1. (a) (10 bodova) Odredite λ tako da vektori $\vec{a} = \lambda\vec{i} - 3\vec{j} + 3\vec{k}$ i $\vec{b} = -3\vec{i} - 2\lambda\vec{j} - 2\vec{k}$ budu okomiti.

   (b) (15 bodova) Odredite jednadžbu ravnine određenu s dva paralelna pravca

$$p_1 \equiv \frac{x-4}{-1} = \frac{y-2}{1} = \frac{z-2}{1} \text{ i } p_2 \equiv \frac{x-2}{-1} = \frac{y-2}{1} = \frac{z-2}{1}.$$

   Skicirajte.

2. (15 bodova) Riješite sustav $\begin{cases} x_1 + x_2 - 3x_3 - x_4 = 0 \\ x_1 - 2x_3 = 0 \\ x_2 - 3x_4 = 0 \\ 3x_1 - 2x_2 - 2x_3 - 2x_4 = 0 \end{cases}.$

3. (20 bodova) Odredite prirodnu domenu, nultočke, intervale rasta i pada, ekstreme, asimptote, te skicirajte graf funkcije

$$f(x) = \frac{4x}{4+x^2}.$$

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4. (15 bodova) Odredite

$$\int e^x \sqrt{e^{e^x} - 1}\, dx.$$

5. (a) (15 bodova) Izračunajte manju površinu omeđenu krivuljama $x^2 + y^2 = 2$ i $y = x^2$.

   (b) (10 bodova) Odredite volumen tijela koje nastaje rotacijom lika omeđenog krivuljom $y = e^x$, pravcem $x = 1$ i koordinatnim osima oko osi x. Skicirajte lik.

1a) $\vec{a} = \lambda\vec{i} - 3\vec{j} + 3\vec{k}$ $\quad\quad$ $\vec{b} = -3\vec{i} - 2\lambda\vec{j} - 2\vec{k}$

$$\vec{a} \perp \vec{b} \iff \vec{a}\cdot\vec{b} = 0$$

$$\vec{a}\cdot\vec{b} = -3\lambda + 6\lambda - 6 = 0$$

$$3\lambda = 6$$

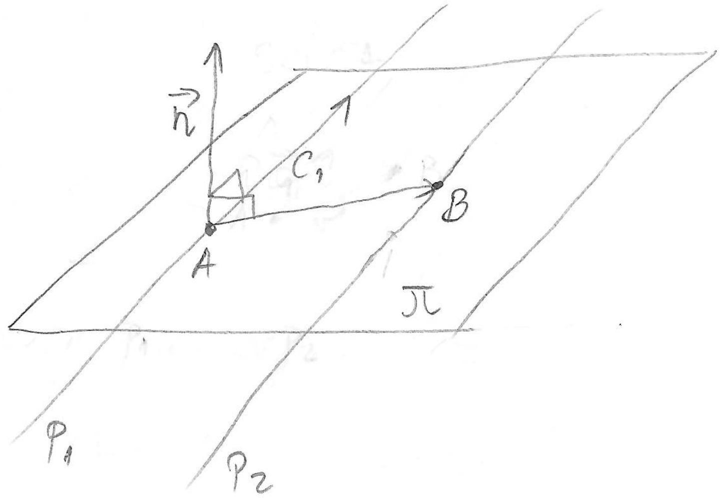
$$\lambda = 2$$

b) $\vec{c_1} = \vec{c_2} = (-1, 1, 1)$

$A(4,2,2) \in p_1$

$B(2,2,2) \in p_2$

$\overrightarrow{AB} = (2,0,0)$



$$\vec{n} = \vec{c_1} \times \overrightarrow{AB} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 1 & 1 \\ 2 & 0 & 0 \end{vmatrix} = \vec{i}\cdot 0 - \vec{j}\cdot(-2) + \vec{k}(-2) = (0, 2, -2)$$

RAVNINA π $\quad\quad$ $2\cdot(y-2) - 2(z-2) = 0$

$$2y - 2z = 0 \Rightarrow \pi \ldots y - z = 0$$

2. $\left[\begin{array}{cccc|c} 1 & 1 & -3 & -1 & 0 \\ 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & 0 & -3 & 0 \\ 3 & -2 & -2 & -2 & 0 \end{array}\right] \begin{array}{l} \\ II-I \\ \\ IV-3I \end{array} \sim \left[\begin{array}{cccc|c} 1 & 1 & -3 & -1 & 0 \\ 0 & -1 & 1 & 1 & 0 \\ 0 & 1 & 0 & -3 & 0 \\ 0 & -5 & 7 & 1 & 0 \end{array}\right] \begin{array}{l} \\ \\ III+II \\ IV-5II \end{array} \sim$

$\sim \left[\begin{array}{cccc|c} 1 & 1 & -3 & -1 & 0 \\ 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 2 & -4 & 0 \end{array}\right] \begin{array}{l} \\ \\ \\ IV-2III \end{array} \sim \left[\begin{array}{cccc|c} 1 & 1 & -3 & -1 & 0 \\ 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

$\Rightarrow$ 1-param. rješenje $\quad x_4 = t$

Treća j: $\quad x_3 - 2t = 0 \Rightarrow x_3 = 2t$

Druga j: $\quad -x_2 + 2t + t = 0 \Rightarrow x_2 = 3t$

Prva j: $\quad x_1 + 3t - 6t - t = 0 \Rightarrow x_1 = 4t$

Rj: $\quad X = \begin{bmatrix} 4t \\ 3t \\ 2t \\ t \end{bmatrix}, \ t \in \mathbb{R}$ //

3. $f(x)=\dfrac{4x}{4+x^2}$ $\qquad D_f=\mathbb{R}$

N.T: $\quad 4x=0 \Rightarrow x=0$

$$f'(x)=\frac{4(4+x^2)-4x(2x)}{(4+x^2)^2}=\frac{16-4x^2}{(4+x^2)^2}$$

S.T. $\quad 16-4x^2=0 \Rightarrow x_{1,2}=\pm 2$

	$-\infty$		-2		2		$+\infty$
f'		$-$		$+$		$-$	
f		$\searrow$	m	$\nearrow$	M	$\searrow$	

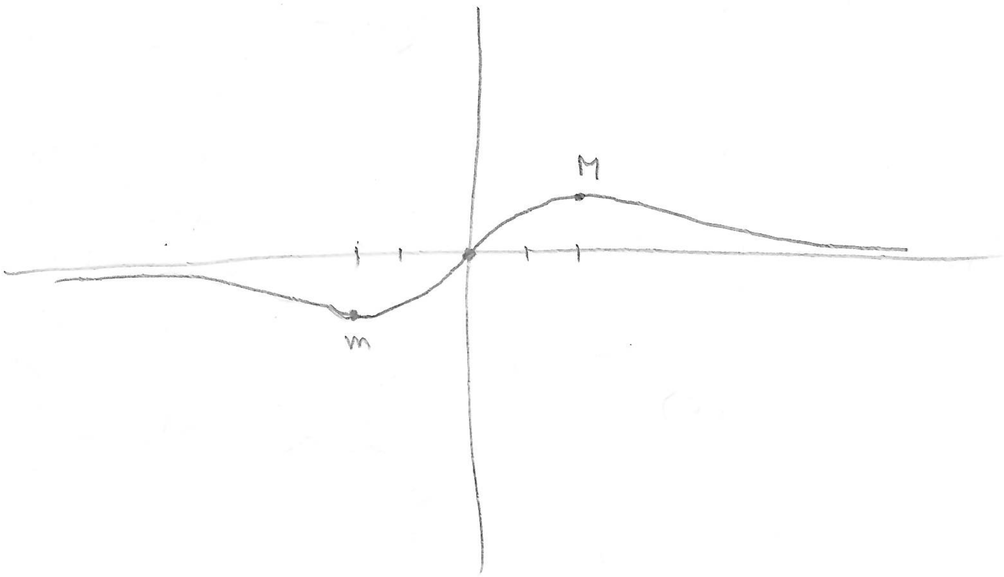
min. u $x=-2 \Rightarrow (-2,-1)$

max u $x=2 \Rightarrow (2,1)$

V.A. NEMA.

H.A. $\lim\limits_{x\to\pm\infty} \dfrac{4x \;/:x^2}{4+x^2\;/:x^2} = \lim\limits_{x\to\pm\infty} \dfrac{\frac{4}{x}}{\frac{4}{x^2}+1} = 0$

$x=0$ JE H.A. $\Rightarrow$ NEMA KOSIH



4. $\int e^x\sqrt{e^{e^x}-1}\,dx = \begin{cases} t=e^x \\ dt=e^x dx \end{cases}$

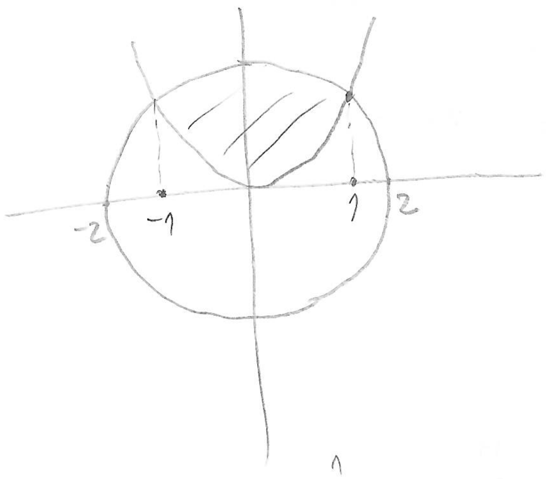
$= \int \sqrt{e^t-1}\,dt = \begin{cases} w^2=e^t-1 \\ 2w\,dw=e^t dt \Rightarrow dt=\frac{2w\,dw}{w^2+1} \end{cases}$

$= 2\int \frac{w^2}{w^2+1}dw = 2\int \frac{w^2+1}{w^2+1}dw - 2\int\frac{1}{w^2+1}$

$= 2w - 2\,\mathrm{arctg}\,w = 2\sqrt{e^t-1} - 2\,\mathrm{arctg}\sqrt{e^t-1}$

$= 2\sqrt{e^{e^x}-1} - 2\,\mathrm{arctg}\sqrt{e^{e^x}-1} + C$

5a.)



PRESJEK

$x^2+y^2=2 \qquad y=x^2$

$y+y^2=2 \Rightarrow y^2+y-2=0$

$y_{1,2}=\frac{1\pm\sqrt{1+8}}{2} = \frac{1\pm 3}{2} = 1, -$

$P = \underbrace{\int_{-1}^{1}\sqrt{2-x^2}}_{(*)} - \underbrace{\int_{-1}^{1}x^2\,dx}_{(\square)}$

$(*) = \begin{cases} x=\sqrt{2}\sin t \\ dx=\sqrt{2}\cos t\,dt \\ x=-1 \Rightarrow t=-\frac{\pi}{4} \\ x=1 \Rightarrow t=\frac{\pi}{4} \end{cases}$

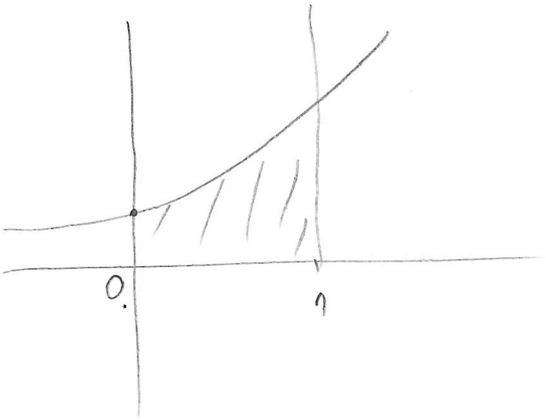
$\int_{-\frac{\pi}{4}}^{\pi/4} 2\cos^2 t\,dt = 2\int_{-\frac{\pi}{4}}^{\pi/4}\frac{1+\cos 2t}{2}dt =$

$= \int_{-\pi/4}^{\pi/4}(1+\cos 2t)dt = t\Big|_{-\frac{\pi}{4}}^{\pi/4} + \frac{1}{2}\sin t\Big|_{-\pi/4}^{\pi/4} = \frac{\pi}{2} + 1$ //

$$(\square) = \int_{-1} x^2\,dx = \frac{1}{3}x^3 \Big|_{-1} = \frac{2}{3}$$

$$P = \frac{\pi}{2} + 1 - \frac{2}{3} = \frac{\pi}{2} + \frac{1}{3}$$

5b)



$$V_x = \pi \int_0^1 e^{2x}\,dx$$

$$= \pi \cdot \frac{1}{2} e^{2x} \Big|_0^1$$

$$= \frac{\pi}{2}(e^2 - 1)$$