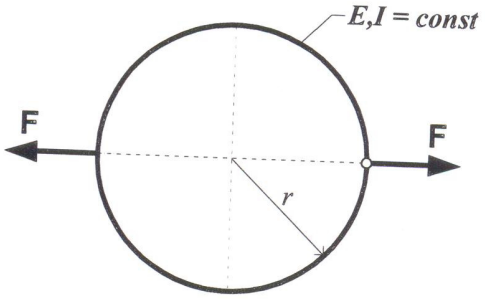
Primjer 2. kolokvija iz "OTPORNOSTI MATERIJALA 2"

1. Za zadani sustav prikazan na slici treba primjenom principa o minimumu potencijalne energije deformacije odrediti i nacrtati dijagrame unutarnjih sila **M, T, N**.

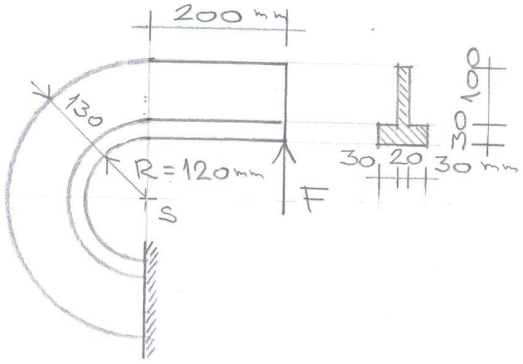
Zadano je: $F = 20\ kN$

$r = 2{,}0\ m$

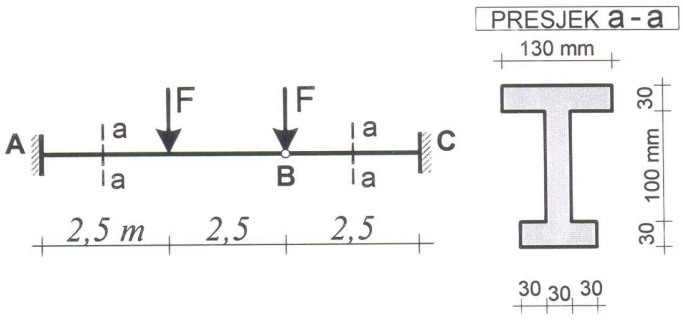
$EI = const$



2. Za zakrivljeni štap prikazan na slici treba odrediti dopuštenu veličinu sile F, ako je dopušteno naprezanje $\sigma_{dop} = 160\ MPa$



3. Za zadani sistem prikazan na slici treba po teoriji plastičnosti odrediti dopušteno opterećenje **F**, ako je granica tečenja materijala $\sigma_T = 250\ MPa$ i koeficijent sigurnosti $k = 1{,}40$.

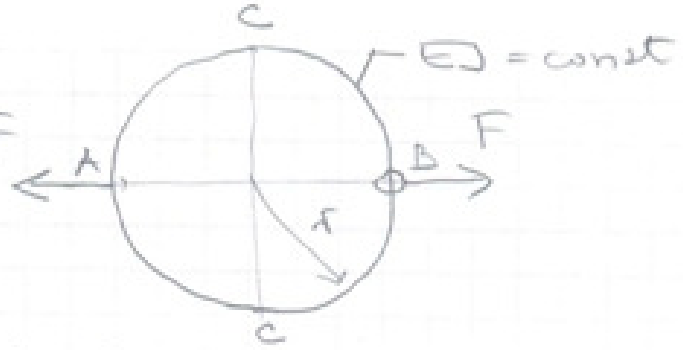


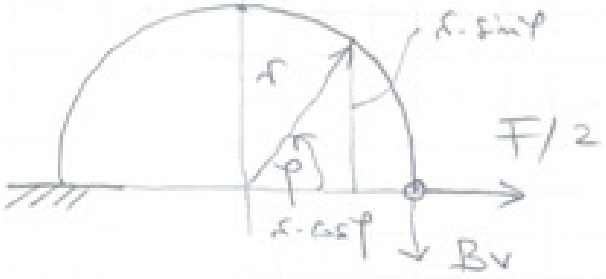
1. ZADATAK

$F = 20\ kN$

$r = 2{,}0\ m$

$EJ = const$





$$\frac{\partial E_p}{\partial B_v} = 0$$

$$M_\varphi = \frac{F}{2} \cdot r \sin\varphi - B_v \cdot r (1 - \cos\varphi)$$

$$\frac{\partial M_\varphi}{\partial B_v} = -r(1 - \cos\varphi)$$

$$\frac{\partial E_p}{\partial B_v} = \frac{1}{EJ} \int_0^{\pi} M_\varphi \frac{\partial M_\varphi}{\partial B_v} \cdot r\, d\varphi = 0$$

$$\int_0^{\pi} \left[\frac{F}{2} \cdot r \sin\varphi - B_v \cdot r(1 - \cos\varphi)\right] \cdot \left[-r(1 - \cos\varphi)\right] \cdot r\, d\varphi = 0$$

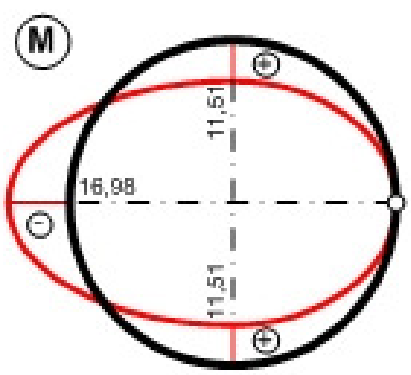
$$\frac{F}{2} \int_0^{\pi} (-\sin\varphi + \sin\varphi\cos\varphi)\, d\varphi + B_v \int_0^{\pi} (1 - 2\cos\varphi + \cos^2\varphi)\, d\varphi = 0$$

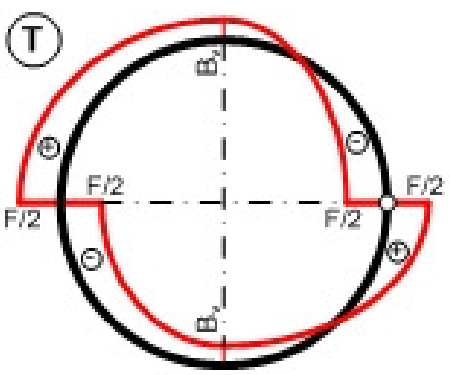
$$\frac{F}{2}(-2 + 0) + B_v\left(\pi - 2 \cdot 0 + \frac{\pi}{2}\right) = 0$$

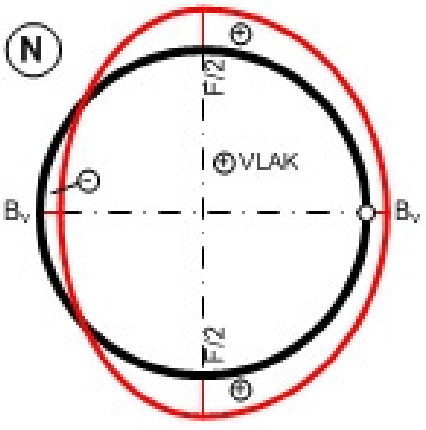
$$-F + B_v \cdot 4{,}712 = 0 \rightarrow B_v = 0{,}2122 \cdot F = \underline{4{,}244\ kN}$$

$$M_C = \frac{F}{2} \cdot r - B_v \cdot r = \underline{11{,}51\ kNm}$$

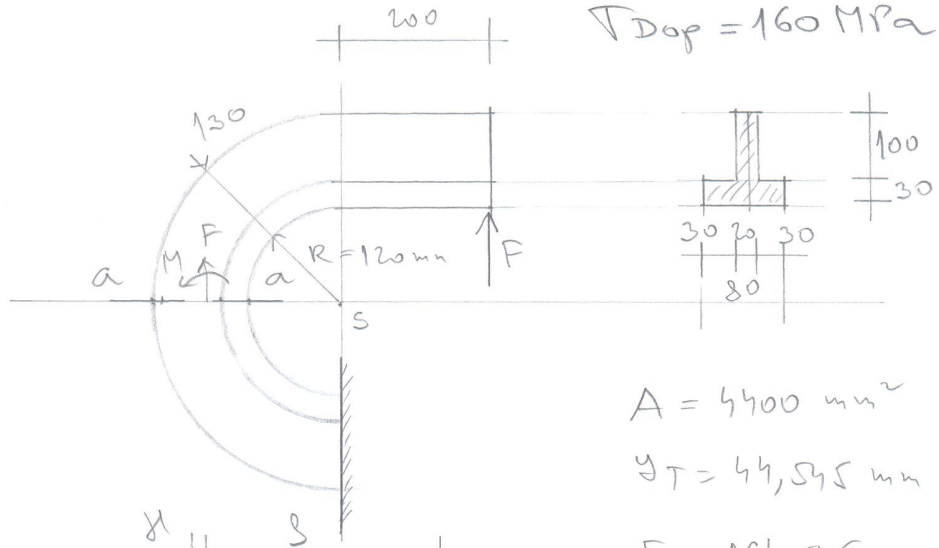
$$M_A = -B_v \cdot 2r = \underline{-16{,}98\ kNm}$$







2. ZADATAK



$\sigma_{Dop} = 160\ MPa$

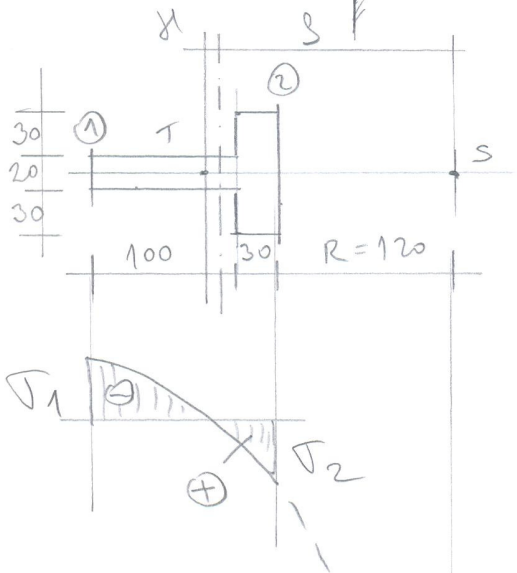
$A = 4400\ mm^2$

$y_T = 44{,}545\ mm$

$r = 164{,}545\ mm$ š.v.z.

$\frac{r}{h} = 1{,}37 < 5$

$M = -F(200 + r) = -364{,}545 \cdot F$ (Nmm)



$$\rho = \frac{A}{80 \cdot \ln\frac{150}{120} + 20 \cdot \ln\frac{250}{150}} = 156{,}7622\ mm < r$$

$y = r - \rho = 7{,}7828\ mm$

$S = y \cdot A = 34244{,}32\ mm$

$y_1 = +92{,}24\ mm$

$y_2 = -36{,}76\ mm$

$$\sigma_1 = \left| \frac{F}{A} - \frac{F \cdot 364{,}545}{34244{,}32} \cdot \frac{y_1}{250} \right| \le \sigma_{Dop}$$

$F(3{,}743 \cdot 10^{-3}) \le 160 \rightarrow F \le 42{,}75\ kN$ MJERODAVNO !

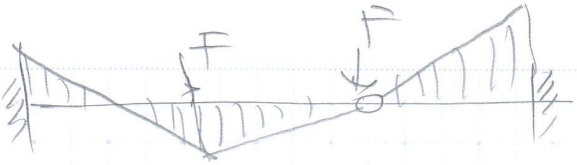
$$\sigma_2 = \left| \frac{F}{A} + \frac{F \cdot 364{,}545}{34244{,}32} \cdot \frac{y_2}{120} \right| \le \sigma_{Dop}$$

$F(3{,}4883 \cdot 10^{-2}) \le 160 \rightarrow F \le 45{,}87\ kN$

$\sigma_1 = -160\ MPa$

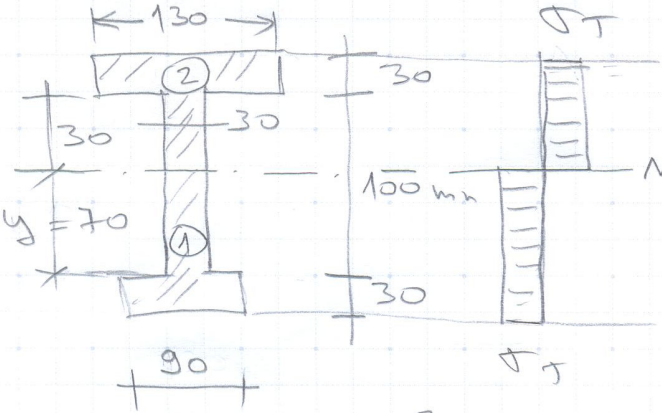
$\sigma_2 = +149{,}12\ MPa$

3. ZADATAK



$\sigma_T = 250$ MPa

$k = 1{,}40$



$A = 9600\ mm^2$

$\frac{A}{2} = 90 \cdot 30 + 30 \cdot y$

$y = 70$ mm

$W_{pl} = S_{①} + S_{②}$

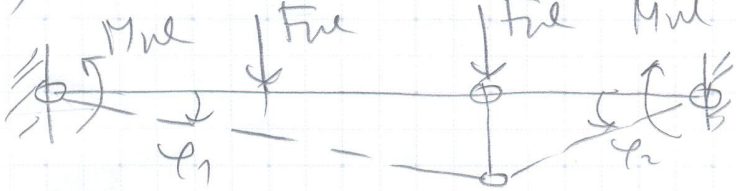
$S_{①} = 303000\ mm^3$

$S_{②} = 189000\ mm^3$

$W_{pl} = 492000\ mm^3$

$M_{pl} = W_{pl} \cdot \sigma_T = 123$ kNm

a)



$\varphi_1 \cdot 5 = \varphi_2 \cdot 2{,}5$

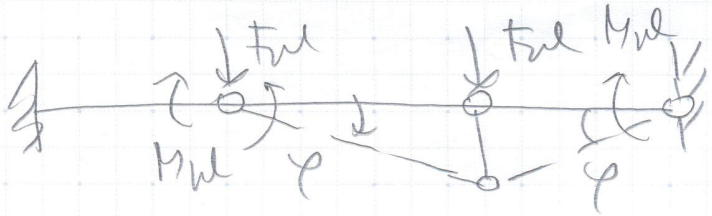
$\varphi_2 = 2 \cdot \varphi_1$

$\sum A = F_{pl} \cdot \varphi_1 \cdot 2{,}5 + F_{pl} \cdot \varphi_1 \cdot 5 - M_{pl} \cdot \varphi_1 - M_{pl} \cdot \varphi_2 = 0$

$F_{pl} \cdot 7{,}5 = M_{pl} \cdot 3 \rightarrow F_{pl} = \frac{3}{7{,}5} M_{pl} = 49{,}2$ kN

MOGUĆE RJEŠENJE!

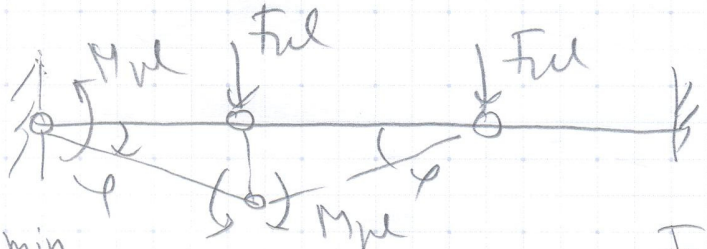
b)



$F_{pl} \cdot \varphi \cdot 2{,}5 - 2M_{pl} \cdot \varphi = 0$

$F_{pl} = \frac{2}{2{,}5} M_{pl} = 98{,}4$ kN

c)



$F_{pl} \cdot \varphi \cdot 2{,}5 - 3M_{pl} \cdot \varphi = 0$

$F_{pl} = \frac{3}{2{,}5} \cdot M_{pl} = 147{,}60$ kN

$F_{pl}^{min} = 49{,}20$ kN

$F_{op} = 35{,}14$ kN

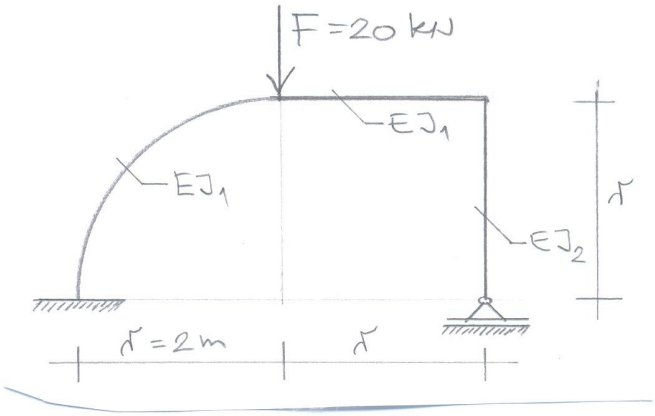
Primjer 2. kolokvija iz "OTPORNOSTI MATERIJALA 2"

1. Za zadani sustav prikazan na slici treba primjenom principa o minimumu potencijalne energije deformacije odrediti i nacrtati dijagrame unutarnjih sila **M, T, N**.

Zadano je: $F = 20\ kN$

$r = 2{,}0\ m$

$I_1 = 2 \cdot I_2 = 10^8\ mm^4$

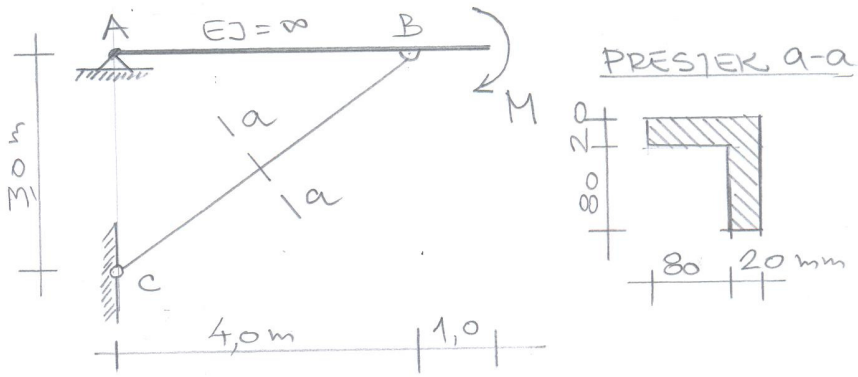


2. Apsolutno kruta greda oslonjena je zglobno u točkama A i B i opterećena momentom M. Treba odrediti koeficijent sigurnosti na izvijanje štapa BC, ako je zadano:

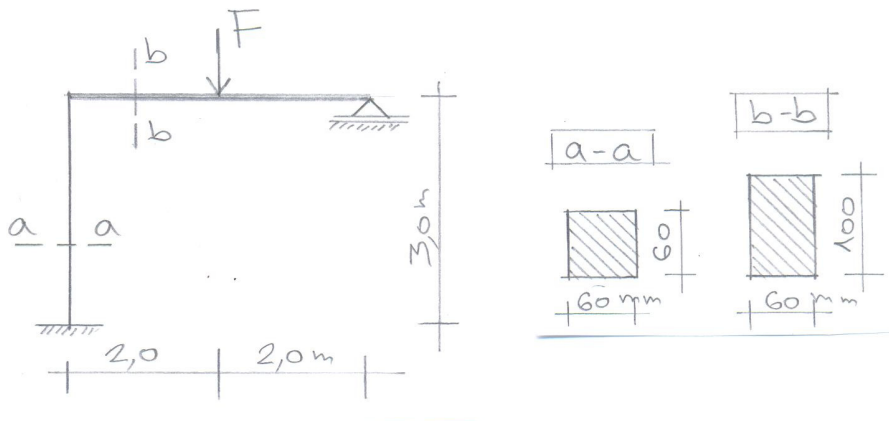
$M = 80\ kNm$

$\sigma_p = 210\ MPa$

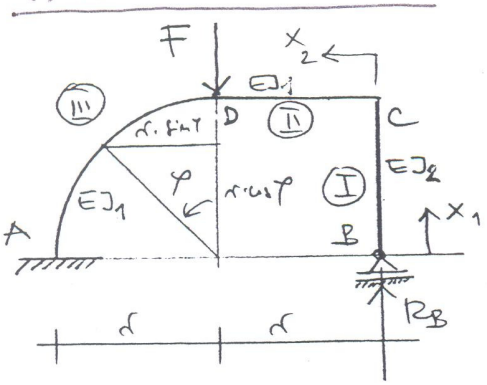
$E = 2{,}1 \cdot 10^5\ MPa$



3. Za zadani sistem prikazan na slici treba po teoriji plastičnosti odrediti dopušteno opterećenje **F**, ako je granica tečenja materijala $\sigma_T = 240\ MPa$ i koeficijent sigurnosti $k = 1{,}60$.



1. ZADATAK



$F = 20\ kN$

$r = 2\ m$

$J_1 = 2 \cdot J_2 = 10^8\ mm^4$

$\frac{\partial E_p}{\partial R_B} = \phi \qquad E_p = E_p^{I} + E_p^{II} + E_p^{III}$

$$\frac{\partial E_p}{\partial R_B} = \frac{1}{EJ_1}\int_0^r M_{x_1}\frac{\partial M_{x_1}}{\partial R_B}dx_1 + \frac{1}{EJ_2}\int_0^r M_{x_2}\frac{\partial M_{x_2}}{\partial R_B}dx_2 + \frac{1}{EJ_2}\int_0^{\pi/2} M_\varphi \cdot \frac{\partial M_\varphi}{\partial R_B} r\,d\varphi = \phi$$

(I) $M_{x_1} = \phi$ (II) $M_{x_2} = R_B \cdot x_2$ (III) $M_\varphi = R_B \cdot r(1+\sin\varphi) - F \cdot r \sin\varphi$

$\frac{\partial M_{x_1}}{\partial R_B} = \phi$ $\frac{\partial M_{x_2}}{\partial R_B} = x_2$ $\frac{\partial M_\varphi}{\partial R_B} = r(1+\sin\varphi)$

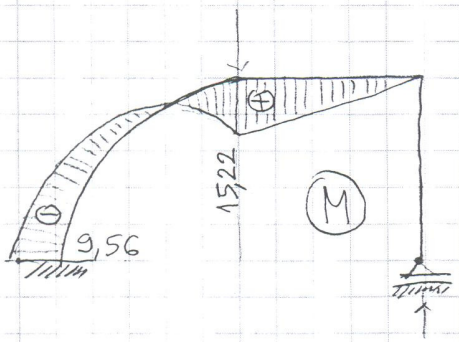
$$\frac{\partial E_p}{\partial R_B} = \int_0^r R_B x_2^2 \cdot dx_2 + \int_0^{\pi/2}\left[R_B \cdot r(1+\sin\varphi) - F \cdot r\sin\varphi\right] \cdot r(1+\sin\varphi)\, r\,d\varphi = \phi$$

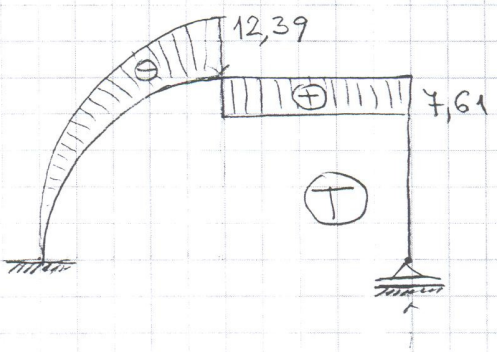
$$R_B \cdot \frac{r^3}{3} + r^3 \int_0^{\pi/2}\left[R_B(1+2\sin\varphi+\sin^2\varphi) - F(\sin\varphi+\sin^2\varphi)\right]d\varphi = \phi$$

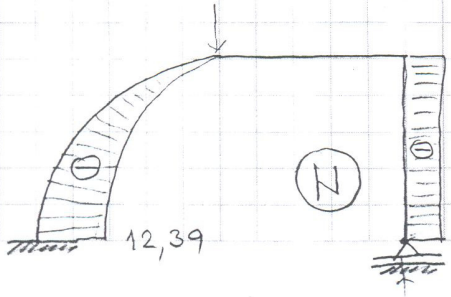
$$\frac{R_B}{3} + R_B\left(\frac{\pi}{2} + 2\cdot 1 + \frac{\pi}{4}\right) = F\left(1+\frac{\pi}{4}\right)$$

$$R_B\left(\frac{1}{3} + \frac{3\pi}{4} + 2\right) = F\left(1+\frac{\pi}{4}\right)$$

$R_B \cdot 4{,}6895 = F \cdot 1{,}7854 \rightarrow R_B = 0{,}381 \cdot F = \underline{\underline{7{,}61\ kN}}$

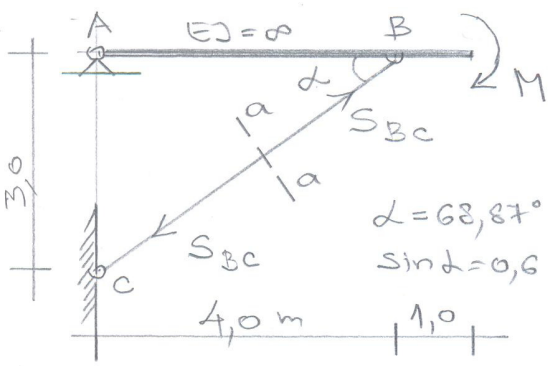




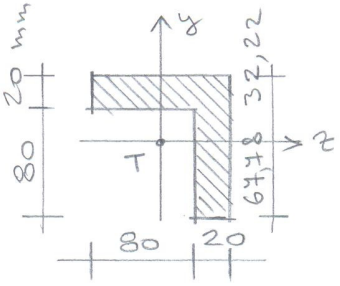


⊖ TLAK

2. ZADATAK



PRESJEK a-a



M = 80 kNm
E = 2,1 · 10⁵ MPa
σ_p = 210 MPa
k_i = ?

$A = 20 \cdot 80 + 20 \cdot 100 = 3600\ mm^2$

$y_T = z_T = (20 \cdot 80 \cdot 40 + 20 \cdot 100 \cdot 90) \cdot \frac{1}{A} = 67,78\ mm$

$J_z = J_y = \frac{20 \cdot 80^3}{12} + 20 \cdot 80 \cdot 27,78^2 + \frac{100 \cdot 20^3}{12} + 100 \cdot 20 \cdot 22,22^2 = 3,14 \cdot 10^6\ mm^4$

$J_{yz} = 20 \cdot 80 (-27,78)(+22,22) + 20 \cdot 100 (+22,22)(-17,78) = -1,778 \cdot 10^6\ mm^4$

$J_{1,2} = \frac{J_y + J_z}{2} \pm \frac{1}{2}\sqrt{(J_z - J_y)^2 + 4 J_{zy}^2} = J_z \pm |J_{zy}|$

$J_{min} = J_z - J_{zy} = \underline{1,362 \cdot 10^6\ mm^4}$ $\quad J_{max} = 4,918 \cdot 10^6\ mm^4$

$l_i = l = 5,0\ m$

$S_{BC}^{KRIT} = \frac{\pi^2 \cdot E \cdot J_{min}}{l_i^2} = \underline{112,92\ kN}$

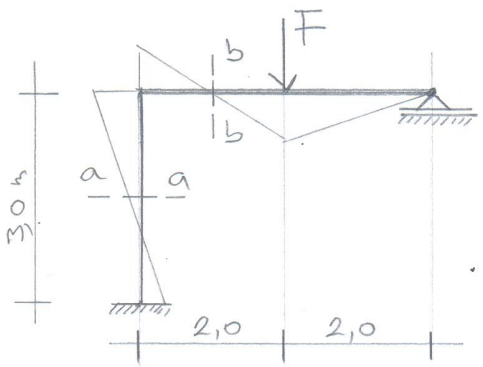
$\Sigma M_A = 0 \quad S_{BC} \cdot \sin\alpha \cdot 4 = M \rightarrow S_{BC} = \underline{33,33\ kN}$

$k_i = \frac{S_{BC}^{KRIT}}{S_{BC}} = \underline{3,39}$

$\lambda = \frac{l_i}{i_{min}} = 257 \qquad i_{min} = \sqrt{\frac{J_{min}}{A}} = 19,45\ mm$

$\lambda_p = \sqrt{\frac{\pi^2 E}{\sigma_p}} = \underline{99,3 < \lambda = 257}$

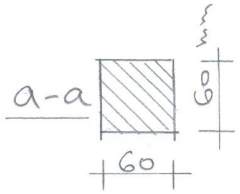
3. ZADATAK



$\sigma_T = 240\ MPa$

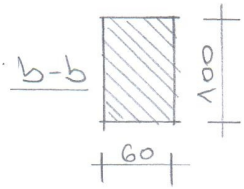
$K = 1,6$

$F_{dop} = ?$



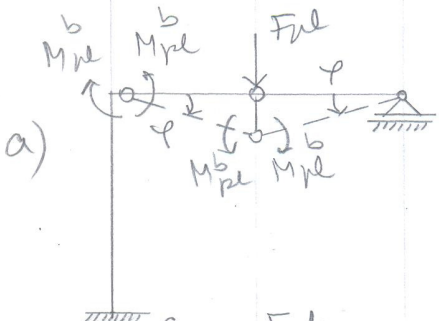
$$W_{pl}^a = \frac{60 \cdot 60^2}{4} = 54000\ mm^3$$

$$M_{pl}^a = 12,96\ kNm$$



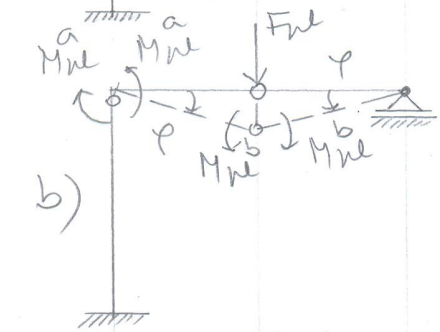
$$W_{pl}^b = \frac{60 \cdot 100^2}{4} = 150000\ mm^3$$

$$M_{pl}^b = 36,0\ kNm$$



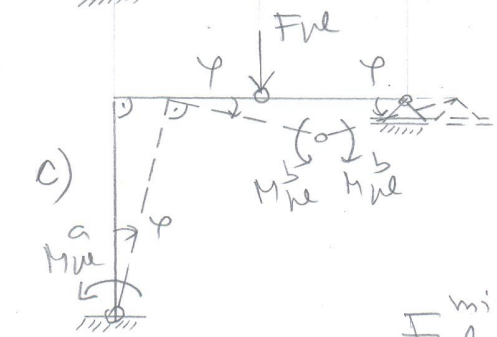
a) $F_{ul} \cdot \varphi \cdot 2 = 3 M_{ul}^b \cdot \varphi$

$F_{ul} = \frac{3}{2} M_{ul}^b = \underline{54,0\ kN}$



b) $F_{ul} \cdot \varphi \cdot 2 = 2 M_{ul}^b \cdot \varphi + M_{ul}^a \cdot \varphi$

$F_{ul} = \underline{42,48\ kN}$ MJERODAVNO!



c) $F_{ul} \cdot \varphi \cdot 2 = 2 M_{ul}^b \cdot \varphi + M_{ul}^a \cdot \varphi$

$F_{ul} = \underline{42,48\ kN}$ MJERODAVNO!

$$F_{ul}^{min} = \underline{42,48\ kN}$$

$$F_{dop} = \frac{F_{ul}^{min}}{K} = \underline{26,55\ kN}$$