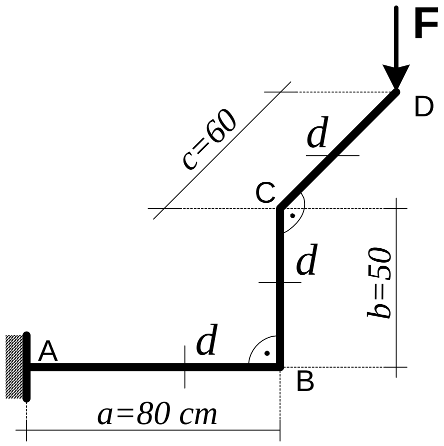
SVEUČILIŠTE U ZAGREBU
GRAĐEVINSKI FAKULTET
ZAVOD ZA TEHNIČKU MEHANIKU

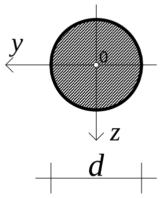
TEORIJE ČVRSTOĆE

ZADATAK 6.

Štap okruglog poprečnog presjeka promjera *"d"* opterećen je vertikalnom silom F = 12 kN.

Treba odrediti promjer štapa po svim teorijama čvrstoće, ako je granica tečenja materijala pri jednoosnom pritisku i jednoosnom rastezanju $\sigma_T = 280\ MPa$. Koeficijent sigurnosti je jednak za pritisak i rastezanje i iznosi $k = 2{,}0$, a Poissonov koeficijent za materijal je $\nu = 0{,}30$.





$$I_y = I_z = \frac{\pi \cdot d^4}{64}$$

$$W_y = \frac{I_y}{\frac{d}{2}} = \frac{\pi \cdot d^3}{32}$$

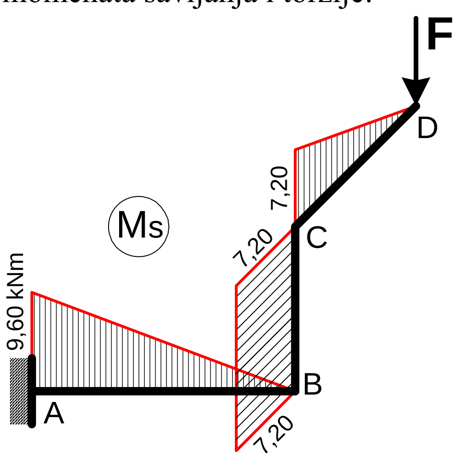
$$I_p = I_y + I_z = \frac{\pi \cdot d^4}{32}$$

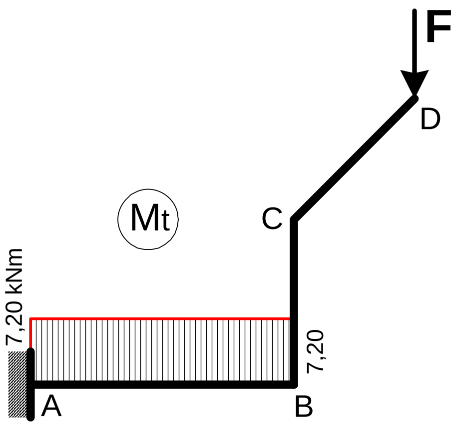
$$W_p = \frac{I_p}{\frac{d}{2}} = \frac{\pi \cdot d^3}{16} = 2 \cdot W_y$$

$$M_s^{CD} = F \cdot c = 7{,}20\ kNm$$
$$M_s^{CB} = M_s^{BC} = F \cdot c = 7{,}20\ kNm$$
$$M_s^{BA} = 0$$
$$M_s^{AB} = F \cdot a = 9{,}60\ kNm$$

$$M_t^{CD} = M_t^{CB} = 0$$
$$M_t^{BA} = M_t^{AB} = F \cdot c = 7{,}20\ kNm$$

Dijagrami momenata savijanja i torzije:

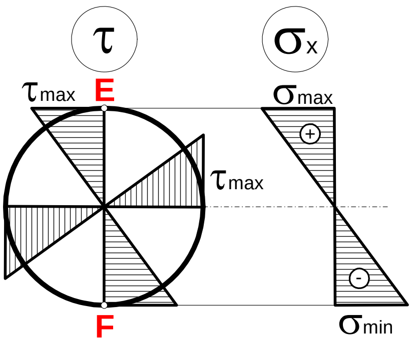


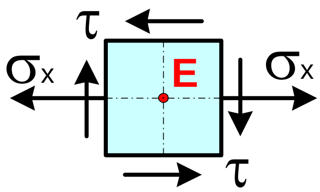


Kritični presjek je presjek **A**, zbog toga što u njemu postoji moment savijanja i moment torzije:

$$M_s = 9{,}60\ kNm$$
$$M_t = 7{,}20\ kNm$$

PRESJEK **A**





Opasne točke presjeka **A** su **E** i **F** u kojima postoji ravninsko stanje naprezanja:

$$\sigma_x = \frac{M_s}{W_y} \qquad\qquad \tau = \frac{M_t}{W_p} = \frac{M_t}{2 \cdot W_y}$$

Glavna naprezanja u točki **E** iznose:

$$\sigma_1 = \frac{\sigma_x}{2} + \frac{1}{2}\sqrt{\sigma_x^2 + 4 \cdot \tau^2} \qquad\qquad \sigma_2 = \frac{\sigma_x}{2} - \frac{1}{2}\sqrt{\sigma_x^2 + 4 \cdot \tau^2}$$

$$\sigma_1 = \frac{M_s}{2 \cdot W_y} + \frac{1}{2}\sqrt{\left(\frac{M_s}{W_y}\right)^2 + 4 \cdot \left(\frac{M_t}{2 \cdot W_y}\right)^2} = \frac{1}{2 \cdot W_y}\left(M_s + \sqrt{M_s^2 + M_t^2}\right)$$

$$\sigma_2 = \frac{M_s}{2 \cdot W_y} - \frac{1}{2}\sqrt{\left(\frac{M_s}{W_y}\right)^2 + 4 \cdot \left(\frac{M_t}{2 \cdot W_y}\right)^2} = \frac{1}{2 \cdot W_y}\left(M_s - \sqrt{M_s^2 + M_t^2}\right).$$

Uvjeti čvrstoće za dimenzioniranje štapa:

a) Po **I. teoriji** najvećih normalnih naprezanja:

$$\sigma_{ek} = \sigma_1 = \frac{1}{2 \cdot W_y} \cdot \left(M_s + \sqrt{M_s^2 + M_t^2}\right) \le \sigma_{dop} \qquad\qquad \sigma_{dop} = \frac{\sigma_T}{k} = \frac{280}{2} = 140 \ \ MPa$$

$$W_y \ge \frac{1}{2 \cdot \sigma_{dop}} \cdot \left(M_s + \sqrt{M_s^2 + M_t^2}\right) = \frac{1}{2 \cdot 140} \cdot \left(9{,}60 \cdot 10^6 + 10^6 \cdot \sqrt{9{,}60^2 + 7{,}20^2}\right) =$$

$$= 77142{,}86 \ \ mm^3$$

$$W_y = \frac{\pi \cdot d^3}{32} \qquad \Rightarrow \qquad d \ge \sqrt[3]{\frac{32 \cdot W_y}{\pi}} = \sqrt[3]{\frac{32 \cdot 77142{,}86}{\pi}} = \underline{92{,}28 \ \ mm}$$

b) Po **II. teoriji** najvećih normalnih deformacija:

$$\sigma_{ek} = \sigma_1 - \nu \cdot \sigma_2 = \frac{1}{W_y} \cdot \left(\frac{1-\nu}{2} \cdot M_s + \frac{1+\nu}{2} \cdot \sqrt{M_s^2 + M_t^2}\right) \le \sigma_{dop}$$

$$W_y \ge \frac{1}{\sigma_{dop}} \cdot \left(\frac{1-\nu}{2} \cdot M_s + \frac{1+\nu}{2} \cdot \sqrt{M_s^2 + M_t^2}\right) = 79714{,}29 \ \ mm^3$$

$$d \ge \sqrt[3]{\frac{32 \cdot W_y}{\pi}} = \underline{93{,}29 \ \ mm}$$

c) Po **III. teoriji** najvećih posmičnih naprezanja:

$$\sigma_{ek} = \sigma_1 - \sigma_2 = \frac{1}{W_y} \cdot \sqrt{M_s^2 + M_t^2} \leq \sigma_{dop}$$

$$W_y \geq \frac{1}{\sigma_{dop}} \cdot \sqrt{M_s^2 + M_t^2} = 85714{,}29 \ mm^3 \qquad d \geq \sqrt[3]{\frac{32 \cdot W_y}{\pi}} = \underline{95{,}58 \ mm}$$

d) Po **IV. teoriji** potencijalne energije deformacija:

$$\sigma_{ek} = \sqrt{\sigma_1^2 + \sigma_2^2 - 2 \cdot \nu \cdot \sigma_1 \cdot \sigma_2} = \frac{1}{W_y} \cdot \sqrt{M_s^2 + \frac{1+\nu}{2} \cdot M_t^2} \leq \sigma_{dop}$$

$$W_y \geq \frac{1}{\sigma_{dop}} \cdot \sqrt{M_s^2 + \frac{1+\nu}{2} \cdot M_t^2} = 80132{,}54 \ mm^3 \qquad d \geq \sqrt[3]{\frac{32 \cdot W_y}{\pi}} = \underline{93{,}46 \ mm}$$

e) Po **V. teoriji** potencijalne energije promjene oblika:

$$\sigma_{ek} = \sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1 \cdot \sigma_2} = \frac{1}{W_y} \cdot \sqrt{M_s^2 + 0{,}75 \cdot M_t^2} \leq \sigma_{dop}$$

$$W_y \geq \frac{1}{W_y} \cdot \sqrt{M_s^2 + 0{,}75 \cdot M_t^2} = 81766{,}22 \ mm^3 \qquad d \geq \sqrt[3]{\frac{32 \cdot W_y}{\pi}} = \underline{94{,}09 \ mm}$$

f) Po **Mohrovoj teoriji** čvrstoće:

$$\sigma_{ek} = \sigma_1 - \mu \cdot \sigma_2 = \frac{1}{W_y} \cdot \left(\frac{1-\mu}{2} \cdot M_s + \frac{1+\mu}{2} \cdot \sqrt{M_s^2 + M_t^2} \right) \leq \sigma_{dop} \qquad \mu = \frac{\sigma_{vl \ dop}}{\sigma_{tl \ dop}} = 1$$

Ako je granica tečenja pri jednoosnom rastezanju jednaka granici tečenja pri jednoosnom pritisku ($\sigma_{vl \ dop} = \sigma_{tl \ dop} = \sigma_{dop}$) onda se uvjeti čvrstoće prema Mohrovoj teoriji podudaraju s uvjetima prema teoriji najvećih posmičnih naprezanja.

$$\sigma_{ek} = \sigma_1 - \sigma_2 = \frac{1}{W_y} \cdot \sqrt{M_s^2 + M_t^2} \leq \sigma_{dop}$$

$$W_y \geq \frac{1}{\sigma_{dop}} \cdot \sqrt{M_s^2 + M_t^2} = 85714{,}29 \ mm^3 \qquad d \geq \sqrt[3]{\frac{32 \cdot W_y}{\pi}} = \underline{95{,}58 \ mm}.$$

Rezultati izračunatih promjera štapa prikazani su u slijedećoj tablici:

	I. teorija čvrstoće	II. teorija čvrstoće	III. teorija čvrstoće	IV. teorija čvrstoće	V. teorija čvrstoće	Mohrova teorija
W_y (mm^3)	77142,86	79714,29	85714,29	80132,54	81766,22	85714,29
d (mm)	**92,28**	**93,29**	**95,58**	**93,46**	**94,09**	**95,58**

Odabrani promjer štapa ***d = 96 mm*** zadovoljava po svim teorijama čvrstoće.