1. (a) **(10 bodova)** Dokažite da su vektori $\vec{a} = \vec{i} + \vec{j} + \vec{k}$, $\vec{b} = 6\vec{i} + 5\vec{j} + 4\vec{k}$ i $\vec{c} = 5\vec{i} + 4\vec{j} + 3\vec{k}$ komplanarni. Prikažite vektor $\vec{c}$ kao linearnu kombinaciju vektora $\vec{a}$ i $\vec{b}$.

   (b) **(10 bodova)** Odredite jednadžbu ravnine π određenu točkama $A(1,3,5)$, $B(2,4,6)$ i $C(7,8,9)$.

2. **(20 bodova)** Riješite sustav $\begin{cases} x_1 + x_2 - 2x_3 = 0 \\ x_1 - x_3 - x_4 = 0 \\ x_1 - x_2 - 2x_4 = 0 \\ 2x_1 - x_2 - x_3 - 3x_4 = 0 \end{cases}$.

3. **(20 bodova)** Odredite prirodnu domenu, intervale rasta i pada, ekstreme, točke infleksije, intervale konveksnosti i konkavnosti, asimptote, te skicirajte graf funkcije

$$f(x) = e^{\frac{1}{x}} - x.$$

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4. **(15 bodova)** Odredite

$$\int \ln\left(\sin x + \sqrt{\sin^2 x + 1}\right) \cos x \, dx.$$

5. (a) **(15 bodova)** Izračunajte površinu lika omeđenog parabolom $y = -x^2 + 4$, njezinom tangentom u točki s apscisom -1 i osi x. Skicirajte lik.

   (b) **(10 bodova)** Izračunajte volumen tijela koje nastaje rotacijom lika omeđenog krivuljom $y = \sin x$ i osi x na segmentu $[0, 2\pi]$, oko osi x. Skicirajte lik.

1. a) $\vec{a}, \vec{b}, \vec{c}$ komplanarni $\Leftrightarrow (\vec{a}\times\vec{b})\cdot\vec{c}=0$

$$(\vec{a}\times\vec{b})\cdot\vec{c} = \begin{vmatrix}1&1&1\\6&5&4\\5&4&3\end{vmatrix} = 1\cdot(-1)-1\cdot(-2)+1\cdot(-1)=0$$

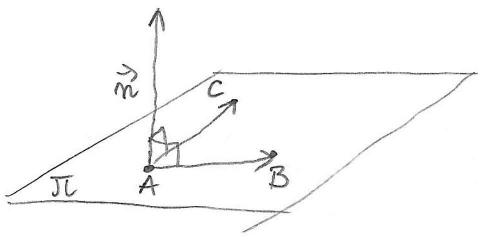
$$\vec{c} = \alpha\vec{a}+\beta\vec{b}$$

$$6\vec{i}+5\vec{j}+4\vec{k} = 5\alpha\vec{i}+4\alpha\vec{j}+3\alpha\vec{k}+\beta\vec{i}+\beta\vec{j}+\beta\vec{k}$$

$$\begin{aligned} 6 &= 5\alpha+\beta \quad / - \\ 5 &= 4\alpha+\beta \\ 4 &= 3\alpha+\beta \end{aligned}$$

$$\boxed{1=\alpha} \Rightarrow \beta = 1 \Rightarrow \vec{c}=\vec{a}+\vec{b} \,//$$

b)



$$\vec{AB} = (1,1,1)$$
$$\vec{AC} = (6,5,4)$$

$$\vec{n} = \vec{AB}\times\vec{AC} = \begin{vmatrix}\vec{i}&\vec{j}&\vec{k}\\1&1&1\\6&5&4\end{vmatrix} = \vec{i}(-1)-\vec{j}(-2)+\vec{k}(-1)$$
$$= -\vec{i}+2\vec{j}-\vec{k} = (-1,2,-1)$$

RAVNINA $\pi(\vec{n}, A) \ldots -1(x-1)+2(y-3)-1(z-5)=0$

$$\pi \ldots -x+2y-z=0 \,//$$

2. $\left[\begin{array}{cccc|c} 1 & 1 & -2 & 0 & 0 \\ 1 & 0 & -1 & -1 & 0 \\ 1 & -1 & 0 & -2 & 0 \\ 2 & -1 & -1 & -3 & 0 \end{array}\right] \begin{array}{l} \\ II-I \\ III-I \\ IV-2I \end{array} \sim \left[\begin{array}{cccc|c} 1 & 1 & -2 & 0 & 0 \\ 0 & -1 & 1 & -1 & 0 \\ 0 & -2 & 2 & -2 & 0 \\ 0 & -3 & 3 & -3 & 0 \end{array}\right] \begin{array}{l} \\ \\ III-2II \\ IV-3II \end{array} \sim$

$\sim \left[\begin{array}{cccc|c} 1 & 1 & -2 & 0 & 0 \\ 0 & -1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

$r(A) = 2 \Rightarrow 4-2 = 2$ -paramet. rj.

$x_3 = u$

$x_4 = v$

$-x_2 + u - v = 0$

$x_2 = u - v$

$x_1 + u - v - 2u = 0$

$x_1 = u + v$

Rj: $\begin{bmatrix} u+v \\ u-v \\ u \\ v \end{bmatrix} = u \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} + v \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}, \; u, v \in \mathbb{R}$

3. $f(x)=e^{\frac{1}{x}}-x$

$D_f=\mathbb{R}\setminus\{0\}$

$f'(x)=e^{\frac{1}{x}}\cdot\frac{-1}{x^2}-1=-\frac{e^{\frac{1}{x}}}{x^2}-1 \Rightarrow -\overbrace{\frac{e^{\frac{1}{x}}}{x^2}}^{>0}=1 \Rightarrow$ NEMA S.T.

$f''(x)=\frac{-e^{\frac{1}{x}}\cdot\frac{-1}{x^2}\cdot x^2+e^{\frac{1}{x}}\cdot 2x}{x^4} \Rightarrow e^{\frac{1}{x}}(1+2x)=0$

$x=-\frac{1}{2}$

	$-\infty$	$-\frac{1}{2}$	0	$+\infty$
f''	$-$	$+$	$+$	
f	$\cap$	$\cup$	$\cup$	

V.A. $\lim\limits_{x\to 0^+}\left(e^{\frac{1}{x}}-x\right)=+\infty-0=+\infty \Rightarrow$ $x=0$ JE D.V.A

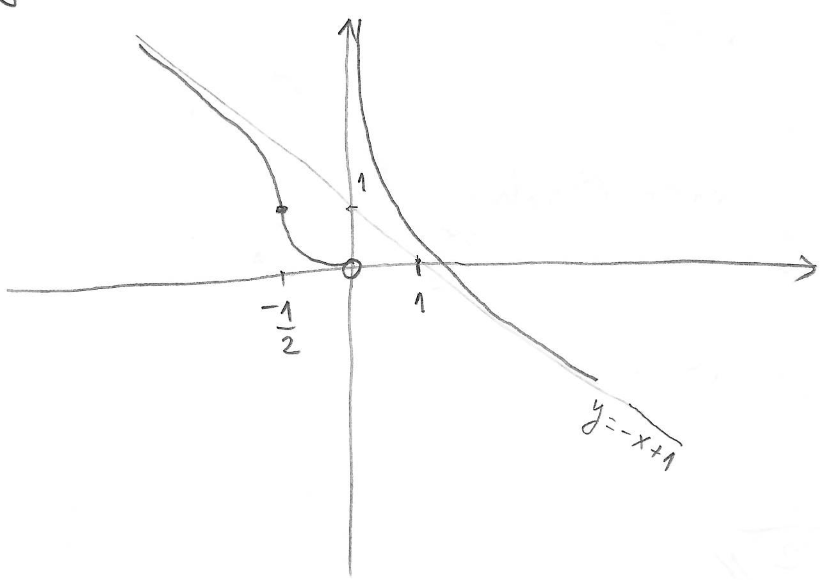
$\lim\limits_{x\to 0^-}\left(e^{\frac{1}{x}}-x\right)=0-0=0$

H.A. $\lim\limits_{x\to\pm\infty}\left(e^{\frac{1}{x}}-x\right)=\mp\infty$ NEMA H.A.

K.A. $\lim\limits_{x\to+\infty}\left(\frac{\overset{\to 1}{e^{\frac{1}{x}}}}{\underset{\to+\infty}{x}}-1\right)=-1$ $\lim\limits_{x\to-\infty}\left(\frac{\overset{\to 0}{e^{\frac{1}{x}}}}{\underset{\to-\infty}{x}}-1\right)=-1$

$\lim\limits_{x\to\pm\infty}\left(e^{\frac{1}{x}}-x+x\right)=\lim\limits_{x\to\pm\infty}e^{\frac{1}{x}}=1$

K.A. $y=-x+1$



4. $\int \ln(\sin x + \sqrt{\sin^2 x+1}) \cos x \, dx = \left\{ \begin{array}{l} t = \sin x \\ dt = \cos x \, dx \end{array} \right\}$

$= \int \ln(t+\sqrt{t^2+1}) dt = \left\{ \begin{array}{l} u = \ln(t+\sqrt{t^2+1}) \\ du = \frac{1}{t+\sqrt{t^2+1}} \cdot \left(1+\frac{1}{\cancel{2}\sqrt{t^2+1}} \cdot \cancel{2}t\right) dt \\ \quad = \frac{1}{t+\sqrt{t^2+1}} \cdot \frac{\sqrt{t^2+1}+t}{\sqrt{t^2+1}} dt \\ \quad = \frac{1}{\sqrt{t^2+1}} dt \qquad \begin{array}{l} dv = dt \\ v = t \end{array} \end{array} \right.$

$= t \cdot \ln(t+\sqrt{t^2+1}) - \int \frac{t}{\sqrt{t^2+1}} dt = \left\{ \begin{array}{l} z = t^2+1 \\ dz = 2t\,dt \Rightarrow t\,dt = \frac{dz}{2} \end{array} \right.$

$= t \cdot \ln(t+\sqrt{t^2+1}) - \frac{1}{2} \int \frac{dz}{\sqrt{z}} =$

$= t \ln(t+\sqrt{t^2+1}) - \frac{1}{2} \cdot 2\sqrt{z} + C$

$= \sin x \cdot \ln(\sin x + \sqrt{\sin^2 x+1}) - \sqrt{\sin^2 x+1} + C$ //

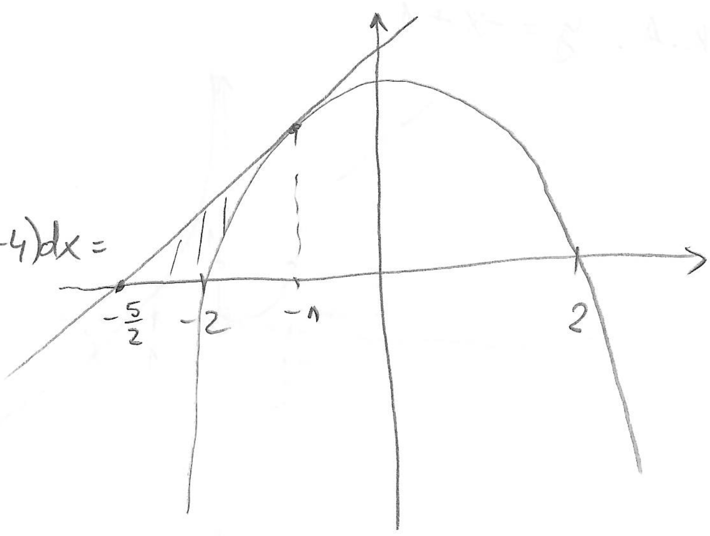
5a) $y = -x^2+4$ $\qquad x_0 = -1 \qquad y' = -2x$

$\Downarrow$

$k = y'(-1) = 2 \qquad y_0 = 3$

t... $y-3 = 2(x+1)$

$y = 2x+5$

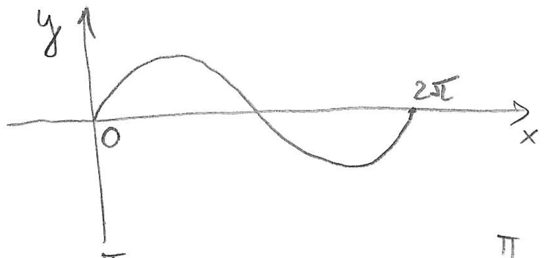


$P = \int_{-\frac{5}{2}}^{-2} (2x+5) dx + \int_{-2}^{-1} (2x+5+x^2-4) dx =$

$= (x^2+5x)\Big|_{-\frac{5}{2}}^{-2} + \left(\frac{1}{3}x^3+x^2+x\right)\Big|_{-2}^{-1}$

$= \frac{1}{4} + \frac{1}{3} = \frac{7}{12}$ //

5b)



$$V_x = 2\pi \int_0^{\pi} \sin^2 x \, dx = 2\pi \int_0^{\pi} \frac{1-\cos 2x}{2} \, dx$$

$$= \pi \left(x - \frac{1}{2} \sin 2x \right) \Big|_0^{\pi}$$

$$= \pi^2$$