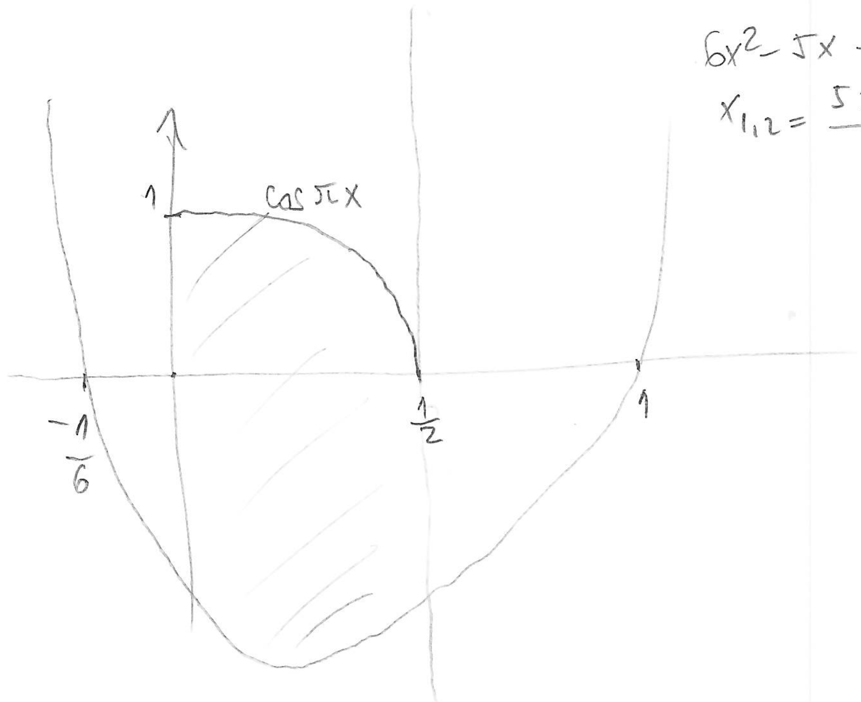
$$= x\ln(1+x^2)\Big|_0^1 - \int_0^1 \frac{2x^2}{1+x^2}\,dx$$

$$= \ln 2 - 2\int_0^1 dx + 2\int_0^1 \frac{dx}{1+x^2} =$$

$$= \ln 2 - 2 + 2\underbrace{\operatorname{arctg} 1}_{=\frac{\pi}{4}} - 2\underbrace{\operatorname{arctg} 0}_{=0}$$

$$= \ln 2 - 2 + \frac{\pi}{4}$$ //

5a.)



$6x^2 - 5x - 1 = 0$

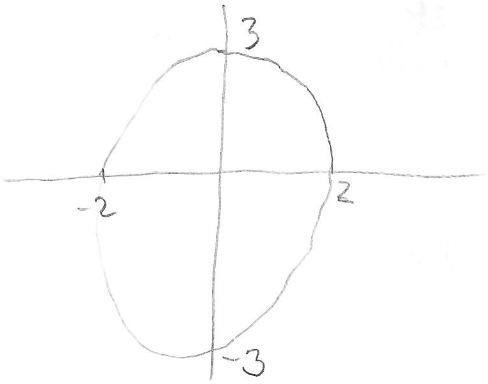
$x_{1,2} = \frac{5 \pm \sqrt{25+4\cdot 6}}{12} = \frac{5\pm 7}{12}$

$= -\frac{1}{6}, 1$ //

$$\int_0^{1/2} \cos\pi x\,dx - \int_0^1 (6x^2-5x-1)\,dx = \frac{\sin\pi x}{\pi}\Big|_0^{1/2} - \left(2x^3 - \frac{5}{2}x^2 - x\right)\Big|_0^1$$

$$= \frac{1}{\pi} + \frac{7}{8}$$ //

5b)



$$\Rightarrow \quad y^2 = 9 - \frac{9x^2}{4}$$

$$y = \pm\sqrt{9 - \frac{9}{4}x^2}$$

$$V_x = \pi \int_a^b f^2(x)\,dx$$

$$V_x = \pi \int_{-2}^{2} \left(9 - \frac{9}{4}x^2\right) dx$$

$$= \pi \left(9x - \frac{3}{4}x^3\right) \Big|_{-2}^{2}$$

$$= \pi \left(18 - \frac{3 \cdot 8}{4}\right) - \pi\left(-18 + \frac{3 \cdot 8}{4}\right)$$

$$= 12\pi + 12\pi = 24\pi \;//$$