1. (15 bodova) Odredite jednadžbu ravnine π koja sadrži točku $T(2,0,1)$ i usporedna je s pravcima

$$p_1 \equiv \frac{x+3}{-1} = \frac{y-4}{2} = \frac{z-2}{1} \quad \text{i} \quad p_2 \equiv \frac{x}{3} = \frac{y-1}{-2} = \frac{z+2}{4}.$$

Skicirajte.

2. (a) (10 bodova) Ispitajte konvergenciju reda

$$\sum_{n=1}^{\infty} \frac{n+2}{n \cdot 3^n}.$$

(b) (12 bodova) Odredite svojstvene vrijednosti i svojstvene vektore matrice $A = \begin{bmatrix} 0 & -2 \\ 1 & 3 \end{bmatrix}$.

3. (23 boda) Odredite prirodnu domenu, nultočke, intervale rasta i pada, ekstreme, asimptote, te skicirajte graf funkcije

$$f(x) = \ln\left(\frac{x-1}{2+x}\right).$$

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4. (18 bodova) Odredite

$$\int \frac{2\cos x}{\sin^4 x - 1}\, dx.$$

5. (a) (8 bodova) Izračunajte površinu lika omeđenog krivuljama

$$y = -x^2 + 3x + 4, \quad y = x^2 - 6x + 5, \quad x = 1 \text{ i } x = 3.$$

Skicirajte lik.

(b) (14 bodova) Izračunajte volumen tijela koje nastaje rotacijom lika omeđenog krivuljom

$$y = \ln x$$

te pravcima

$$y = 1 \text{ i } x = 3e,$$

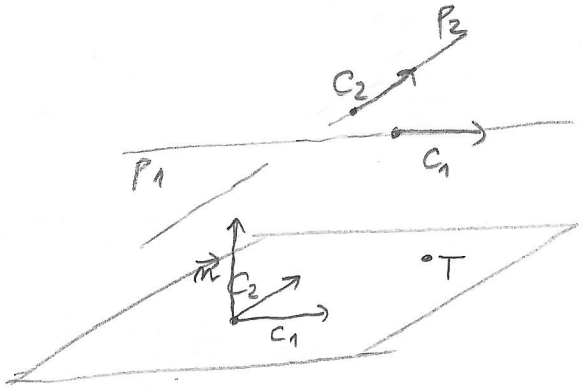
oko osi y.
Skicirajte tijelo.

1.

$$\vec{n} = \vec{c}_1 \times \vec{c}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 2 & 1 \\ 3 & -2 & 4 \end{vmatrix} =$$

$$= \vec{i}(10) - \vec{j}(-7) + \vec{k}(-4)$$

$$= (10, 7, -4)$$



$\pi(\vec{n}, T) \ldots \quad 10(x-2) + 7(y-0) - 4(z-1) = 0$

$\pi \ldots 10x + 7y - 4z - 16 = 0$

2a) $a_n = \dfrac{n+2}{n \cdot 3^n}$ D'Al.

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{n+3}{(n+1)3^{n+1}}}{\frac{n+2}{n3^n}} = \lim_{n\to\infty} \frac{n(n+3)}{3(n+1)(n+2)} = \lim_{n\to\infty} \frac{n^2+3n}{3(n^2+3n+2)} = \frac{1}{3}$$

Red konvergira prema D'Al. kriteriju.

b) $\begin{bmatrix} -\lambda & -2 \\ 1 & 3-\lambda \end{bmatrix} = -\lambda(3-\lambda) + 2 = 0$

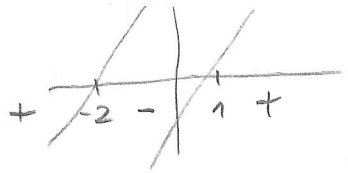
$= \lambda^2 - 3\lambda + 2$ $\qquad \lambda_{1,2} = \dfrac{3 \pm \sqrt{9-8}}{2} = 2, 1$

$\lambda = 2$ $\quad \begin{bmatrix} -2 & -2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \quad x_1 + x_2 = 0 \Rightarrow v = \begin{bmatrix} -t \\ t \end{bmatrix}, t \neq 0$

$\lambda = 1$ $\quad \begin{bmatrix} -1 & -2 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad x_1 + 2x_2 = 0 \Rightarrow v = \begin{bmatrix} -2t \\ t \end{bmatrix}, t \neq 0$

3.) DOMENA:

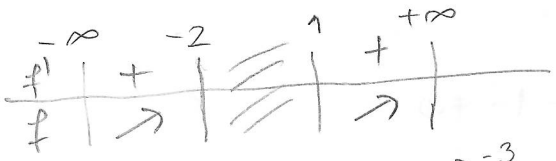
$\frac{x-1}{x+2} > 0$



$D_f = \langle -\infty, -2 \rangle \cup \langle 1, +\infty \rangle$

N.T. $\ln\left(\frac{x-1}{x+2}\right) = 0$ $\quad \frac{x-1}{x+2} = 1 \Rightarrow$ NEMA N.T.

$f'(x) = \frac{x+2}{x-1} \cdot \frac{x+2-x+1}{(x+2)^2} = \frac{3}{(x-1)(x+2)}$ NEMA S.T.



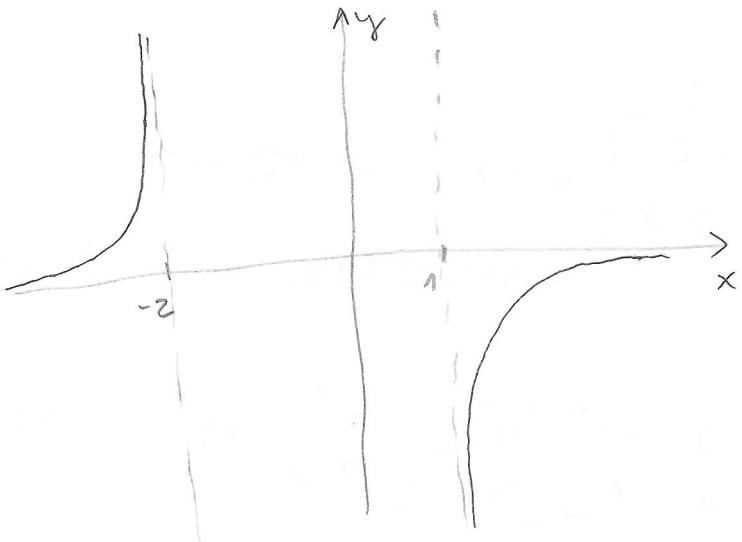
$\Rightarrow$ RASTE NA CIJELOJ DOMENI

V.A. $\lim_{x \to -2^-} \ln \underbrace{\frac{x-1}{x+2}}_{\to 0^-} = +\infty \Rightarrow x = -2$ JE L.V.A.

$\lim_{x \to 1^+} \ln \underbrace{\frac{x-1}{x+2}}_{\to 3} = -\infty \Rightarrow x = 1$ JE D.V.A

H.A. $\lim_{x \to \pm\infty} \ln \frac{x+2}{x-1} = \ln 1 = 0 \Rightarrow y = 0$ JE H.A.

$\Rightarrow$ NEMA KOSIH



4. $\int \frac{2\cos x}{\sin^4 x - 1}\,dx = \left\{\begin{matrix} t = \sin x \\ dt = \cos x\,dx \end{matrix}\right\} = 2\int \frac{dt}{t^4-1} = 2\int \frac{dt}{(t-1)(t+1)(t^2+1)} = (*)$

$$\frac{1}{(t-1)(t+1)(t^2+1)} = \frac{A}{t-1} + \frac{B}{t+1} + \frac{Ct+D}{t^2+1} \quad / \cdot (t-1)(t+1)(t^2+1)$$

$$1 = A(t^3+t^2+t+1) + B(t^3-t^2+t-1) + \underbrace{(Ct+D)(t^2-1)}_{Ct^3-Ct+Dt^2-D}$$

$$1 = t^3(A+B+C) + t^2(A-B+D) + t(A+B-C) + (A-B-D)$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 1 & 0 \\ 1 & 1 & -1 & 0 & 0 \\ 1 & -1 & 0 & -1 & 1 \end{array}\right] \begin{matrix} \\ \text{II}-\text{I} \\ \text{III}-\text{I} \\ \text{IV}-\text{I} \end{matrix} \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 0 \\ 0 & -2 & -1 & 1 & 0 \\ 0 & 0 & -2 & 0 & 0 \\ 0 & -2 & -1 & -1 & 1 \end{array}\right] \begin{matrix} \\ \\ 1:(-2) \\ \text{IV}-\text{II} \end{matrix} \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 0 \\ 0 & -2 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -2 & 1 \end{array}\right]$$

$-2D = 1 \Rightarrow \quad D = \frac{-1}{2} \qquad C = 0$

$-2B - \frac{1}{2} = 0 \Rightarrow \quad B = \frac{-1}{4}$

$A - \frac{1}{4} = 0 \Rightarrow \quad A = \frac{1}{4}$

$$(*) = 2\cdot\frac{1}{4}\int \frac{dt}{t-1} - 2\cdot\frac{1}{4}\int \frac{dt}{t+1} - 2\cdot\frac{1}{2}\int \frac{dt}{t^2+1} =$$

$$= \frac{1}{2}\ln|t-1| - \frac{1}{2}\ln|t+1| - \operatorname{arctg} t + C$$

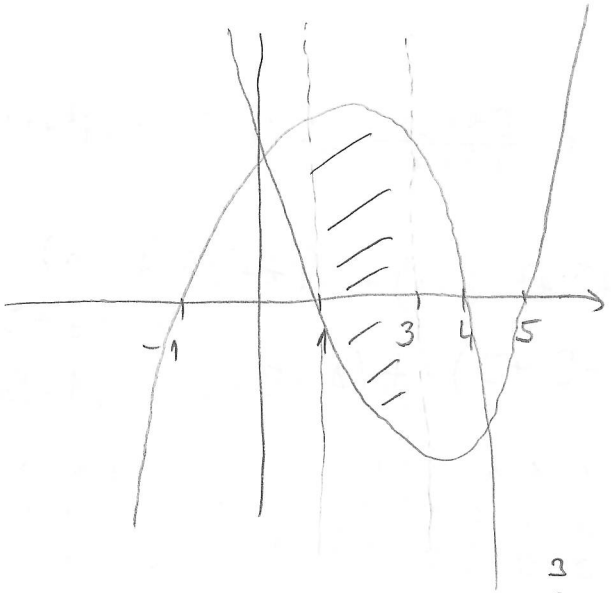
$$= \frac{1}{2}\ln|\sin x - 1| - \frac{1}{2}\ln|\sin x + 1| - \operatorname{arctg}(\sin x) + C$$

5a) $-x^2+3x+4=0$

$x_{1,2}=\frac{-3\pm\sqrt{9+16}}{-2}=4,-1$

$x^2-6x+5=0$

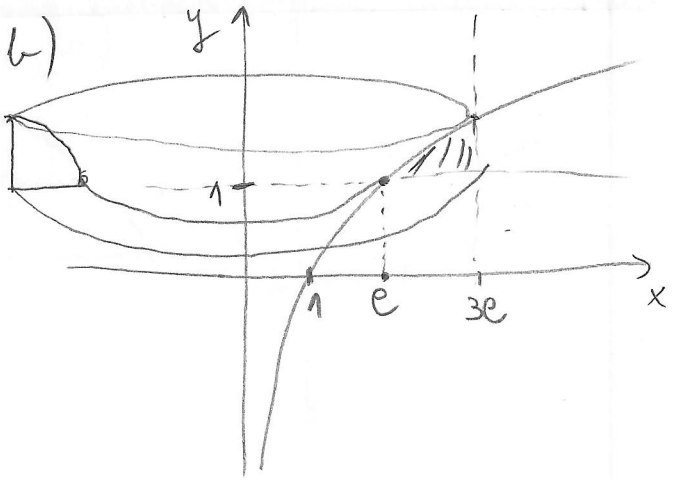
$x_{1,2}=\frac{6\pm\sqrt{36-20}}{2}=1,5$



$$P=\int_1^3(-x^2+3x+4-x^2+6x-5)dx=\int_1^3(-2x^2+9x-1)dx=\left(-\frac{2}{3}x^3+\frac{9}{2}x^2-x\right)\Big|_1^3$$

$$=\left(-18+\frac{81}{2}-3\right)-\left(-\frac{2}{3}+\frac{9}{2}-1\right)=\frac{50}{3}$$

b)



$$V_y=\int_e^{3e}2\pi(x\ln x)\,dx=\begin{cases}u=\ln x & x\,dx=dv\\ du=\frac{1}{x}dx & v=\frac{1}{2}x^2\end{cases}$$

$$=2\pi\left(\frac{1}{2}x^2\ln x\Big|_e^{3e}-\frac{1}{2}\int_e^{3e}x\,dx\right)$$

$$=\pi\left(9e^2\ln(3e)-e^2-\frac{1}{2}x^2\Big|_e^{3e}\right)$$

$$=\pi\left(9e^2\ln 3e-e^2-\frac{1}{2}\cdot(9e^2-e^2)\right)$$

$$=\pi\cdot e^2(9\ln 3e-5)$$