1. (a) (15 bodova) Odredite međusobni položaj pravca $p \equiv \begin{cases} -x+y=0 \\ y-z+1=0 \end{cases}$ i ravnine $\pi \equiv y-2z=0$. Skicirajte.

   (b) (10 bodova) Neka je $\vec{a}=2\vec{m}-\vec{n}$ i $\vec{b}=\vec{m}-2\vec{n}$, gdje je $|\vec{m}|=2$, $|\vec{n}|=4$ i $\sphericalangle(\vec{m},\vec{n})=\frac{\pi}{3}$. Izračunajte skalarni produkt vektora $\vec{a}$ i $\vec{b}$.

2. (15 bodova) Riješite sustav $\begin{cases} 2x+2y-3z=0 \\ x+3y+\ \ z=0 \\ x+\ \ y+4z=0 \end{cases}$.

3. (20 bodova) Odredite prirodnu domenu, nultočke, intervale rasta i pada, ekstreme, asimptote, te skicirajte graf funkcije

$$f(x)=x^3e^{-x^2}.$$

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4. (a) (6 bodova) Odredite

$$\int t\ln t\,dt.$$

   (b) (9 bodova) Odredite

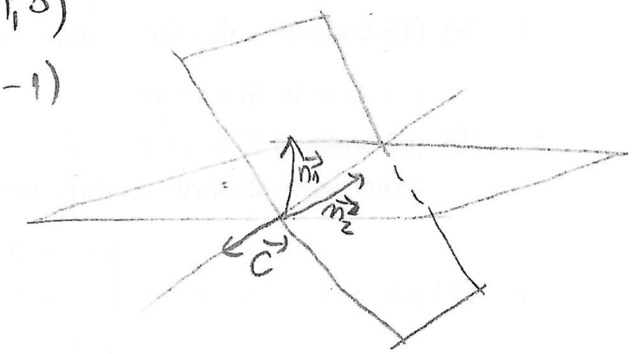
$$\int \frac{\operatorname{tg} x \ln(\operatorname{tg} x)}{\cos^2 x}\,dx.$$

5. (a) (10 bodova) Izračunajte površinu omeđenu krivuljama $y=\frac{8}{x^2+4}$ i $y=\frac{x^2}{4}$.

   (b) (15 bodova) Izračunajte duljinu luka krivulje $y=\ln\frac{e^x-1}{e^x+1}$ od točke $A(1,y_1)$ do točke $B(2,y_2)$.

1a) 15 $\vec{c} \perp \vec{n_1}, \vec{n_2}$, $\vec{n_1} = (-1, 1, 0)$

$\vec{n_2} = (0, 1, -1)$



$\vec{c} = \vec{n_1} \times \vec{n_2}$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix}$$

$$= \vec{i}(-1) - \vec{j}(1) + \vec{k}(-1) = -\vec{i} - \vec{j} - \vec{k} = (-1, -1, -1)$$

ODABERIMO NEKU TOČKU NA p npr., za $z = 0$

$\Rightarrow \quad y = -1 \Rightarrow x = -1$

$$p \equiv \frac{x+1}{-1} = \frac{y+1}{-1} = \frac{z}{-1} \equiv \begin{cases} x = -t - 1 \\ y = -t - 1 \\ z = -t \end{cases}$$

$p \cap \pi = ?$

$$\left.\begin{aligned} \pi \equiv \; & y - 2z = 0 \\ & -t - 1 + 2t = 0 \\ & t = 1 \end{aligned}\right\} \quad P(-2, -2, -1)$$

b) 10 $\vec{a} = 2\vec{m} - \vec{n}$, $\vec{b} = \vec{m} - 2\vec{n}$, $\vec{m} \cdot \vec{n} = |\vec{m}| \cdot |\vec{n}| \cdot \cos\measuredangle$

$= 2 \cdot 4 \cdot \frac{1}{2} = 4$

$\vec{a} \cdot \vec{b} = (2\vec{m} - \vec{n})(\vec{m} - 2\vec{n}) =$

$= 2|\vec{m}|^2 - 4\vec{m}\cdot\vec{n} - \vec{m}\cdot\vec{n} + 2|\vec{n}|^2$

$= 2 \cdot 4 - 5 \cdot 4 + 2 \cdot 16$

$= 24$

2) 15

$$\left(\begin{array}{ccc|c} 2 & 2 & -3 & 0 \\ 1 & 3 & 1 & 0 \\ 1 & 1 & 4 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} \boxed{1} & 1 & 4 & 0 \\ 1 & 3 & 1 & 0 \\ 2 & 2 & -3 & 0 \end{array}\right) /(-1) /(-2) \sim \left(\begin{array}{ccc|c} 1 & 1 & 4 & 0 \\ 0 & 2 & -3 & 0 \\ 0 & \boxed{1} & -7 & 0 \end{array}\right) /(-2) /(-1)$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & 11 & 0 \\ 0 & 0 & 11 & 0 \\ 0 & 1 & -7 & 0 \end{array}\right) \Rightarrow \begin{array}{l} x + 11z = 0 \Rightarrow \boxed{x=0} \\ 11z = 0 \Rightarrow \boxed{z=0} \\ y - 7z = 0 \Rightarrow \boxed{y=0} \end{array}$$

Rješenje je $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$.

3. (20) $f(x)=x^3e^{-x^2}$ $\qquad D_f=\mathbb{R}$

N.T $\quad x=0$

$$f'(x)=3x^2e^{-x^2}+x^3\cdot(-2x)\,e^{-x^2}$$
$$=e^{-x^2}\left(3x^2-2x^4\right)=e^{-x^2}\left(x^2\cdot(3-2x^2)\right)$$
$$\Rightarrow \text{S.T.}\quad x_1=0,\quad x_{2,3}=\pm\sqrt{\frac{3}{2}}$$

$(e^{-x^2}>0)$

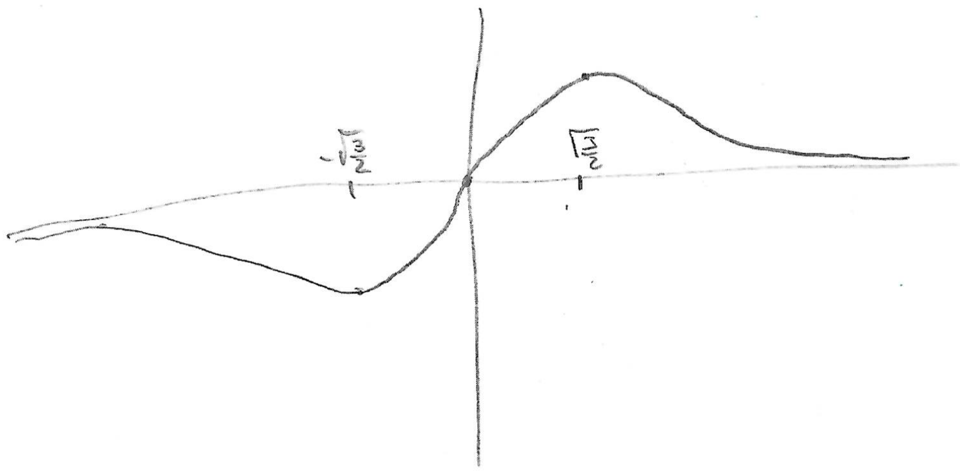
	$-\infty$	$-\sqrt{\frac{3}{2}}$	0	$\sqrt{\frac{3}{2}}$	$+\infty$
$f'(x)$	$-$	$+$	$+$	$-$	
$f(x)$	$\searrow$	$\nearrow$	$\nearrow$	$\searrow$	
		m		M	

V.A. nema

H.A. $\lim\limits_{x\to\pm\infty}\frac{x^3}{e^{x^2}}=\left(\frac{\infty}{\infty}\right)\overset{L'H}{=}\lim\limits_{x\to\pm\infty}\frac{3x^2}{2xe^{x^2}}=\frac{3}{2}\lim\limits_{x\to\pm\infty}\frac{x}{e^{x^2}}$

$$\overset{L'H}{=}\frac{3}{2}\lim_{x\to\pm\infty}\frac{1}{2xe^{x^2}}=0$$ $\quad y=0$ JE H.A.

NEMA KOSIH



4a) 6 $\int t\ln t\,dt = \left\{\begin{matrix} \ln t = u & dv = t\,dt \\ \frac{1}{t}dt = du & v = \frac{1}{2}t^2 \end{matrix}\right\}$

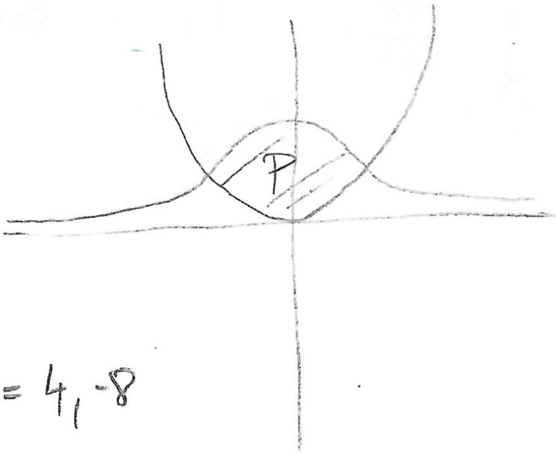
$= \frac{1}{2}t^2\ln t - \frac{1}{2}\int t\,dt = \frac{1}{2}t^2\ln t - \frac{1}{4}t^2 + C$

b) 9 $\int \frac{tg x \ln(tg x)}{\cos^2 x}dx = \left\{\begin{matrix} t = tg x \\ dt = \frac{1}{\cos^2 x}dx \end{matrix}\right.$

$= \int t\ln t\,dt = (a)\text{ dio} = \frac{1}{2}t^2\ln t - \frac{1}{4}t^2 + C$

$= \frac{1}{2}tg^2 x\ln(tg x) - \frac{1}{4}tg^2 x + C$

5a) 10 PRESJEK:



$\frac{8}{x^2+4} = \frac{x^2}{4}$

$x^4 + 4x^2 - 32 = 0$

$x^2_{1,2} = \frac{-4 \pm \sqrt{16+128}}{2} = \frac{-4\pm 12}{2} = 4, -8$

$x^2 = 4 \qquad \cancel{x^2 = -8}$

$x_1 = -2, \; x_2 = 2$

$\boxed{* \int \frac{dx}{x^2+a^2} = \frac{1}{a}\text{arctg}\frac{x}{a}}$

$P = \int_{-2}^{2}\left(\frac{8}{x^2+4} - \frac{x^2}{4}\right)dx = 4\,\text{arctg}\frac{x}{2}\Big|_{-2}^{2} - \frac{x^3}{12}\Big|_{-2}^{2}$

$= 4(\underbrace{\text{arctg}\,1}_{=\frac{\pi}{4}} - \underbrace{\text{arctg}(-1)}_{=-\frac{\pi}{4}}) - \frac{1}{12}(8-(-8)) =$

$= 2\pi - \frac{4}{3}$ //

b) 15 $y' = \frac{e^x+1}{e^x-1} \cdot \frac{e^x(e^x+1) - e^x(e^x-1)}{(e^x+1)^2}$

$$= \frac{2e^x}{e^{2x}-1}$$

$$\int_1^2 \sqrt{1+\frac{4e^{2x}}{e^{4x}-2e^{2x}+1}}\,dx = \int_1^2 \sqrt{\frac{e^{4x}+2e^{2x}+1}{e^{4x}-2e^{2x}+1}}\,dx = \int_1^2 \frac{e^{2x}+1}{e^{2x}-1}\,dx =$$

$$= \int_1^2 \frac{2\,e^{2x}}{e^{2x}-1}\,dx - \int_1^2 \frac{e^{2x}-1}{e^{2x}-1}\,dx = \begin{cases} t = e^{2x}-1 \\ dt = 2e^{2x}\,dx \end{cases} \quad \begin{matrix} t = e^4-1 \\ t = e^2-1 \end{matrix}$$

$$= \int_{e^2-1}^{e^4-1} \frac{dt}{\ln t} - \underbrace{\int_1^2 dx}_{1} = \ln|e^4-1| - \ln|e^2-1| - 1 \;//$$