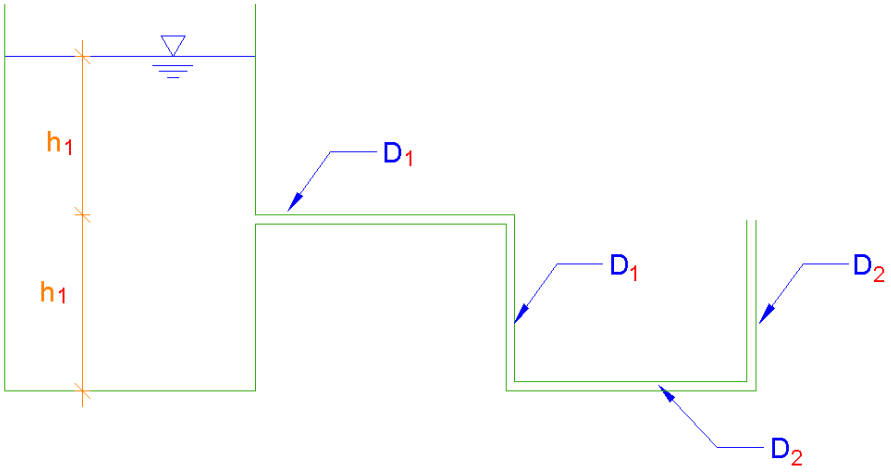
BJ IDEALNA TEKUĆINA

1. Za sistem prema slici (slučaj a i b) potrebno je odrediti protok Q, minimalni tlak u cjevovodu te nacrtati energetsku i piezometarsku liniju.
   Zadano je: $\rho = 1000$ kg/m^3; $D_1 = 0{,}2$ m; $D_2 = 0{,}3$ m; $h_1 = 2$ m (a); $h_1 = 1$ m (b)

   a)



$$v_2 = \sqrt{2gh_1} = \sqrt{2 \cdot 9{,}81 \cdot 2} = 6{,}26\ m/s$$

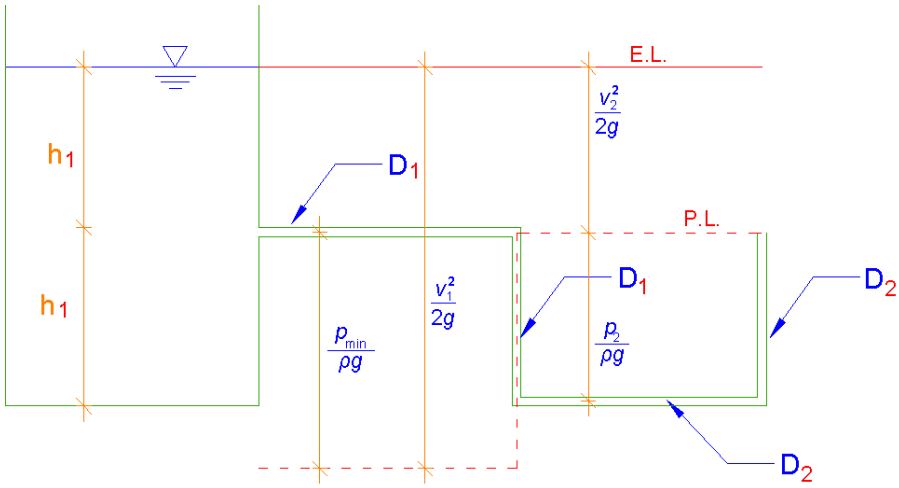
$$Q = v_2 \cdot \frac{D_2^{\,2}\pi}{4} = 6{,}26 \cdot \frac{0{,}3^2\pi}{4} = 0{,}44\ m^3/s$$

$$v_1 = \frac{Q}{A_1} = \frac{Q}{\frac{D_1^2\pi}{4}} = \frac{4Q}{D_1^2\pi} = \frac{4 \cdot 0{,}44}{0{,}2^2\pi} = 14{,}09\ m/s$$

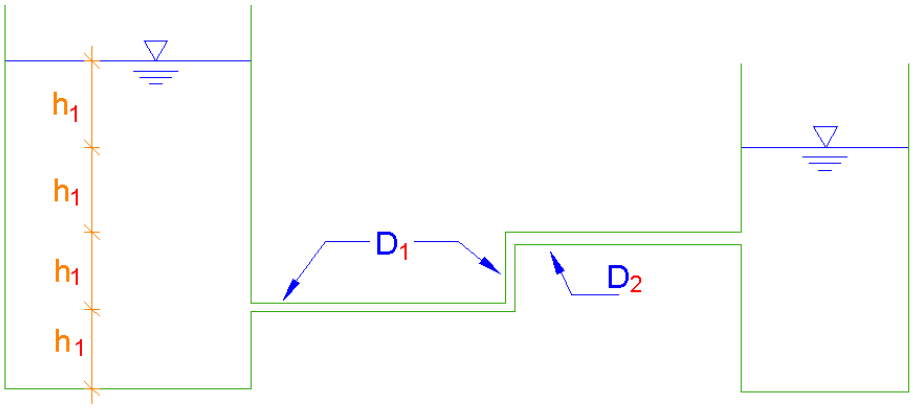
$$\frac{v_1^2}{2g} = \frac{14{,}09^2}{2g} = 10{,}12\ m \quad (\textit{kinetička energija u cijevi 1 izražena u visinskom obliku})$$

$$\frac{p_1}{\rho g} = \frac{p_{\min}}{\rho g} = h_1 - \frac{v_1^2}{2g} = 2 - 10{,}12 = -8{,}12\ m$$

$$p_{\min} = -8{,}12 \cdot \rho \cdot g - 79{,}66\ kPa$$



b)



$$v_2 = \sqrt{2gh_1} = \sqrt{2 \cdot 9,81 \cdot 1} = 4,43\ m/s$$

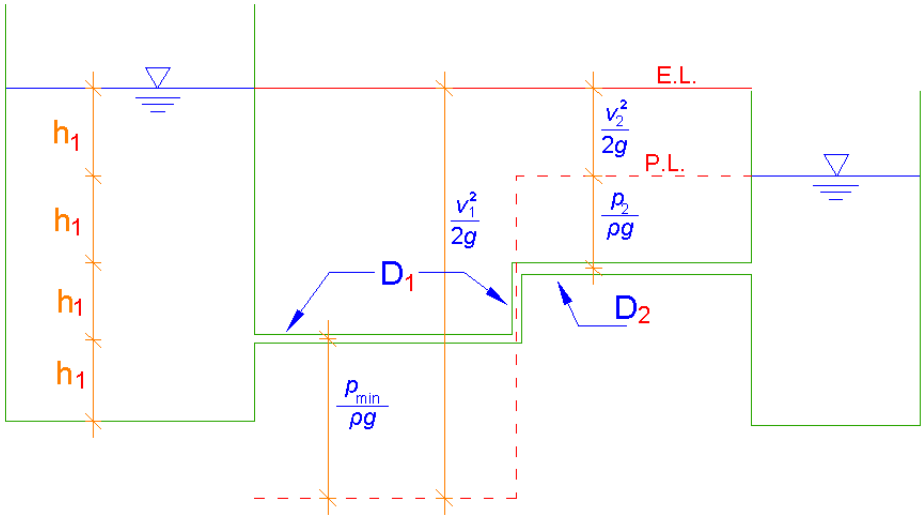
$$Q = v_2 \cdot A_2 = v_2 \cdot \frac{D_2^2 \pi}{4} = 4,43 \cdot \frac{0,3^2 \pi}{4} = 0,31\ m^3/s$$

$$v_1 = \frac{Q}{A_1} = \frac{4Q}{D_1^2 \pi} = \frac{4 \cdot 0,31}{0,2^2 \pi} = 9,97\ m/s$$

$$\frac{v_1^2}{2g} = 5,06\ m$$

$$\frac{p_{\min}}{\rho g} = 3 - 5,06 = -2,06\ m$$

$$p_{\min} = -20,21\ kPa$$

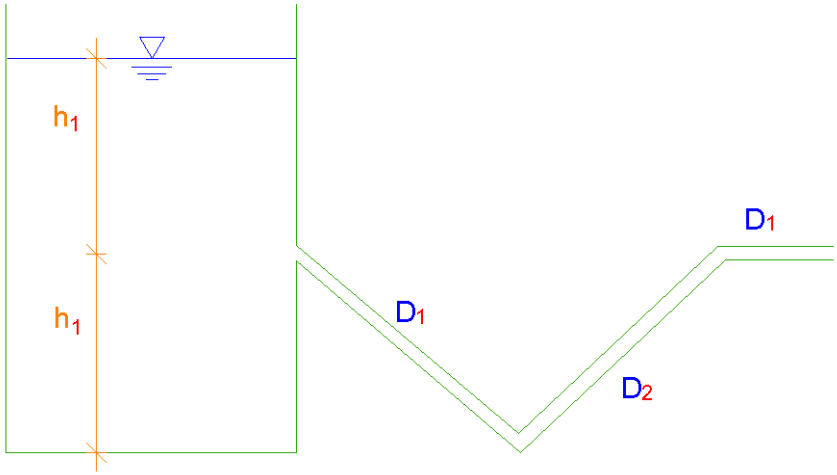


Napomena: u slučaju a) vrijednost kinetičke energije i potencijalne energije tlaka na dionici s promjerom D_1, nisu nacrtane u mjerilu u odnosu na zadanu vrijednost h_1.

2. Potrebno je odrediti protok idealne tekućine Q, brzine v_1 i v_2 te minimalni tlak p_{min} za sustav prikazan na slici. Nacrtati energetsku i piezometarsku liniju za cjevovod na slici (slučaj a i b).

   Zadano je: $\rho = 1000$ kg/m^3; $D_1 = 0{,}5$ m; $D_2 = 0{,}4$ m; $h_1 = 2$ m

   a)



$$v_1 = \sqrt{2gh_1} = \sqrt{2 \cdot 9{,}81 \cdot 2} = 6{,}26\ m/s$$

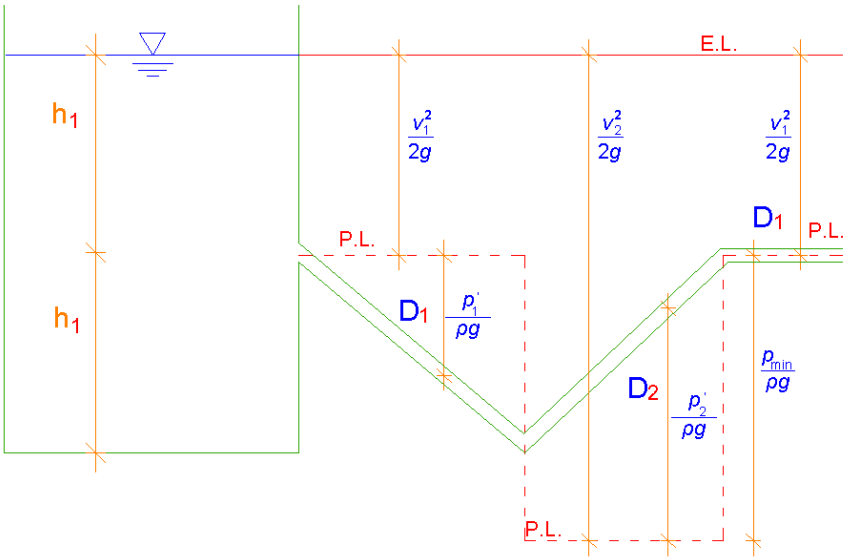
$$Q = v_1 \cdot \frac{D_1^2 \pi}{4} = 1{,}23\ m^3/s$$

$$v_2 = \frac{4Q}{D_2^2 \pi} = 9{,}78\ m/s$$

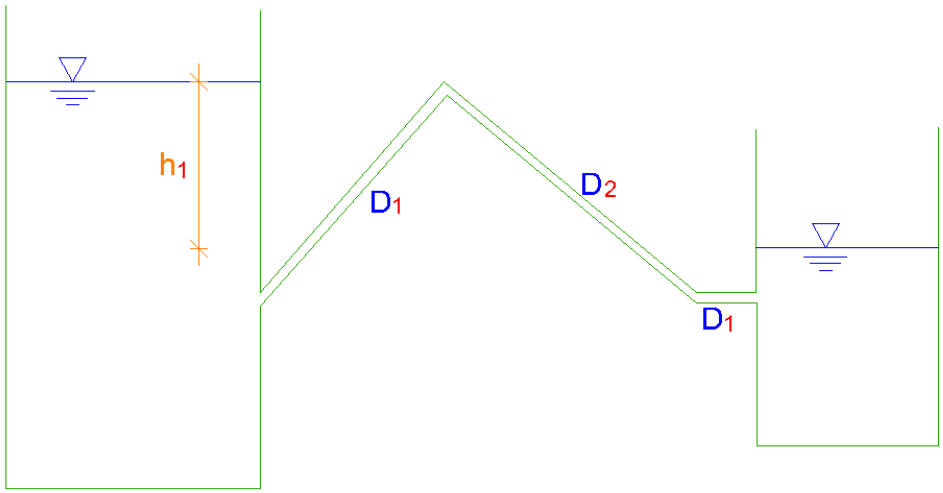
$$\frac{v_2^2}{2g} = 4{,}88\ m$$

$$\frac{p_{\min}}{\rho g} = 2 - 4{,}88 = -2{,}88\ m$$

$$p_{\min} = -28{,}25\ kPa$$



b)



$$v_1 = \sqrt{2gh_1} = \sqrt{2 \cdot 9,81 \cdot 2} = 6,26\ m/s$$

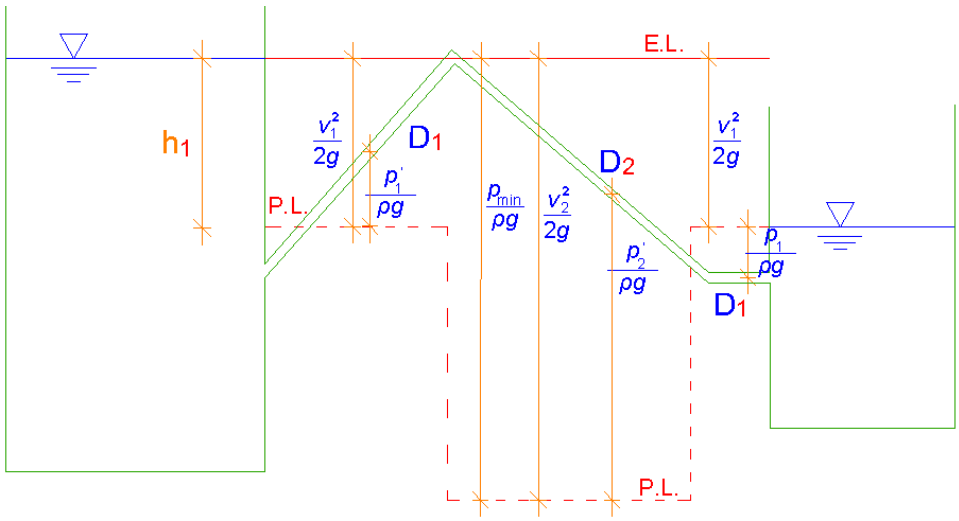
$$Q = v_1 \cdot \frac{D_1^2 \pi}{4} = 1,23\ m^3/s$$

$$v_2 = \frac{4Q}{D_2^2 \pi} = 9,78\ m/s$$

$$\frac{v_2^{\ 2}}{2g} = 4,88\ m$$

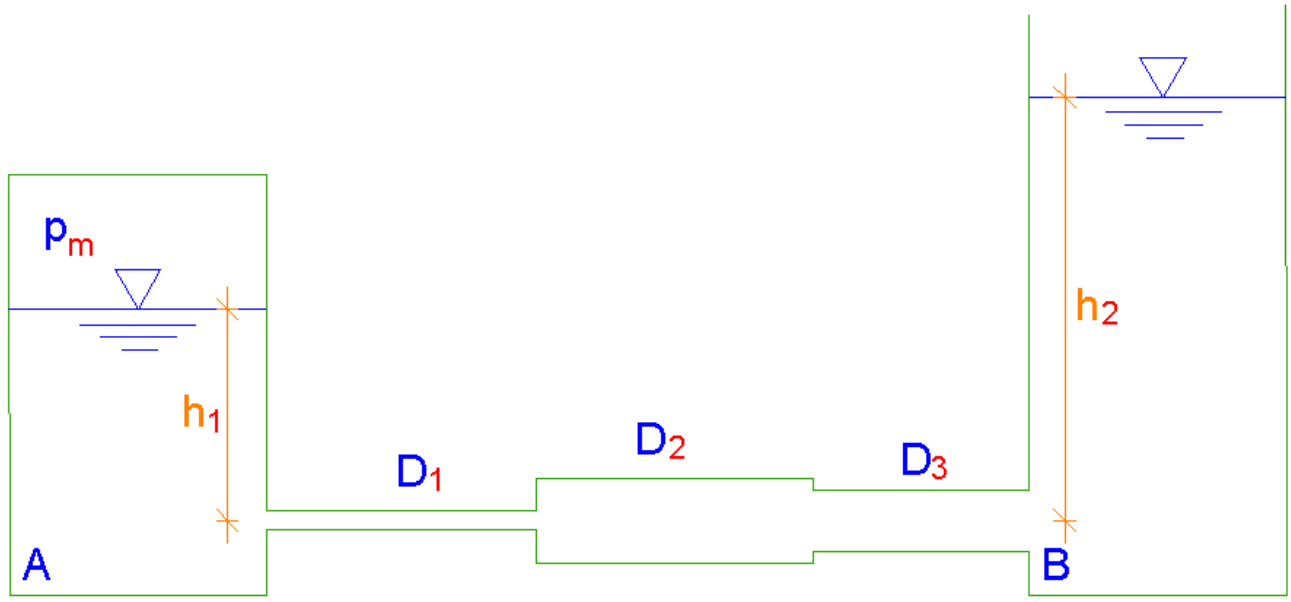
$$\frac{p_{\min}}{\rho g} = -4,88\ m$$

$$p_{\min} = -47,87\ kPa$$



3. Za stacionarno istjecanje iz komore *A* u komoru *B* potrebno je odrediti protok *Q*, brzine v_1, v_2 i v_3 koje se postižu u djelovima cjevovoda različitih promjera, te pripadne tlakove p_1, p_2 i p_3. Tekućina je idealna i gustoće ρ = 1000 kg/m^3. Nacrtati piezometarsku i energetsku liniju, te označiti područja s negativnim i pozitivnim vrijednostima tlakova.

Zadano: $p_m = 29{,}43$ kPa; $D_1 = 110$ mm; $D_2 = 200$ mm; $D_3 = 150$ mm; $h_1 = 1$ m; $h_2 = 2$ m

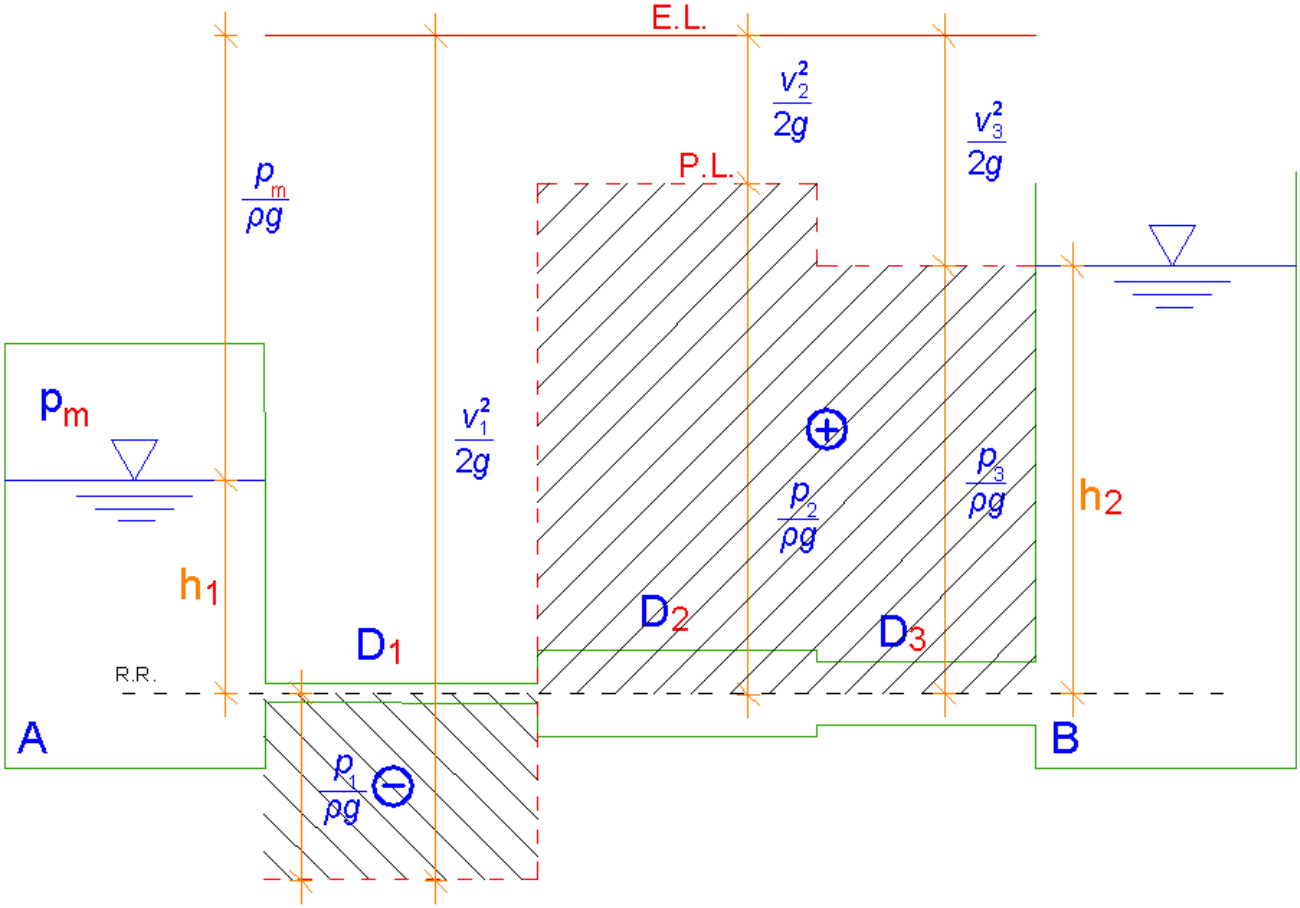


$$v_3 = \sqrt{2g\left[\left(h_1 + \frac{p_m}{\rho g}\right) - h_2\right]} = \sqrt{2g\left[\left(1 + \frac{29{,}43}{1 \cdot g}\right) - 2\right]} = 6{,}26\ m/s \qquad \frac{v_3^2}{2g} = 2{,}0\ m$$

$$Q = v_3 \cdot A_3 = v_3 \cdot \frac{D_3^2 \cdot \pi}{4} = 0{,}11\ m^3/s$$

$$v_2 = \frac{Q}{A_2} = \frac{4Q}{D_2^2 \pi} = 3{,}50\ m/s \qquad \frac{v_2^2}{2g} = 0{,}62\ m$$

$$v_1 = \frac{Q}{A_1} = \frac{4Q}{D_1^2 \pi} = 11{,}57\ m/s \qquad \frac{v_1^2}{2g} = 6{,}82\ m$$

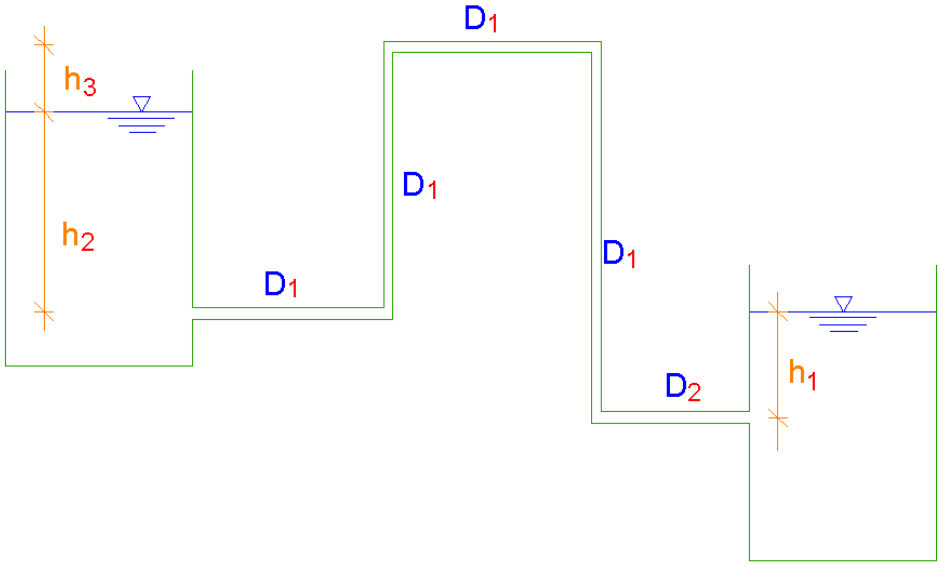


Napomena: nisu svi elementi energetske i piezometarske linije nacrtani u mjerilu u odnosu na zadane vrijednosti.

U slučaju da je geometrijska linija (os cjevovoda) iznad piezometarske linije, to je područje negativnog tlaka (podtlaka).

4. Potrebno je odrediti protok kroz cjevovod sa slike, te vrijednost i mjesto minimalnog tlaka uz pretpostavku strujanja idealne tekućine. Nacrtati energetsku i piezometarsku liniju te naznačiti područja negativnog i pozitivnog tlaka u sustavu.

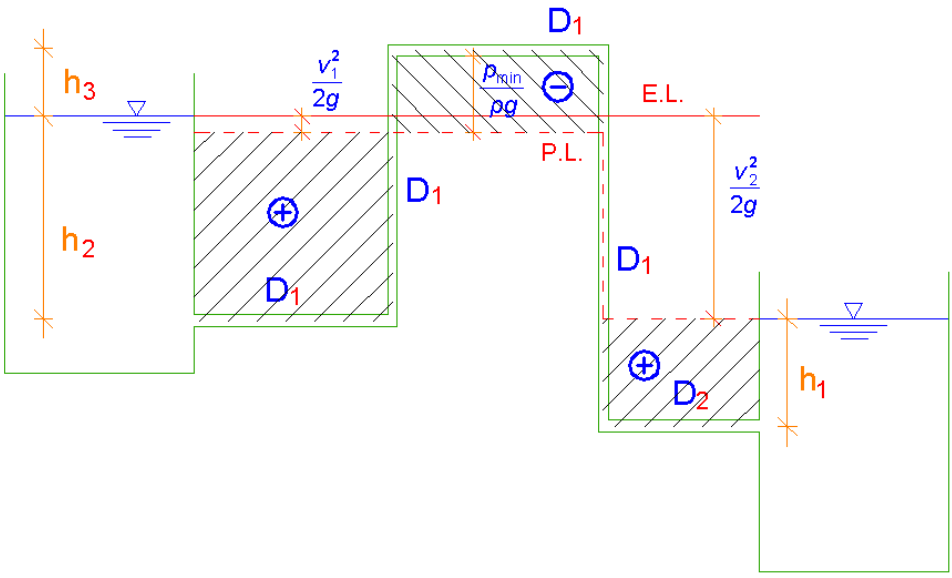
Zadano je: $\rho = 1000\ \text{kg/m}^3$; $D_1 = 0{,}2\ \text{m}$; $D_2 = 0{,}1\ \text{m}$; $h_1 = 2\ \text{m}$; $h_2 = 4\ \text{m}$; $h_3 = 1\ \text{m}$



$$v_2 = \sqrt{2g \cdot h_2} = \sqrt{2g \cdot 4} = 8{,}86\ m/s \qquad \frac{v_2^2}{2g} = 4{,}0\ m$$

$$Q = A_2 \cdot v_2 = \frac{D_2^2 \pi}{4} \cdot v_2 = \frac{0{,}1^2 \pi}{4} \cdot 8{,}86 = 0{,}07\ m^3/s$$

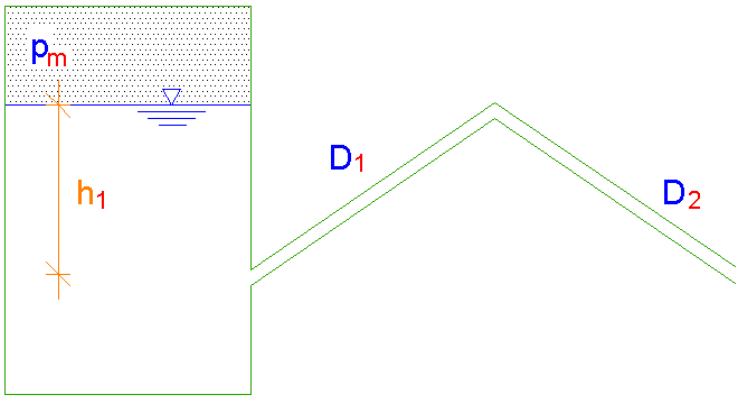
$$v_1 = \frac{Q}{A_1} = \frac{4 \cdot 0{,}07}{D_1^2 \pi} = 2{,}21\ m/s \qquad \frac{v_1^2}{2g} = 0{,}25\ m$$



$$p_{\min} = -\rho g \left(h_3 + \frac{v_1^2}{2g}\right) = -12{,}26\ kPa$$

5. Potrebno je odrediti protok, brzine v_1 i v_2 kroz cjevovod sa slike, te vrijednost i položaj minimalnog tlaka uz pretpostavku strujanja idealne tekućine. Nacrtati energetsku i piezometarsku liniju.

Zadano je: $\rho = 1000$ kg/m^3; $p_m = 9{,}81$ kPa; $h_1 = 1$ m; $D_1 = 0{,}25$ m; $D_2 = 0{,}3$ m

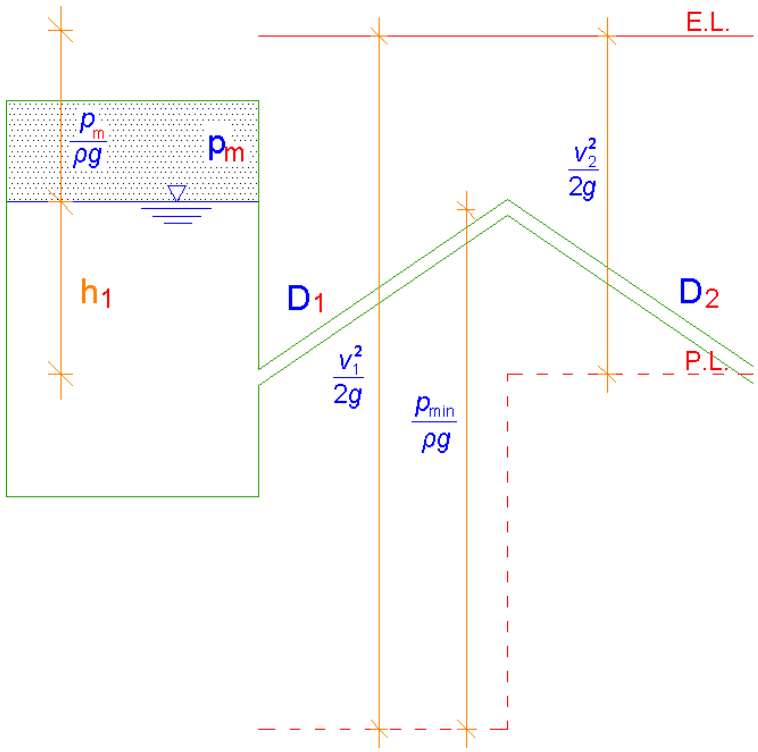


$$A_1 = \frac{D_1^2\pi}{4} = \frac{0{,}25^2\pi}{4} = 0{,}049\ m^2 \qquad A_2 = \frac{D_2^2\pi}{4} = \frac{0{,}3^2\pi}{4} = 0{,}071\ m^2$$

$$v_2 = \sqrt{2g\cdot\left(h_1 + \frac{p_m}{\rho g}\right)} = \sqrt{2g\cdot\left(1 + \frac{9{,}81}{1\cdot g}\right)} = 6{,}26\ m/s$$

$$Q = v_2 \cdot A_2 = 6{,}26\cdot 0{,}071 = 0{,}444\ m^3/s$$

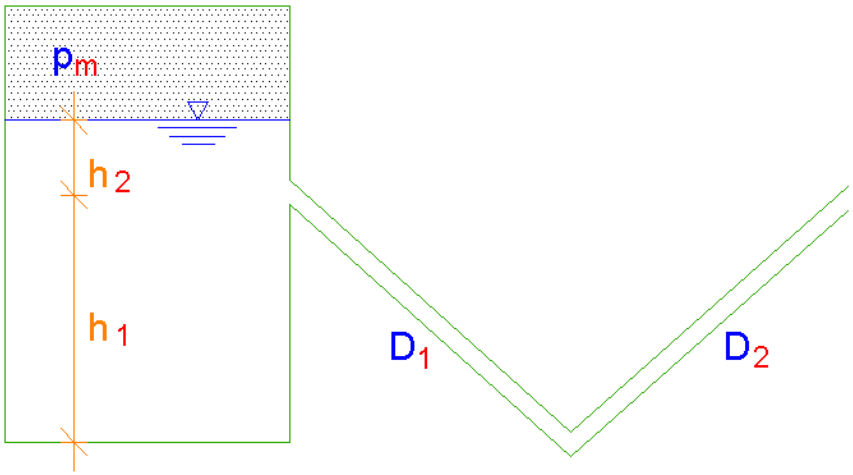
$$v_1 = \frac{Q}{A_1} - \frac{0{,}444}{0{,}049} = 9{,}06\ m/s \qquad \frac{v_1^2}{2g} = 4{,}18\ m$$



$$\frac{p_{\min}}{\rho g} = \frac{v_1^2}{2g} - \frac{p_m}{\rho g} = 4{,}18 - 1 = 3{,}18\ m \qquad \rightarrow \qquad p_{\min} = -\rho g\cdot 3{,}18 = -31{,}2\ kPa$$

6. Potrebno je odrediti protok, brzine v_1 i v_2 kroz cjevovod sa slike, te vrijednost i položaj mjesta maksimalnog tlaka uz predpostavku strujanja idealne tekućine. Nacrtati energetsku i piezometarsku liniju.

Zadano je: $\rho = 1000$ kg/m^3; $p_m = 9{,}81$ kPa; $h_1 = 3$ m; $h_2 = 1$ m; $D_1 = 0{,}3$ m; $D_2 = 0{,}25$ m

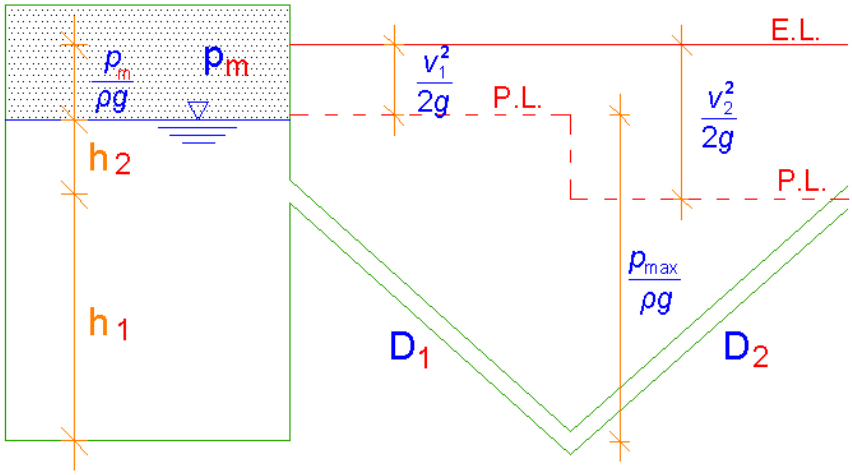


$$A_1 = \frac{D_1^2\pi}{4} = \frac{0{,}3^2\pi}{4} = 0{,}071\,m^2 \qquad A_2 = \frac{D_2^2\pi}{4} = \frac{0{,}25^2\pi}{4} = 0{,}049\,m^2$$

$$v_2 = \sqrt{2g\cdot\left(h_2 + \frac{p_m}{\rho g}\right)} = \sqrt{2g\cdot\left(1 + \frac{9{,}81}{1\cdot g}\right)} = 6{,}26\,m/s$$

$$Q = v_2 \cdot A_2 = 6{,}26\cdot 0{,}049 = 0{,}307\,m^3/s$$

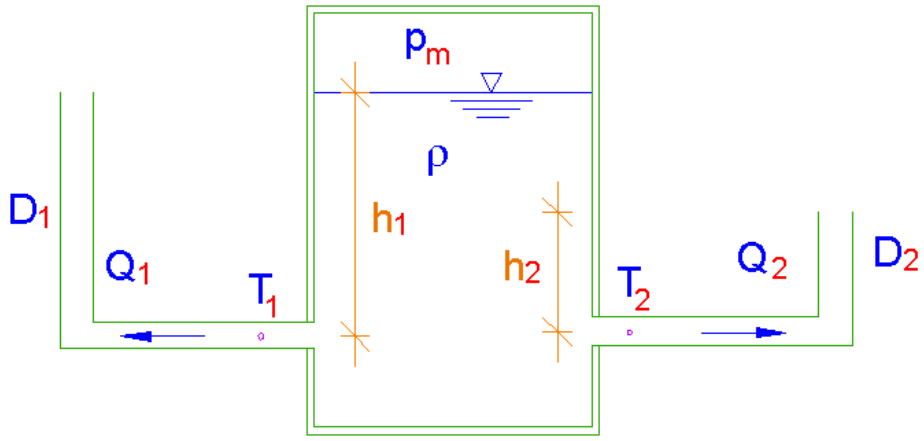
$$v_1 = \frac{Q}{A_1} = 4{,}32\,m/s \qquad \frac{v_1^2}{2g} = 0{,}95\,m$$



$$p_{\max} = \rho g\left(h_1 + h_2 + \frac{p_m}{\rho g} - \frac{v_1^2}{2g}\right) = \rho g\,(5 - 0{,}95) = 39{,}7\,kPa$$

7. Potrebno je odrediti odnos promjera lijeve (D_1) i promjera desne (D_2) grane cjevovoda priključenog na komoru sa slike, a da bi se omogućio jednaki istjecajni protok idealne tekućine iz obje grane ($Q_1 = Q_2$). Potrebno je izračunati tlakove u točkama T_1 i T_2 te nacrtati energetsku i piezometarsku liniju za obje grane cjevovoda.

Zadano je: $\rho = 1000\ \text{kg/m}^3$; $p_m = 9{,}81\ \text{kPa}$; $h_1 = 2\ \text{m}$; $h_2 = 1\ \text{m}$



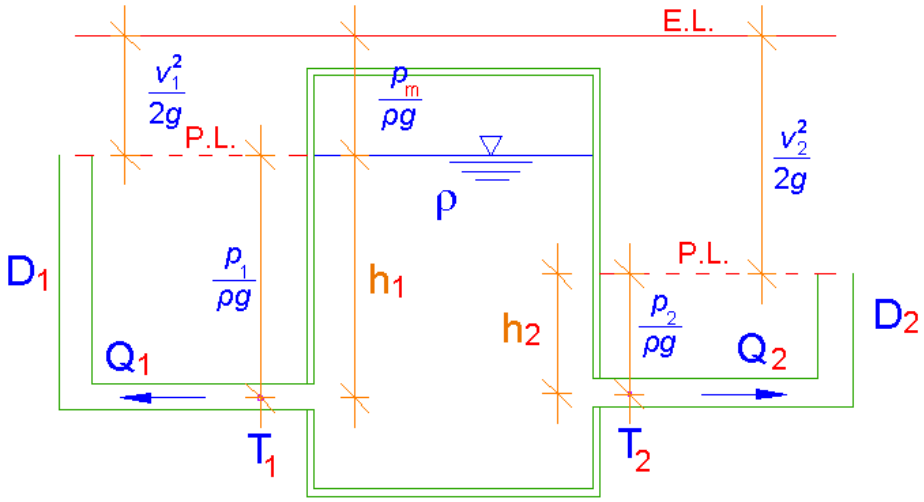
$$Q_1 = Q_2$$

$$v_1 \cdot \frac{D_1^2\pi}{4} = v_2 \cdot \frac{D_2^2\pi}{4}$$

$$\sqrt{2g \cdot \frac{p_m}{\rho g}} \cdot \frac{D_1^2\pi}{4} = \sqrt{2g \cdot \left(\frac{p_m}{\rho g} + (h_2 - h_1)\right)} \cdot \frac{D_2^2\pi}{4}$$

$$\sqrt{2g \cdot 1} \cdot \frac{D_1^2\pi}{4} = \sqrt{2g \cdot 2} \cdot \frac{D_2^2\pi}{4}$$

$$D_1^2 = \sqrt{2} \cdot D_2^2 \quad \rightarrow \quad \frac{D_1}{D_2} = \sqrt[4]{2} = 1{,}19$$

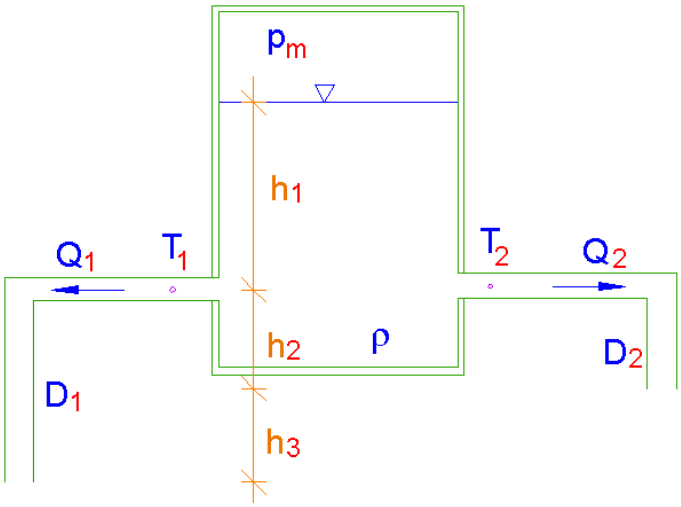


$$\frac{p_1}{\rho g} = h_1 = 2\ m \quad \rightarrow \quad p_1 = 19{,}62\ kPa$$

$$\frac{p_2}{\rho g} = h_2 = 1\ m \quad \rightarrow \quad p_2 = 9{,}81\ kPa$$

8. Potrebno je odrediti odnos promjera lijeve (D_1) i promjera desne (D_2) grane cjevovoda priključenog na komoru sa slike, a da bi se omogućio jednaki istjecajni protok idealne tekućine iz obje grane ($Q_1 = Q_2$). Potrebno je izračunati tlakove u točkama T_1 i T_2 te nacrtati energetsku, piezometarsku i geometrijsku liniju za obje grane cjevovoda.

Zadano je: $\rho = 1000$ kg/m^3; $p_m = -9{,}81$ kPa; $h_1 = 2$ m; $h_2 = 1$ m; $h_3 = 1$ m



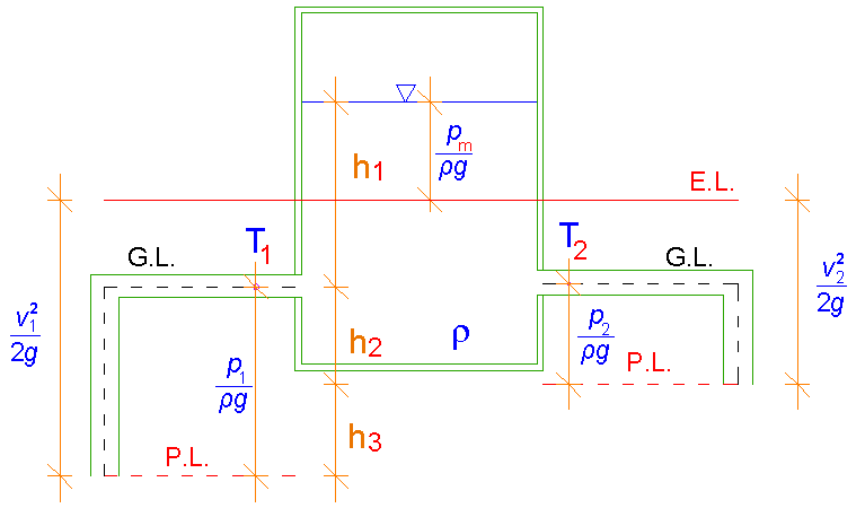
$$Q_1 = Q_2$$

$$v_1 \cdot \frac{D_1^2\pi}{4} = v_2 \cdot \frac{D_2^2\pi}{4}$$

$$\sqrt{2g \cdot \left((h_1 + h_2 + h_3) - \frac{p_m}{\rho g} \right)} \cdot \frac{D_1^2\pi}{4} = \sqrt{2g \cdot \left((h_1 + h_2) - \frac{p_m}{\rho g} \right)} \cdot \frac{D_2^2\pi}{4}$$

$$\sqrt{2g \cdot 3} \cdot \frac{D_1^2\pi}{4} = \sqrt{2g \cdot 2} \cdot \frac{D_2^2\pi}{4}$$

$$D_1^2 = \sqrt{\frac{2}{3}} \cdot D_2^2 \quad \rightarrow \quad \frac{D_1}{D_2} = \sqrt[4]{\frac{2}{3}} = 0{,}9$$

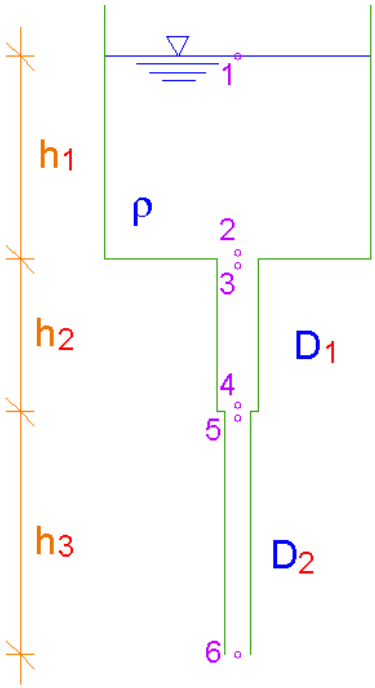


$$\frac{p_1}{\rho g} = -2 \text{ m} \quad \rightarrow \quad p_1 = -19{,}62 \ kPa$$

$$\frac{p_2}{\rho g} = -1 \text{ m} \quad \rightarrow \quad p_2 = -9{,}81 \ kPa$$

9. Potrebno je odrediti protok kroz vertikalni cjevovod na slici te brzine v_1 i v_2 u cijevima različitog promjera D_1 i D_2. Nacrtati geometrijsku, piezometarsku i energetsku liniju. Odrediti vrijednosti potencijalne energije položaja, potencijalne energije tlaka i kinetičke energije u točkama 1-6 izražene u visinskom obliku.

Zadano je: $\rho = 1000\ \text{kg/m}^3$; $h_1 = 2$ m; $h_2 = 1{,}5$ m; $h_3 = 2{,}5$ m; $D_1 = 0{,}2$ m; $D_2 = 0{,}15$ m



$$v_{D2} = \sqrt{2g \cdot (h_1 + h_2 + h_3)} = \sqrt{2g \cdot (2 + 1{,}5 + 2{,}5)} = 10{,}85\ m/s$$

$$v_{D1} \cdot \frac{D_1^2 \pi}{4} = v_{D2} \cdot \frac{D_2^2 \pi}{4} \qquad \rightarrow \qquad v_{D1} = v_{D2} \cdot \frac{D_2^2}{D_1^2} = 10{,}85 \cdot \frac{0{,}15^2}{0{,}2^2} = 6{,}1\ m/s$$

$z_1 = h_1 + h_2 + h_3 = 6\ m$ $\quad$ (*referentna ravnina je postavljena u točku* 6)

$\frac{p_1}{\rho g} = 0\ m$ $\quad$ (*atmosferskom tlaku u tehničkim proračunima pridružujemo vrijednost* 0)

$\frac{v_1^2}{2g} = 0\ m$ $\quad$ (*brzina se uzima u obzir tek u cjevovdu – pretpostavlja se da nema spuštanja vodnog lica*)

$z_2 = h_2 + h_3 = 4\ m$

$\frac{p_2}{\rho g} = h_1 = 2\ m$

$\frac{v_2^2}{2g} = 0\ m$

$$z_3 = z_2 = 4\,m$$

$$\frac{p_3}{\rho g} = \frac{p_2}{\rho g} - \frac{v_{D1}^2}{2g} = 2 - \frac{6,1^2}{2g} = 2 - 1,90 = 0.10\,m$$

$$\frac{v_3^2}{2g} = \frac{v_{D1}^2}{2g} = 1,90\,m$$

$$z_4 = h_3 = 2,50\,m$$

$$\frac{p_4}{\rho g} = \frac{p_3}{\rho g} + h_3 = 0,1 + 1,5 = 1,60\,m$$

$$\frac{v_4^2}{2g} = \frac{v_3^2}{2g} = 1,90\,m$$

$$z_5 = z_4 = 2,50\,m$$

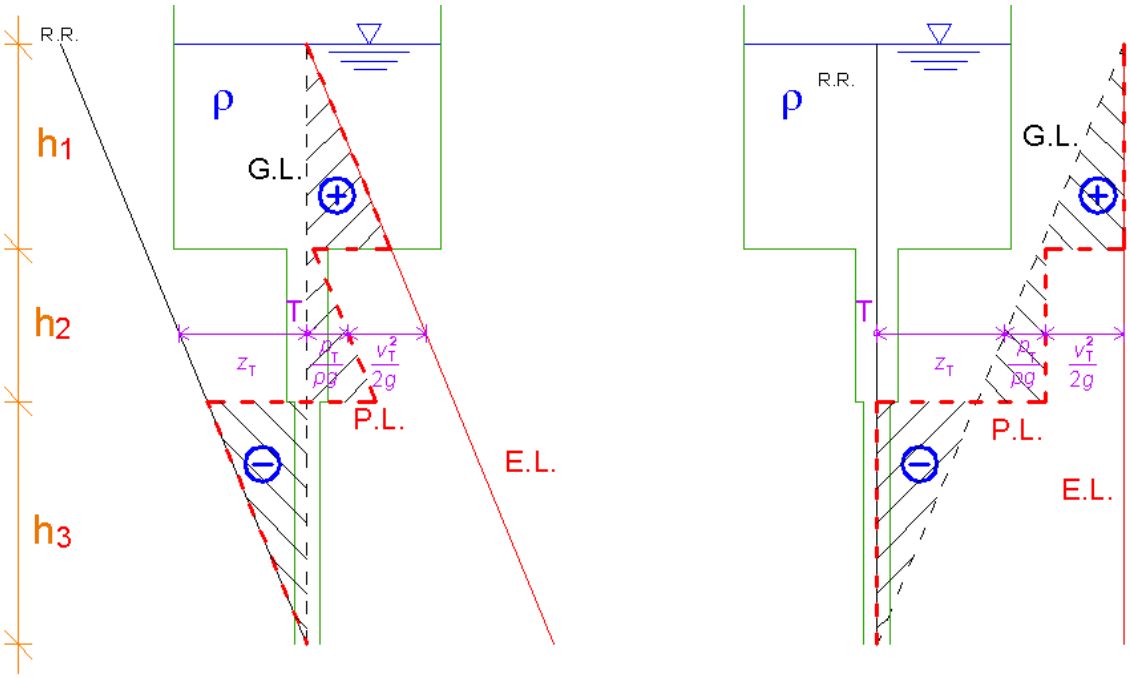
$$\frac{p_5}{\rho g} = \frac{p_4}{\rho g} + \frac{v_{D1}^2}{2g} - \frac{v_{D2}^2}{2g} = 1,60 + 1,90 - \frac{10,85^2}{2g} = 1,60 + 1,90 - 6,0 = -2,50\,m$$

$$\frac{v_5^2}{2g} = \frac{v_{D2}^2}{2g} = 6,0\,m$$

$$z_6 = 0\,m$$

$$\frac{p_6}{\rho g} = 0\,m \quad (atmosferski\ tlak)$$

$$\frac{v_5^2}{2g} = \frac{v_{D2}^2}{2g} = 6,0\,m$$



Napomena: način crtanja na lijevoj strani je fizikalno ispravniji jer je geometrijska linija smještena na središnju strujnicu cjevovda. Drugi način (desno) malo zornije prikazuje odnose potenc. i kin. energije.