Conference on Diophantine m-tuples and related problems III

September 14 – 16, 2022, Faculty of Civil Engineering, University of Zagreb, Zagreb, Croatia

Book of Abstracts

Supported by

Croatian Science Foundation

Faculty of Civil Engineering, Zagreb, Croatia

Preface

This brochure consists of the program of presentations and abstracts of papers presented at the *Conference on Diophantine m-tuples and related problems III*. The first two conferences from this series were held at Purdue University Northwest, Westville/Hammond, Indiana, USA. This one is supported by the *Croatian Science Foundation under the project no. IP-2018-01-1313* and the *Faculty of Civil Engineering, University of Zagreb, Zagreb, Croatia*. It is dedicated to Diophantine m-tuples and related problems, and other aspects of Diophantine equations as well. It is held at the Faculty of Civil Engineering, University of Zagreb, Zagreb, Croatia, from September 14 to September 16, 2022.

The scientific program of this conference consists of invited talks and short communications of international and Croatian experts in number theory and algebra.

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Nikola Adžaga, University of Zagreb, Croatia
Ivan Soldo, University of Osijek, Croatia
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László Szalay, University of Sopron, Hungary

Conference Language: English

Participants

Kouèssi Norbert Adédji, Université d'Abomey Calavi/IMSP, Bénin
Nikola Adžaga, University of Zagreb, Croatia
Alen Andrašek, University of Zagreb, Croatia
Djamel Bellaouar, University 08 Mai 1945 Guelma, Algeria
Marija Bliznac Trebješanin, University of Split, Croatia
Sanda Bujačić Babić, University of Rijeka, Croatia
Goran Dražić, University of Zagreb, Croatia
Andrej Dujella, University of Zagreb, Croatia
Christian Elsholtz, Graz University of Technology, Austria
Alan Filipin, University of Zagreb, Croatia
Zrinka Franušić, University of Zagreb, Croatia
Gautam Gangopadhyay, Vilnius University, Lithuania
Bo He, Aba Teachers University, Sichuan, P. R. China
Mirela Jukić Bokun, University of Osijek, Croatia
Florian Luca, Wits University, Johannesburg, South Africa
Vinko Petričević, University of Zagreb, Croatia
Supawadee Prugsapitak, Prince of Songkla University Hatyai, Thailand
Prasanta Kumar Ray, Sambalpur University, India
Katerina Santicola, University of Warwick, UK
Ivan Soldo, University of Osijek, Croatia
Gökhan Soydan, Bursa Uludağ University, Turkey
Artatrana Suna, Sambalpur University, India
László Szalay, University of Sopron, Hungary
Alain Togbé, Purdue University Northwest, USA

Program

The conference is held at the Faculty of Civil Engineering, University of Zagreb, Fra Andrije Kačića-Miošića 26. All talks take place in room 121 on the first floor.

Wednesday, September 14, 2022

9:00 - 9:50	Registration
9:50 - 10:00	Opening
Morning session	
10:00 - 10:50	*On Diophantine pairs* Alain Togbé
10:50 - 11:20	Coffee break
11:20 - 11:50	$D(4)$*-triples with two largest elements in common* Marija Bliznac Trebješanin
11:50 - 12:20	*On the regularity of the* $D(4)$*-quadruples in* $\mathbb{Z}[X][i]$ Sanda Bujačić Babić
12:20 - 14:00	Lunch break
Afternoon session 1	
14:00 - 14:30	*On* $D(-1)$*-tuples in the ring* $\mathbb{Z}[\sqrt{-k}], k > 0$ Ivan Soldo
14:30 - 15:00	*Asymptotics of* $D(q)$*-pairs and triples* Goran Dražić
15:00 - 15:30	Coffee break
Afternoon session 2	
15:30 - 16:00	*Applications of Pellian equations of a special type* Mirela Jukić Bokun
16:00 - 16:30	*Representation of integers of the form* $x^2 + ky^2 - lz^2$ Supawadee Prugsapitak

Thursday, September 15, 2022

Morning session	
10:00 - 10:50	*$D(n)$-tuples for several n's* Andrej Dujella
10:50 - 11:20	Coffee break
11:20 - 11:50	*$D(n)$-quintuples with square elements* Vinko Petričević
11:50 - 12:20	*Diophantine triples with non-uniform recurrences* László Szalay
12:20 - 14:00	Lunch break
Afternoon session 1	
14:00 - 14:50	*On X and Y coordinates of Pell equations in various sequences* Florian Luca
14:50 - 15:20	*On the comparison between the value of the divisor function to its value at Euler's function* Djamel Bellaouar
15:20 - 15:50	Coffee break
Afternoon session 2	
15:50 - 16:40	*On the regularity of Diophantine triple with degree* 1 Bo He
16:40 - 17:10	*The extension of the $D(-k)$-triple $\{1, k, k+1\}$ to a quadruple* Kouèssi Norbert Adédji
19:00	Conference dinner

Friday, September 16, 2022

Morning session	
10:00 - 10:50	*On the solutions of a class of generalized Fermat equations signature* $(2, 2n, 3)$ Gökhan Soydan
10:50 - 11:20	Coffee break
11:20 - 11:50	*Longer gaps between values of binary quadratic forms* Christian Elsholtz
11:50 - 12:20	*Integer points on elliptic curves induced by a family of Diophantine triples involving balancing numbers* Artatrana Suna
12:20 - 12:30	Closing
12:30 - 14:00	Lunch break

Abstracts of talks

Invited lectures

$D(n)$-tuples for several n's

Andrej Dujella

University of Zagreb, Croatia

duje@math.hr

(joint work with Nikola Adžaga, Dijana Kreso, Vinko Petričević and Petra Tadić)

For an integer n, a set of distinct nonzero integers $\{a_1, a_2, ..., a_m\}$ such that $a_i a_j + n$ is a perfect square for all $1 \leq i < j \leq m$, is called a Diophantine m-tuple with the property $D(n)$ or $D(n)$-m-tuple. When considering $D(n)$-tuples, usually an integer n is fixed in advance. However, we may ask if a set can have the property $D(n)$ for several different n's. For example, $\{8, 21, 55\}$ is a $D(1)$-triple and $D(4321)$-triple. In a joint work with Adžaga, Kreso and Tadić, we presented several families of Diophantine triples which are $D(n)$-sets for two distinct n's with $n \neq 1$. In a joint work with Petričević, we proved that there are infinitely many (essentially different) quadruples which are simultaneously $D(n_1)$-quadruples and $D(n_2)$-quadruples with $n_1 \neq n_2$. Moreover, the elements in some of these quadruples are squares, so they are also $D(0)$-quadruples. E.g. $\{54^2, 100^2, 168^2, 364^2\}$ is a $D(8190^2)$, $D(40320^2)$ and $D(0)$-quadruple. In this talk, we will describe methods used in constructions of mentioned triples and quadruples.

On the regularity of Diophantine triple with degree 1

Bo He

Aba Teachers University, Sichuan, P. R. China

bhe@live.cn

(joint work with Alain Togbé)

Let a, b and r be positive integers with $ab + 1 = r^2$. In this talk, we show that for sufficiently large r, the diophantine quadruple $\{a, b, c = 4r(r \pm a)(b \pm r), d\}(c < d)$ is regular.

On X and Y coordinates of Pell equations in various sequences

Florian Luca

Wits University, Johannesburg, South Africa

florian.luca@wits.ac.za

(joint work with F. S. Zottor)

In this talk, we will first survey various results concerning the presence of X or Y coordinates in various interesting sequences of integers like squares, factorials, Fibonacci numbers, rep-digits (in various bases), k-generalized Fibonacci numbers, etc. Then we will give the main ideas of the proof of a recent result which asserts that for all nonsquare integers $d > 2$, there are at most two values of Y satisfying $X^2 - dY^2 = \pm 1$ which are also Fibonacci numbers. This does not hold when $d = 2$ for which $F_1 = F_2 = 1 = Y_1$, $F_3 = 2 = Y_2$, $F_5 = 5 = Y_3$ are the Y-coordinates of the first three solutions of the corresponding Pell equation.

On the solutions of a class of generalized Fermat equations signature $(2, 2n, 3)$

Gökhan Soydan

Department of Mathematics, Bursa Uludağ University, Bursa, Turkey

gsoydan@uludag.edu.tr

(joint work with Karolina Chałupka and Andrzej Dąbrowski)

Fix nonzero integers A, B and C. For given positive integers p, q, r satisfying $1/p + 1/q + 1/r < 1$, the generalized Fermat equation

$$Ax^p + By^q = Cz^r \tag{1}$$

has only finitely many primitive integer solutions. Modern techniques coming from Galois representations and modular forms (methods of Frey–Hellegouarch curves and variants of Ribet's level-lowering theorem, and of course, the modularity of elliptic curves or abelian varieties over the rationals or totally real number fields) allow to give partial (sometimes complete) results concerning the set of solutions to (1). Recently, two survey papers concerning solving the equation (1) when $ABC = 1$ have been published. See [1, 2] for details.

In this talk, we give some results on the solutions of the Diophantine equations

$$ax^2 + y^{2n} = 4z^3, \quad x, y, z \in \mathbb{Z},\ \gcd(x, y) = 1,\ n \in \mathbb{N}_{\geq 2},$$

and

$$x^2 + ay^{2n} = 4z^3, \quad x, y, z \in \mathbb{Z},\ \gcd(x, y) = 1,\ n \in \mathbb{N}_{\geq 2},$$

where the class number of $\mathbb{Q}(\sqrt{-a})$ with $a \in \{7, 11, 19, 43, 67, 163\}$ is 1.

One of our motivations was to extend the former results (and methods) of Bruin [3], Chen [6] and Dahmen [8], by considering (1) with (A, B, C)'s different from $(1, 1, 1)$ (assuming for simplicity that the class number of $\mathbb{Q}(\sqrt{-AB})$ is one).

Another motivation was to extend our previous results on the Diophantine equation

$$ax^2 + b^{2n} = 4y^k, \quad x, y \in \mathbb{Z},\ n, k \in \mathbb{N},\ k \geq 5 \text{ odd } prime, \gcd(x, y) = 1.$$

were given in [7]. This talk consists of the results in [4, 5].

[1] M. A. Bennett, I. Chen, S. R. Dahmen, S. Yazdani (2015). Generalized Fermat equations: a miscellany, Int. J. Number Theory, 11, 1–28.

[2] M. A. Bennett, P. Mihăilescu, S. Siksek (2016). The generalized Fermat equation, Open Problems in Mathematics (J. F. Nash, Jr. and M. Th. Rassias eds), 173-205, Springer, New York.

[3] N. Bruin (1999). The Diophantine Equations $x^2 \pm y^4 = \pm z^6$ and $x^2 + y^8 = z^3$, Compos. Math., 118, 305–321.

[4] K. Chałupka, A. Dąbrowski, G. Soydan (2022). On a class of generalized Fermat equations of signature $(2, 2n, 3)$, J. Number Theory, 234, 153–178.

[5] K. Chałupka, A. Dąbrowski, G. Soydan (2022). On a class of generalized Fermat equations of signature $(2, 2n, 3)$-II, submitted.

[6] I. Chen (2007). On the equation $s^2 + y^{2p} = \alpha^3$, Math. Comput., 262, 1223–1227.

[7] A. Dąbrowski, N. Günhan, G. Soydan (2020). On a class of Lebesgue-Ljunggren-Nagell type equations, J. Number Theory, 215, 149–159.

[8] S. R. Dahmen (2011). A refined modular approach to the Diophantine equation $x^2 + y^{2n} = z^3$, Int. J. Number Theory, 7, 1303–1316.

On Diophantine pairs

Alain Togbé

Purdue University Northwest, USA

atogbe@pnw.edu

(joint work with N. Adedji, B. He and A. Pintér)

A set of m distinct positive integers $\{a_1, \ldots, a_m\}$ is called a Diophantine m-tuple if $a_i a_j + 1$ is a perfect square. In general, let n be an integer, a set of m positive integers $\{a_1, \ldots, a_m\}$ is called a Diophantine m-tuple with the property $D(n)$ or a $D(n)$-m-tuple (or a P_n-set of size m), if $a_i a_j + n$ is a perfect square. Diophantus studied sets of positive rational numbers with the same property, particularly he found the set of four positive rational numbers $\left\{\frac{1}{16}, \frac{33}{16}, \frac{17}{4}, \frac{105}{16}\right\}$. But the first Diophantine quadruple was found by Fermat. That is the set $\{1, 3, 8, 120\}$. Moreover, Baker and Davenport proved that the set $\{1, 3, 8, 120\}$ cannot be extended to a Diophantine quintuple. The problem of the extendibility of Diophantine m-tuples is of a big interest.

During this talk, we will give a very quick history of some results obtained on Diophantine pairs and discuss of the recent result of the Diophantine pair $\{a, 3a\}$ and others.

Short communications

The extension of the $D(-k)$-triple $\{1, k, k+1\}$ to a quadruple

Kouèssi Norbert Adédji

Université d'Abomey Calavi/IMSP, Bénin

adedjnorb1988@gmail.com

(joint work with A. Filipin and A. Togbé)

Let n be a nonzero integer. We call the set of m distinct positive integers a $D(n)$-m-tuple, or m-tuple with the property $D(n)$, if the product of any two of its distinct elements increased by n is a perfect square. Let k be a positive integer. In recently N. Adžaga, A. Filipin and Y. Fujita conjectured that if $\{k, k+1, c, d\}$ is a $D(-k)$-quadruple with $c < d$, then $c = 1$ and $d = 4k+1$, in which case $3k+1$ must be a square. In this talk, we confirm the above conjecture for k of the form $k = \ell^2 - 1$, where ℓ is a positive integer such that $\ell \geq 3$.

On the comparison between the value of the divisor function to its value at Euler's function

Djamel Bellaouar

Department of Mathematics, University 08 Mai 1945 Guelma, Guelma, Algeria

bellaouar.djamel@univ-guelma.dz

Let $d(n)$ denote the number of positive integers dividing the positive integer n, and let $\varphi(n)$ denote Euler's function representing the number of numbers less than and prime to n. Let us define

$$S := \{n \in \mathbb{N} : d(n) = d(\varphi(n))\},$$

$$S^+ := \{n \in \mathbb{N} : d(n) > d(\varphi(n))\},$$

and

$$S^- := \{n \in \mathbb{N} : d(n) < d(\varphi(n))\}.$$

We also define for any $k \geq 1$ the subsets $S_k \subseteq S, S_k^+ \subseteq S^+$ and $S_k^- \subseteq S^-$ as follows:

$$S_k = S \cap \mathbb{W}_k,\ S_k^+ = S^+ \cap \mathbb{W}_k \text{ and } S_k^- = S^- \cap \mathbb{W}_k,$$

where $\mathbb{W}_k = \{n \in \mathbb{N} : \omega(n) = k\}$ and $\omega(n)$ denotes the number of distinct prime factors of n. In this talk, we study the sets S_k, S_k^+ and S_k^- for $k \geq 1$. More precisely, we prove that these sets are infinite.

$D(4)$-triples with two largest elements in common

Marija Bliznac Trebješanin

Faculty of Science, University of Split, Split, Croatia

marbli@pmfst.hr

We outline current results about $D(4)$-m-tuples and consider two new conjectures concerning $D(4)$-quadruples. Some special cases that support their validity are proven. The main result is the proof that $\{a, b, c\}$ and $\{a + 1, b, c\}$ cannot both be $D(4)$-triples.

On the regularity of the $D(4)$-quadruples in $\mathbb{Z}[X][i]$

Sanda Bujačić Babić

Faculty of Mathematics, University of Rijeka, Rijeka, Croatia

sbujacic@math.uniri.hr

(joint work with Marija Bliznac Trebješanin)

A set $\{a, b, c, d\}$ of four distinct nonzero polynomials in $\mathbb{Z}[i][X]$ is called a polynomial Diophantine $D(4)$-quadruple if the product of any two of its distinct elements increased by 4 is a square of a polynomial in $\mathbb{Z}[i][X]$.

In this work we prove that every polynomial $D(4)$-quadruple in $\mathbb{Z}[i][X]$ is regular, or in other words that the equation

$$(a + b - c - d)^2 = (ab + 4)(cd + 4)$$

holds for every polynomial $\{a, b, c, d\}$ $D(4)$-quadruple in $\mathbb{Z}[i][X]$. Moreover, we compare the strategies used in this case with the cases of polynomial Diophantine quadruples in $\mathbb{R}[X]$ and $\mathbb{Z}[X][i]$ introduced in [1, 2] and point out some differences in ideas and approaches.

[1] A. Filipin, A. Jurasić, *A polynomial variant of a problem of Diophantus and its consequences*, Glas. Mat. Ser. III 54 (2019), 21-52.

[2] A. Filipin, A. Jurasić, *Diophantine quadruples in* $\mathbb{Z}[i][X]$, Period. Math. Hungar. **82** (2021), 198–212.

Asymptotics of $D(q)$-pairs and triples

Goran Dražić

Faculty of Food Technology and Biotechnology, University of Zagreb, Zagreb, Croatia

goran.drazic@pbf.unizg.hr

(joint work with Nikola Adžaga, Andrej Dujella and Attila Pethő)

For an integer q, a *$D(q)$-m-tuple* is a set of m distinct nonzero integers $\{a_1, a_2, \dots, a_m\}$ such that $a_i a_j + q$ is a square for all $1 \leqslant i < j \leqslant m$.

Denote by $D_{m,q}(N) := |\{D \subset \{1, 2, \dots, N\} \colon D \text{ is a } D(q) - m - \text{tuple}\}|$, which is the number of $D(q)$-m-tuples such that all elements of the m-tuple are natural numbers less than or equal to N.

We prove the asymptotic behavior of $D_{2,q}(N)$ and $D_{3,q}(N)$ for a prime q is linearly dependent with respect to N, where the constant depends on the value at 1 of the L-function of a certain Dirichlet character mod $4q$.

We give an algorithm to determine the asymptotic behavior of $D_{2,q}(N)$ when q is squarefree and discuss some simpler cases when q is not squarefree.

Longer gaps between values of binary quadratic forms

Christian Elsholtz

Graz University of Technology, Graz, Austria

(joint work with Rainer Dietmann, Alexander Kalmynin, Sergei Konyagin and James Maynard)

We prove new lower bounds on large gaps between integers which are sums of two squares, or are represented by *any* binary quadratic form of discriminant D, improving results of Richards. Let $s_1, s_2, \dots$ be the sequence of positive integers, arranged in increasing order, that are representable by *any* binary quadratic form of fixed discriminant D, then

$$\limsup_{n \to \infty} \frac{s_{n+1} - s_n}{\log s_n} \gg \frac{|D|}{\varphi(|D|) \log |D|},$$

improving a lower bound of $\frac{1}{|D|}$ of Richards. In the special case of sums of two squares, we improve Richards's bound of $1/4$ to $\frac{390}{449} = 0.868\dots$.

We also generalize Richards's result in another direction: If d is composite we show that there exist constants C_d such that for all integer values of x none of the values $p_d(x) = C_d + x^d$ is a sum of two squares. Let d be a prime. For all $k \in \mathbf{N}$ there exists a smallest positive integer y_k such that none of the integers $y_k + j^d, 1 \leq j \leq k$, is a sum of two squares. Moreover,

$$\limsup_{k\to\infty} \frac{k}{\log y_k} \gg \frac{1}{\sqrt{\log d}}.$$

(IMRN 2022. The pdf of the paper is on the speaker's webpage.)

Applications of Pellian equations of a special type

Mirela Jukić Bokun

Department of Matematics, University of Osijek, Osijek, Croatia

mirela@mathos.hr

(joint work with Ivan Soldo)

We consider the solvability of the Pellian equation

$$x^2 - (d^2 + 1)y^2 = -m, \tag{2}$$

in the case $d = n^k, m = n^{2l-1}$, where k, l are positive integers and n is a composite positive integer. Moreover, we completely solve the problem of the solvability of equation (2) in integers in the case $d = pq$, $m = pq^2$, p, q are primes. By combining that results with other known results on the existence of Diophantine m-tuples, we proved results on the extensibility of some parametric families of $D(-1)$-pairs to quadruples in the ring $\mathbb{Z}[\sqrt{-t}], t > 0$.

$D(n)$-quintuples with square elements

Vinko Petričević

University of Zagreb, Zagreb, Croatia

vpetrice@math.hr

(joint work with Andrej Dujella and Matija Kazalicki)

For an integer n, a set of m distinct nonzero integers $\{a_1, a_2, \ldots, a_m\}$ such that $a_i a_j + n$ is a perfect square for all $1 \leq i < j \leq m$, is called a $D(n)$-m-tuple. We have shown that there are infinitely many essentially different $D(n)$-quintuples with square elements.

In this presentation, we will show computational background of this search.

Representation of integers of the form $x^2 + ky^2 - lz^2$

Supawadee Prugsapitak

Algebra and Applications Research Unit, Division of Computational Science Faculty of Science, Prince of Songkla University Hatyai, Songkhla 90110 Thailand

supawadee.p@psu.ac.th

(joint work with Nattaporn Thongngam)

In this talk, we will introduce a k-special where k is a positive integer as follows: A positive integer l is k-special if for every integer n there exist non-zero integers x, y, and z such that $n = x^2 + ky^2 - lz^2$. We will show that 1 is k-special if and only if k is not divisible by 4. Furthermore, for any odd integer k, we will provide infinite classes of k-special numbers and $2k$-special numbers.

On $D(-1)$-tuples in the ring $\mathbb{Z}[\sqrt{-k}], k > 0$

Ivan Soldo

Department of Matematics, University of Osijek, Osijek, Croatia

isoldo@mathos.hr

(joint work with Yasutsugu Fujita)

Suppose that n is a non-zero integer and R is a commutative ring. A set of m non-zero elements in R such that the product of any two distinct elements plus n is a perfect square in R is called a $D(n)$-m-tuple in R. We prove that there does not exist a $D(-1)$-quadruple $\{a, b, c, d\}$ in the ring $\mathbb{Z}[\sqrt{-k}]$, $k \geq 2$ with positive integers $a < b \leq 8a - 3$ and negative integers c and d. By using that result we show that such a $D(-1)$-pair $\{a, b\}$ cannot be extended to a $D(-1)$-quintuple $\{a, b, c, d, e\}$ in the ring $\mathbb{Z}[\sqrt{-k}]$ with integers c, d and e. Moreover, we apply the obtained result to the $D(-1)$-pair $\{p^i, q^j\}$ with an arbitrary different primes p, q and positive integers i, j.

Integer points on elliptic curves induced by a family of Diophantine triples involving balancing numbers

Artatrana Suna

Sambalpur University, Odisha, India

prasantamath@suniv.ac.in

(joint work with Prasanta Kumar Ray)

A Diophantine m-tuple is a set $\{a_1, a_2, ..., a_m\}$ of positive integers or rational numbers such that $a_i a_j + 1$ is a perfect square whenever $1 \leq i < j \leq m$. There was an old conjecture denying the existence of such a set having five elements of positive integers, *i.e.* a Diophantine quintuples, recently proved by He *et al.* [5]. If a Diophantine triple $\{a, b, c\}$ is extended to a quadruple

$\{a, b, c, d\}$, a stronger version of the above conjecture claims that the fourth element must be unique. It was first verified for the quadruple $\{1, 3, 8, 120\}$ [1]. For many parametric families of Diophantine triple the conjecture has been substantiated. If inclusion of x to the Diophantine triple $\{a, b, c\}$ makes it a quadruple then an elliptic curve $E : y^2 = (ax + 1)(bx + 1)(cx + 1)$ gets induced to it. Many works have been done in finding integer points on the elliptic curves induced by some Diophantine triples (see [3, 4, 6]) In this work, we take Diophantine triples of the sequence of balancing numbers $\{B_k\}$ [2], which is defined recursively by $B_k = 6B_{k-1} - B_{k-2}$ for $k \geq 2$ with $(B_0, B_1) = (0, 1)$. There are two other number sequences closely related to the sequence of balancing numbers. Lucas balancing sequence $\{C_k\}$ is defined by $C_k = 6C_{k-1} - C_{k-2}$ with $C_0 = 1$ and $C_1 = 3$, whereas cobalancing sequence $\{b_k\}$ satisfies $b_k = 6b_{k-1} - b_{k-2} + 2$ having initial terms $b_0 = b_1 = 0$. The sequence of balancing numbers satisfies $B_{k-1}B_{k+1} + 1 = B_k^2$ [7] and $B_kB_{k+1} = b_{k+1}(b_{k+1} + 1) = \frac{(B_k+b_{k+1})(B_k+b_{k+1}+1)}{2}$ [8], hence $\{B_{k-1}, 4B_k, B_{k+1}\}$ is Diophantine triple for $k \geq 2$. For $k \geq 2$ consider this Diophantine triples and denote the induced elliptic curve by $\mathfrak{B}_k$. In this study, we obtain the structure of the torsion subgroup of $\mathfrak{B}_k$ and find all integer points on it under the assumption that $rank(\mathfrak{B}_k(\mathbb{Q})) = 1$ with some other conditions.

[1] A. Baker, H. Davenport, *The equations* $3x^2 - 2 = y^2$ *and* $8x^2 - 7 = z^2$, Quart. J. Math. Oxford Ser. **20(2)** (1969), 129–137.

[2] A. Behera, G. K. Panda, *On the square roots of triangular numbers*, Fib. Quart. **37** (1999), 98–105.

[3] A. Dujella, *A parametric family of elliptic curves*, Acta Arith. **94** (2000), 87-101.

[4] A. Dujella, *Diophantine m-tuples and elliptic curves*, J. Theor. Nr. Bordx **13** (2001), 111-124.

[5] B. He, A. Togbé, V. Ziegler, *There is no Diophantine quintuple*, Trans. Amer. Math. Soc. **371(9)** (2019), 6665-6709.

[6] F. Najman, *Integer points on two families of elliptic curves*, Publ. Math. Debrecen **75** (2009), 401-418.

[7] G. K. Panda, *Some fascinating properties of balancing numbers,* Proceeding of the Eleventh International Conference on Fibonacci Numbers and Their Applications, (W. A.Webb, Ed.), Congr. Numer., (2009), 185–189.

[8] G. K. Panda, P. K. Ray, *Some links of balancing and cobalancing numbers with Pell and associated Pell numbers*, Bull. Inst. Math. Acad. Sin. (N.S.) **6** (2011), 41–72.

Diophantine triples with non-uniform recurrences

László Szalay

University of Sopron, Sopron, Hungary

szalay.laszlo@uni-sopron.hu

(joint work with Florian Luca)

In the presentation, we consider Diophantine triples with values in three different binary recurrence sequences. These are the Fibonacci and Pell sequences and the sequence of one more of powers of a given prime p. The novelty of the result is the appearance of three different sequences, as up to now the analogous problem had been investigated only for one sequence. This research was supported by the National Research, Development and Innovation Office Grant 2019-2.1.11-TÉT-2020-00165.