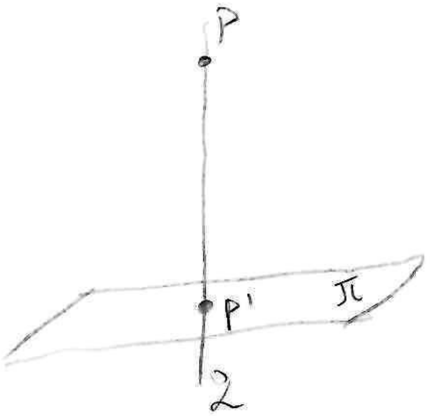
1a) $\sphericalangle(p_1,p_2) = \sphericalangle(\vec{c_1},\vec{c_2})$ $\quad \vec{c_1} = (2,-2,1)$, $\vec{c_2} = (0,-1,1)$

$$\cos\varphi = \frac{\vec{c_1}\,\vec{c_2}}{|\vec{c_1}||\vec{c_2}|} = \frac{2+1}{\sqrt{9}\cdot\sqrt{2}} = \frac{\sqrt{2}}{2} \Rightarrow \varphi = \frac{\pi}{4}$$

b)



$q \perp \pi,\ P \in q \Rightarrow q \equiv \frac{x-1}{2} = \frac{y-3}{-1} = \frac{z-4}{-1}$

$$q \equiv \begin{cases} x = 2t+1 \\ y = -t+3 \\ z = -t+4 \end{cases}$$

$q \cap \pi = P'$ $\quad 2(2t+1) - (-t+3) - (-t+4) = 7$

$6t = 12$

$t = 2 \Rightarrow P'(5,1,2)$

$$d(P,P') = \sqrt{(5-1)^2+(1-3)^2+(2-4)^2} = \sqrt{24} = 2\sqrt{6}$$

2.

$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & -3 & 1 \\ 2 & -1 & -1 & 0 & 2 \\ 1 & 2 & 3 & 1 & 3 \\ 0 & 1 & 1 & 1 & -1 \end{array}\right] \begin{array}{l} \\ II-2I \\ III-I \\ \\ \end{array} \sim \left[\begin{array}{cccc|c} 1 & 0 & -1 & -3 & 1 \\ 0 & -1 & 1 & 6 & 0 \\ 0 & 2 & 4 & 4 & 2 \\ 0 & 1 & 1 & 1 & -1 \end{array}\right] \begin{array}{l} \\ \\ III+2II \\ IV+II \end{array} \sim$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & -1 & -3 & 1 \\ 0 & -1 & 1 & 6 & 0 \\ 0 & 0 & 6 & 16 & 2 \\ 0 & 0 & 2 & 7 & -1 \end{array}\right] \begin{array}{l} \\ \\ \\ 3\cdot IV-III \end{array} \sim \left[\begin{array}{cccc|c} 1 & 0 & -1 & -3 & 1 \\ 0 & -1 & 1 & 6 & 0 \\ 0 & 0 & 6 & 16 & 2 \\ 0 & 0 & 0 & 5 & -5 \end{array}\right]$$

$r(A) = r(A_p) = n = 4$ JEDINSTVENO RJEŠENJE

$5x_4 = -5 \quad x_4 = -1$

$6x_3 = 18 \quad x_3 = 3$

$-x_2 + 3 - 6 = 0$

$x_2 = -3$

$x_1 - 3 + 3 = 1$

$x_1 = 1$

RJ $\begin{bmatrix} 1 \\ -3 \\ 3 \\ -1 \end{bmatrix}$

3. $f(x) = \dfrac{1}{x^2 - x - 2} + \dfrac{1}{2}$

DOMENA: Nazivnik $\neq 0$ $\Rightarrow$ Ne smije biti $x^2 - x - 2 = 0$

$\Leftrightarrow x_{1,2} = \dfrac{1 \pm \sqrt{1 - 4(-2)}}{2}$ $\boxed{\begin{matrix} x_1 = -1 \\ x_2 = 2 \end{matrix}}$

$D_f = \mathbb{R} \setminus \{-1, 2\}$

NULTOČKE: $f(x) = 0 \Leftrightarrow \dfrac{1}{x^2 - x - 2} + \dfrac{1}{2} = 0 \quad / \cdot 2(x^2 - x - 2)$

$\cancel{2} + x^2 - x - \cancel{2} = 0$

$x(x-1) = 0 \Rightarrow \left.\begin{matrix} x_1 = 0 \\ x_2 = 1 \end{matrix}\right\}$ NULTOČKE

EKSTREMI, INTERVALI RASTA/PADA $\qquad \left(\dfrac{1}{g}\right)' = -\dfrac{g'}{g^2}$

$f'(x) = -\dfrac{2x - 1}{(x^2 - x - 2)^2} = 0$

$\Rightarrow \boxed{x = \dfrac{1}{2}} \leftarrow$ EKSTREM

$f\left(\dfrac{1}{2}\right) = \dfrac{1}{\frac{1}{4} - \frac{1}{2} - 2} + \dfrac{1}{2} = \dfrac{1}{\frac{1 - 2 - 8}{4}} + \dfrac{1}{2} = -\dfrac{4}{9} + \dfrac{1}{2} \approx 0.06$

$f'(x) = \dfrac{1 - 2x}{(x^2 - x - 2)^2}$

	$-\infty$ … -1	-1 … $\frac{1}{2}$	$\frac{1}{2}$ … 2	2 … $+\infty$
f'	+	+	−	−
f	↗	↗	↘	↘
			M	

Interval rasta je $\langle -\infty, -1 \rangle \cup \langle -1, \frac{1}{2} \rangle$, a pada $\langle \frac{1}{2}, 2 \rangle \cup \langle 2, +\infty \rangle$

ASIMPTOTE:

Vertikalne: $\lim\limits_{x \to -1} \left(\dfrac{1}{x^2 - x - 2} + \dfrac{1}{2} \right) = "\dfrac{1}{0}" + \dfrac{1}{2} = \infty$

$\lim\limits_{x \to 2} \left(\dfrac{1}{x^2 - x - 2} + \dfrac{1}{2} \right) = "\dfrac{1}{0}" + \dfrac{1}{2} = \infty$

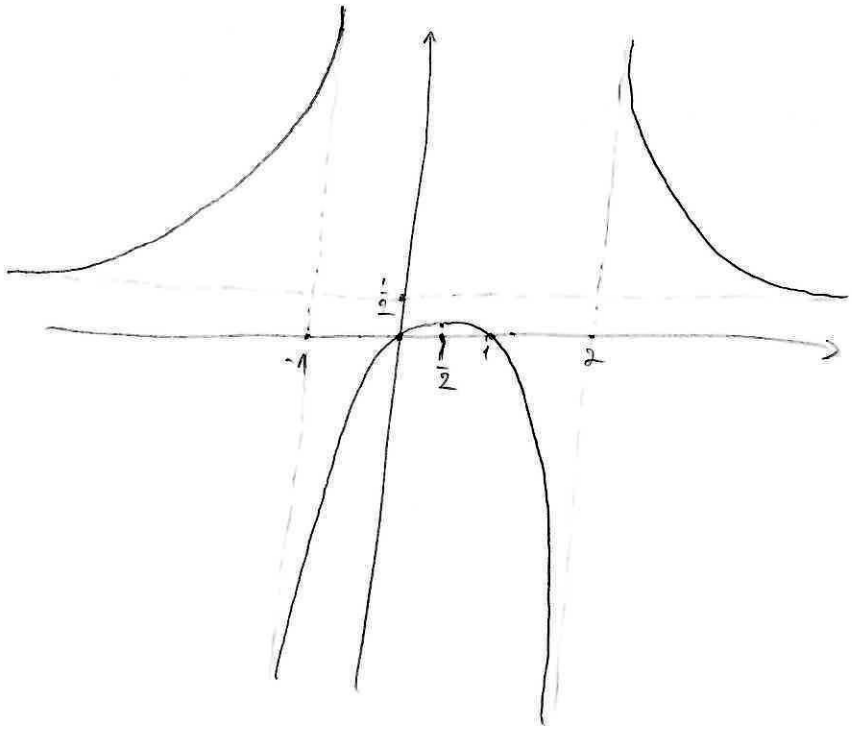
$\Rightarrow$ Pravci $x = -1$, $x = 2$ su vertikalne asimptote.

Horizontalne asimptote:

$$\lim_{x\to\pm\infty}\left(\frac{1}{x^2-x-2}+\frac{1}{2}\right)=\left(\frac{1}{+\infty}+\frac{1}{2}\right)=0+\frac{1}{2}=\frac{1}{2}$$

$\Rightarrow$ Pravac $y=\frac{1}{2}$ je horizontalna asimptota (ujedno i kosa).

Skica



$$4)\int \frac{\sin^3 x \cos x \, dx}{(\sin^2 x-2)(\sin^4 x+2)} = \begin{cases} t = \sin^2 x \, dx \\ dt = 2\sin x \cos x \, dx \end{cases}$$

$$\cancel{\int \frac{1+\sin x-\sin^2 x}{\sin x(\sin^2 x+1)} \cos x \, dx}$$

$$= \frac{1}{2}\int \frac{t \, dt}{(t-2)(t^2+2)} = (*)$$

$$\frac{t}{(t-2)(t^2+2)} = \frac{A}{t-2} + \frac{Bt+C}{t^2+2}$$

$$t = A(t^2+2) + (Bt+C)(t-2)$$

$$t = t^2(A+B) + t(-2B+C) + 2A-2C$$

$$\begin{aligned} A+B&=0 \\ -2B+C&=1 \\ 2A-2C&=0 \end{aligned} \qquad A=-B=C=1/3$$

$$(*) \quad \frac{1}{2}\int \frac{\frac{1}{3}dt}{t-2} - \frac{1}{2}\int \frac{\frac{1}{3}t\,dt}{t^2+2} + \frac{1}{2}\int \frac{\frac{1}{3}dt}{t^2+2} = \begin{cases} w = t^2+2 \\ dw = 2t\,dt \end{cases}$$

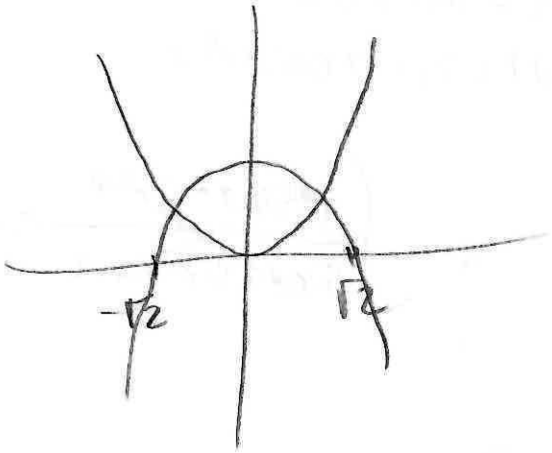
$$= \frac{1}{6}\ln|t-2| - \frac{1}{12}\int \frac{dw}{w} + \frac{1}{6}\cdot\frac{1}{\sqrt{2}}\operatorname{arctg}\frac{t}{\sqrt{2}}$$

$$= \frac{1}{6}\ln|t-2| - \frac{1}{12}\ln|t^2+2| + \frac{1}{6\sqrt{2}}\operatorname{arctg}\frac{t}{\sqrt{2}}$$

$$= \frac{1}{6}\ln|\sin^2 x-2| - \frac{1}{12}\ln|\sin^4 x+2| + \frac{1}{6\sqrt{2}}\operatorname{arctg}\frac{\sin^2 x}{\sqrt{2}} + C$$

5

5.a)



$y = x^2$

$y = 2 - x^2$

$x^4 = 2 - x^2$

$x^4 + x^2 - 2 = 0 \qquad t = x^2$

$t^2 + t - 2 = 0$

$t_{1,2} = \dfrac{-1 \pm \sqrt{1+8}}{2} = -2, 1$

$x^2 \neq -2 \qquad x^2 = 1 \Rightarrow \qquad x = \pm 1$

$$P = \int_{-1}^{1} (2 - x^2 - x^4)\, dx = \left(2x - \frac{1}{3}x^3 - \frac{1}{5}x^5\right)\Bigg|_{-1}^{1}$$

$$= \left(2 - \frac{1}{3} - \frac{1}{5}\right) - \left(-2 + \frac{1}{3} + \frac{1}{5}\right)$$

$$= 4 - \frac{2}{3} - \frac{2}{5} = \frac{60 - 10 - 6}{15} = \frac{44}{15}$$

5b)

$$V_x = \pi \int_a^b f^2(x)\,dx$$

$$V_x = \pi \int_0^{\pi/2} \sin^2 x \cos^2 x \, dx = \frac{1}{4}\pi \int_0^{\pi/2} \sin^2 2x \, dx =$$

$$= \frac{1}{4}\pi \int_0^{\pi/2} \frac{1-\cos 4x}{2} \, dx = \frac{1}{8}\pi \int_0^{\pi/2} (1-\cos 4x)\,dx$$

$$= \frac{1}{8}\pi \left(x - \frac{1}{4}\sin 4x\right)\Big|_0^{\pi/2} = \frac{1}{8}\pi \left(\pi/2 - \frac{1}{4}\cdot 0\right)$$

$$= \frac{\pi^2}{16}$$