Matematika 1 - pismeni ispit

18. 6. 2025.

Ime i prezime: _____

1.	2.	3.	4.	5.	$\sum$

Ocjena pismenog ispita: _____

1. a) (10 bodova) Nađite kut između pravaca

$$p_1 \ldots \frac{x-1}{-1} = \frac{y+4}{-4} = \frac{z-2}{1} \quad \text{i} \quad p_2 \ldots \frac{x}{4} = \frac{y}{-5} = \frac{z-2}{11}.$$

b) (10 bodova) Odredite presjek dviju ravnina

$$\pi_1 \ldots x + y + z = 1 \quad \text{i} \quad \pi_2 \ldots 2x - y + 3z = 4.$$

2. a) (13 bodova) Nađite svojstvene vrijednosti matrice

$$A = \begin{bmatrix} 3 & 0 & -4 \\ 0 & 3 & 4 \\ 0 & 0 & 2 \end{bmatrix}$$

i odredite svojstvene vektore za najveću svojstvenu vrijednost.

b) (7 bodova) Ispitajte konvegenciju reda

$$\sum_{n=1}^{\infty} \frac{n^2+1}{2^n}.$$

3. (20 bodova) Odredite domenu, nultočke, točke ekstrema, intervale rasta i pada, asimptote te skicirajte graf funkcije

$$f(x) = \frac{x^2+x+1}{x+1}.$$

4. (15 bodova) Izračunajte

$$\int \frac{e^x+1}{e^{2x}+1} dx.$$

5. a) (12 bodova) Odredite površinu lika omeđenog krivuljama $y = \frac{x^2-4}{3}$ i $y = x + 2$.

b) (13 bodova) Odredite volumen tijela koje nastaje rotacijom lika omeđenog krivuljom $y = e^{-x}$, koordinatnim osima i pravcem $x = 1$ oko y-osi. Skicirajte.

1. a) $\vec{c_1} = (-1,-4,1)$ i $\vec{c_2} = (4,-5,11)$

$$\cos\varphi = \frac{\vec{c_1}\vec{c_2}}{|\vec{c_1}||\vec{c_2}|} = \frac{-4+20+11}{\sqrt{18}\cdot\sqrt{162}} = \frac{27}{\sqrt{18}\cdot\sqrt{162}} = \frac{1}{2}$$

$9\cdot 2 \quad 81\cdot 2$

$$\varphi = \frac{\pi}{3}$$

b) $\pi_1 \cap \pi_2 = p = (T, \vec{c})$

$$\vec{c} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 2 & -1 & 3 \end{vmatrix} = \vec{i}(3+1) - \vec{j}(3-2) + \vec{k}(-1-2)$$

$$= 4\vec{i} - \vec{j} - 3\vec{k}$$

$z = 0$

$$\begin{matrix} x+y=1 \\ 2x-y=4 \end{matrix} \quad /+$$

$3x = 5 \quad x = \frac{5}{3} \quad y = -\frac{2}{3}$

$$p \equiv \frac{x-\frac{5}{3}}{4} = \frac{y+\frac{2}{3}}{-1} = \frac{z}{-3} \;//$$

2. $$\det(A-\lambda I) = \begin{vmatrix} 3-\lambda & 0 & -4 \\ 0 & 3-\lambda & 4 \\ 0 & 0 & 2-\lambda \end{vmatrix} = (3-\lambda)^2(2-\lambda)$$

$\lambda_{1,2} = 3, \lambda_3 = 2$ //

$\lambda = 3$

$$A - \lambda I = \left[\begin{array}{ccc|c} 0 & 0 & -4 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & -1 & 0 \end{array}\right]$$

$z = 0 \quad x = u \quad y = v$

$$v = \begin{bmatrix} u \\ v \\ 0 \end{bmatrix}, u, v \in \mathbb{R} \setminus \{0\}$$

b) D'Al.

$$\lim_{n\to+\infty}\frac{a_{n+1}}{a_n}=\frac{\frac{n^2+2n+2}{2^{n+1}}}{\frac{n^2+1}{2^n}}=\lim_{n\to+\infty}\frac{n^2+2n+2}{2n^2+2}=\frac{1}{2}$$

Red konvergira.

3. $D_f=\mathbb{R}\setminus\{-1\}$ N.T. $x^2+x+1=0$

$$x_{1,2}=\frac{-1\pm\sqrt{1-4}}{2}\notin\mathbb{R}$$

Nema realnih n.t.

$$f'(x)=\frac{(2x+1)(x+1)-(x^2+x+1)}{(x+1)^2}=\frac{x^2+2x}{(x+1)^2}$$

S.T. $x=0$ $x=-2$

	$-\infty$		-2		-1		0		$+\infty$
f'		$+$	0	$-$	$\nexists$	$-$	0	$+$	
f		↗	M	↘		↘	m	↗	

M(-2, -3)

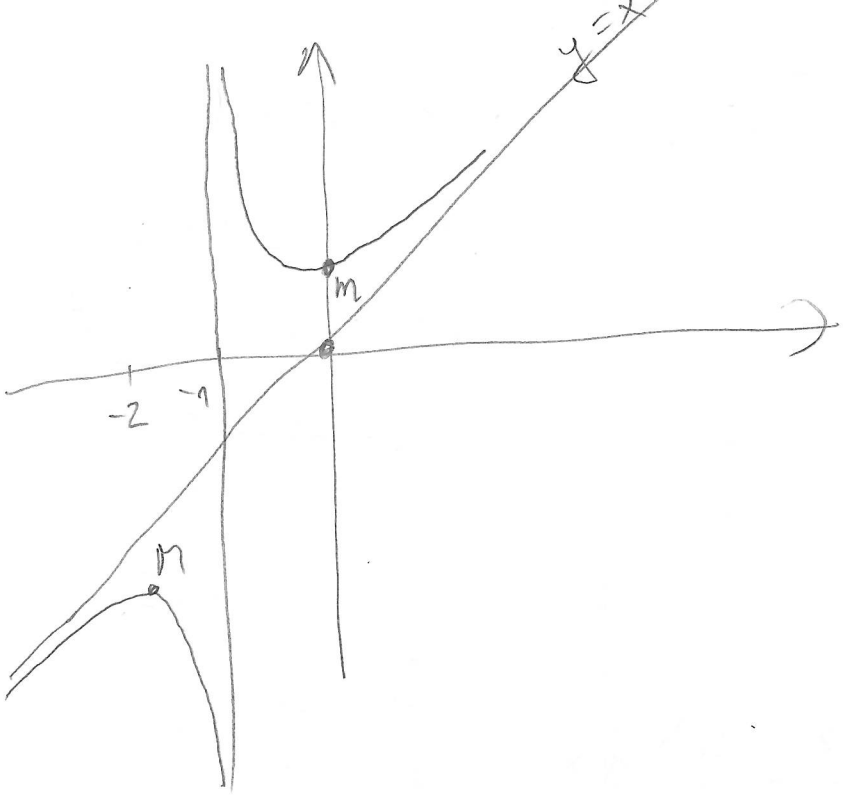
m(0, 1)

$$\lim_{x\to-1^-}\frac{x^2+x+1}{x+1}=\frac{1}{0^-}=-\infty \qquad \lim_{x\to-1^+}\frac{x^2+x+1}{x+1}=+\infty$$

K.A. $\lim\limits_{x\to\pm\infty}\frac{f(x)}{x}=1$

$$l=\lim_{x\to\pm\infty}\left(\frac{x^2+x+1}{x+1}-\frac{x^2+x}{x+1}\right)=\lim_{x\to\pm\infty}\frac{-1}{x+1}=0$$

K.A $y=x$ //



4. $\int \frac{e^x+1}{e^{2x}+1}\,dx = \left\{ \begin{array}{l} t = e^x \\ dt = e^x dx \Rightarrow dx = \frac{dt}{t} \end{array} \right\}$

$= \int \frac{t+1}{(t^2+1)\,t} \cdot dt =$

$\frac{t+1}{(t^2+1)(t)} = \frac{At+B}{t^2+1} + \frac{C}{t}$

$t+1 = At^2 + Bt + Ct^2 + C$

$A + C = 0$

$B = 1$

$C = 1 \qquad A = -1$

$= \int \frac{-t+1}{t^2+1}\,dt + \int \frac{dt}{t}$

$= -\int \frac{t}{t^2+1}\,dt + \int \frac{dt}{t^2+1} + \int \frac{dt}{t} =$

$\left\{ \begin{array}{l} u = t^2+1 \\ du = 2t\,dt \Rightarrow t\,dt = \frac{1}{2}du \end{array} \right.$

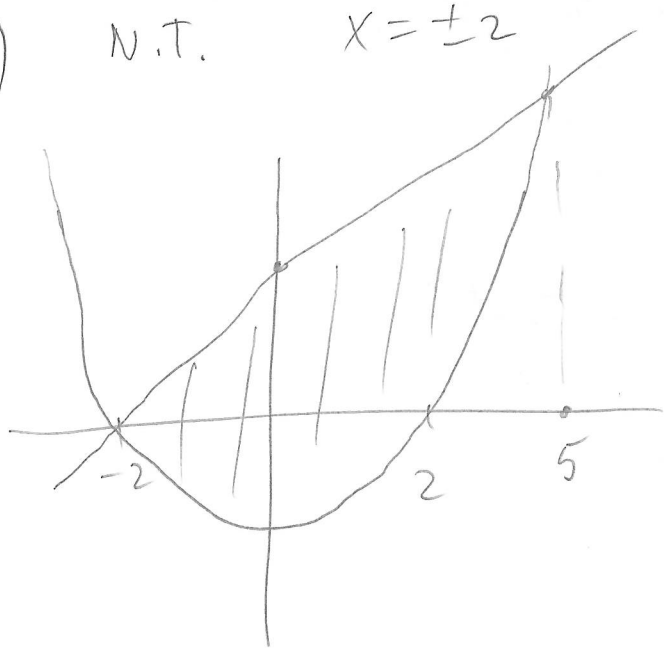
$= -\frac{1}{2}\int \frac{du}{u} + \int \frac{dt}{t^2+1} + \int \frac{dt}{t} =$

$= -\frac{1}{2}\ln|t^2+1| + \operatorname{arctg} t + \ln|t|$

$= -\frac{1}{2}\ln|e^x+1| + \operatorname{arctg} e^x + \ln e^x + C$

x

5a) N.T. $x = \pm 2$



Sjecišta

$x^2 - 4 = 3x + 6$

$x^2 - 3x - 10 = 0$

$x_{1,2} = \frac{3 \pm \sqrt{9+40}}{2} = \frac{3 \pm 7}{2}$

$= -2{,}5$

$$P = \int_{-2}^{5} \left(x + 2 - \frac{x^2}{9} + \frac{4}{3}\right) dx = \int_{-2}^{5} \left(-\frac{x^2}{3} + x + \frac{10}{3}\right) dx$$

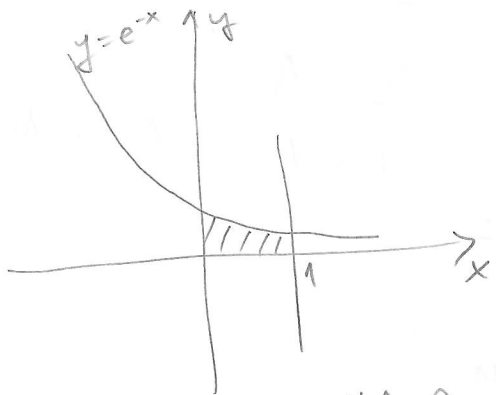
$\frac{75}{12}$

$$= \left(-\frac{x^3}{9} + \frac{x^2}{2} + \frac{10}{3}x\right)\Big|_{-2}^{5} = \frac{-125}{9} + \frac{25}{2} + \frac{50}{3} - \frac{8}{9} - 2 + \frac{2}{3}$$

$\frac{21}{2}$

$$= \frac{343}{18}$$

b)



$$V_y = 2\pi \int_0^1 x f(x)\, dx$$

$$= 2\pi \int_0^1 x e^{-x} dx =$$

$$= \left\{ \begin{matrix} u = x & dv = e^{-x} dx \\ du = dx & v = -e^{-x} \end{matrix} \right\} = \left(-xe^{-x}\Big|_0^1 + \int_0^1 e^{-x} dx\right) 2\pi$$

$$= \left(e^{-1} - e^{-x}\Big|_0^1\right) 2\pi$$

$$= \left(2e^{-1} + 1\right) 2\pi$$