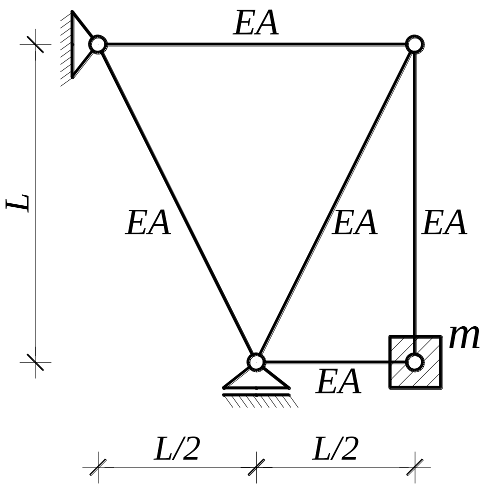
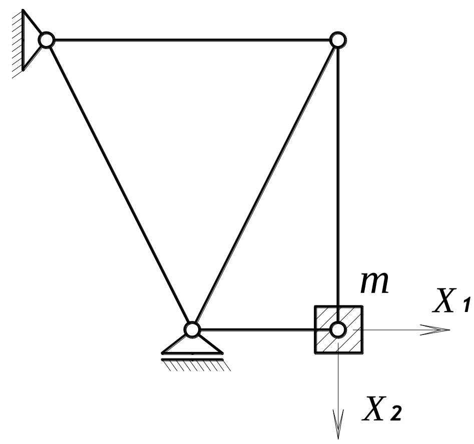
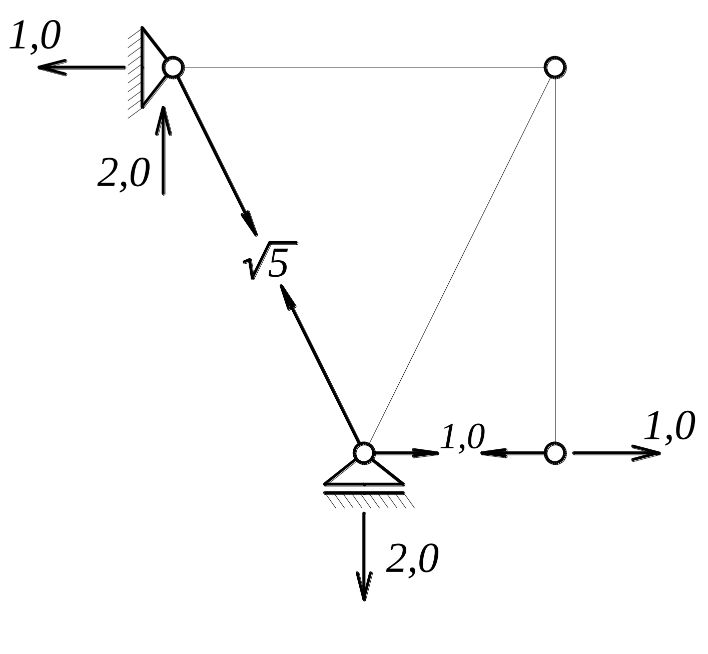
RAVNINSKA REŠETKA:

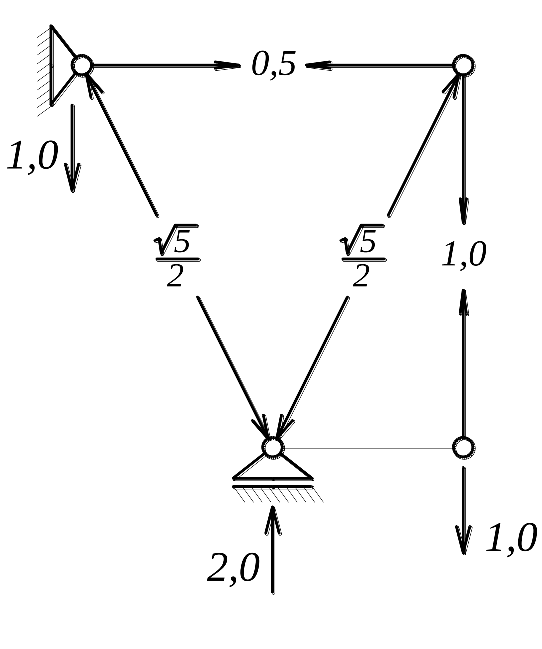


Ako se zanemari masa štapova i uzmemo u obzir samo koncentriranu masu u čvoru, dinamički stupnjevi slobode su:



Određivanje koeficijenata fleksibilnosti:





$$\delta_{11}=\frac{1}{EA}\left[1\cdot 1\cdot\frac{L}{2}+\sqrt{5}\cdot 1\cdot\sqrt{5}\cdot\frac{\sqrt{5}}{2}L\right]=\frac{PL}{2EA}\left(1+5\sqrt{5}\right)$$

$$\delta_{12}=\frac{1}{EA}\left[\sqrt{5}\cdot 1\cdot\frac{\sqrt{5}}{2}\cdot\left(-\frac{\sqrt{5}}{2}L\right)\right]=-\frac{5\sqrt{5}PL}{4EA}$$

$$\delta_{22}=\frac{1}{EA}\left[1\cdot 1\cdot L+2\cdot\frac{\sqrt{5}}{2}\cdot 1\cdot\frac{\sqrt{5}}{2}\cdot\frac{\sqrt{5}}{2}L+\frac{1}{2}\cdot\frac{1}{2}\cdot L\right]=\frac{5PL}{4EA}\left(1+\sqrt{5}\right)$$

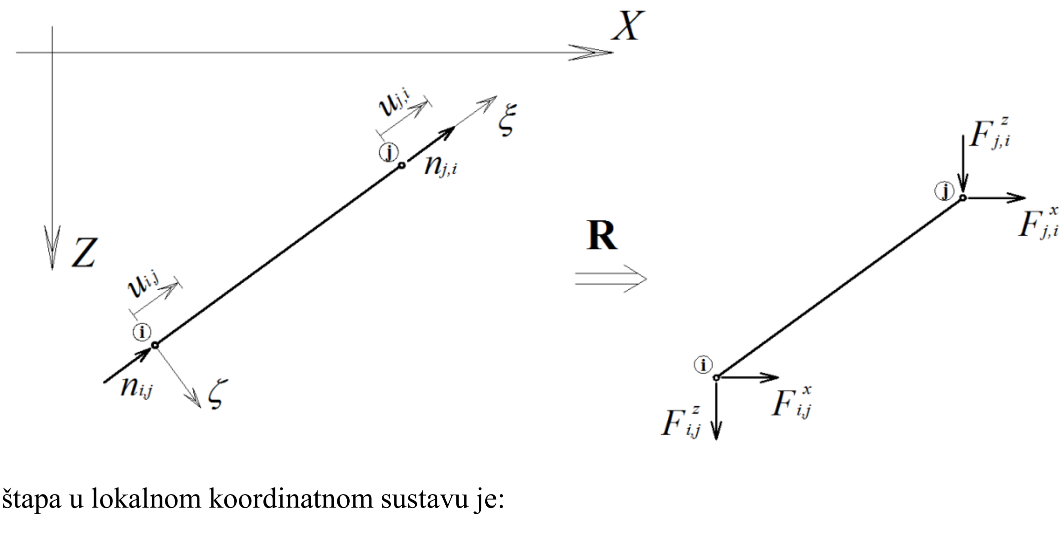
$$[F]=\begin{bmatrix}\frac{L\left(1+5\sqrt{5}\right)}{2EA} & -\frac{5\sqrt{5}L}{4EA}\\ -\frac{5\sqrt{5}L}{4EA} & \frac{5L\left(1+\sqrt{5}\right)}{4EA}\end{bmatrix}$$

Matrica krutosti može se odrediti inverzijom matrice fleksibilnosti:

$$[K]=[F]^{-1}=\begin{bmatrix}\frac{4\left(5\sqrt{5}-11\right)EA}{3L} & \frac{4\left(9\sqrt{5}-20\right)EA}{3L}\\ \frac{4\left(9\sqrt{5}-20\right)EA}{3L} & \frac{8\left(41\sqrt{5}-91\right)EA}{3L}\end{bmatrix}$$

<u>Određivanje matrice krutosti metodom pomaka:</u>

podsjetnik na metodu pomaka: lokalna matrica krutosti aksijalno opterećenog štapa i prijelaz iz lokalnog u globalni koordinatni sustav.



Uzdužna sila štapa u lokalnom koordinatnom sustavu je:

$$n_{i,j}=-n_{j,i}=k^a_{(i,j)}\left(u_{i,j}-u_{j,i}\right)$$

Ako je $\alpha_{(i,j)}$ kut između globalne osi X i lokalne osi ξ, matrica transformacije iz globalnog u lokalni sustav je (kod štapova rešetke nemamo momenta na krajevima štapa): $\mathbb{r}=\begin{bmatrix}c & -s\\ s & c\end{bmatrix}$ $\quad s=\sin\alpha_{(i,j)}$, $c=\cos\alpha_{(i,j)}$

Za transformaciju vektora sila i vektora pomaka na krajevima elemenata oblikujemo matricu: $\mathbb{R} = \begin{bmatrix} \mathbb{r} & 0 \\ 0 & \mathbb{r} \end{bmatrix}$.

Budući nam je potrebna matrica koja vrši transformaciju iz lokalnog u globalni sustav određujemo inverznu matricu $\mathbb{R}^{-1}$ (matrica rotacije je ortogonalna $\mathbb{R}^{-1} = \mathbb{R}^{\mathrm{T}}$):

$$\mathbb{r}^{-1} = \begin{bmatrix} c & s \\ -s & c \end{bmatrix}, \qquad \mathbb{R}^{-1} = \begin{bmatrix} \mathbb{r}^{-1} & 0 \\ 0 & \mathbb{r}^{-1} \end{bmatrix}.$$

Transformacija vektora sile iz lokalnog u globalni koordinatni sustav:

$$f_{i,j}^{g} = \mathbb{R}^{-1} f_{i,j}$$
$$f_{i,j}^{g} = \mathbb{R}^{-1} k_{i,j} u_{i,j}$$

$u_{i,j}$ su pomaci u lokalnom koordinatnom sustavu, te ih matricom transformacije prikazujemo u globalnom sustavu:

$$u_{i,j}^{g} = \mathbb{R}^{-1} u_{i,j}$$
$$u_{i,j} = \mathbb{R}\, u_{i,j}^{g}$$
$$f_{i,j}^{g} = \boxed{\mathbb{R}^{-1} k_{i,j}\, \mathbb{R}}\, u_{i,j}^{g}$$
$$k_{i,j}^{g} = \mathbb{R}^{-1} k_{i,j}\, \mathbb{R}$$

$k_{i,j}^{g}$ je matrica krutosti elementa u globalnom koordinatnom sustavu.

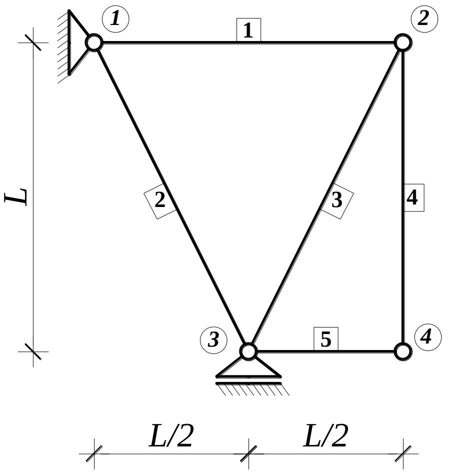
Marica krutosti štapa koji prenosi samo **uzdužnu silu** u lokalnom koordinatnom sustavu je:

$$k_{i,j} = \frac{EA}{L_e} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Transformacija u globalni koordinatni sustav:

$$k_{i,j}^{g} = \mathbb{R}^{-1} k_{i,j}\, \mathbb{R} = \frac{EA}{L_e} \begin{bmatrix} c^2 & -c\,s & -c^2 & c\,s \\ -c\,s & s^2 & c\,s & -s^2 \\ -c^2 & c\,s & c^2 & -c\,s \\ c\,s & -s^2 & -c\,s & s^2 \end{bmatrix}$$

Označavamo čvorove rešetke i elemente:



Matrice krutosti štapova u globalnom koordinatnom sustavu:

$c = 1;\ s = 0;$

$$k_{1,2} = \frac{EA}{L}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$c = \frac{1}{\sqrt{5}};\ s = -\frac{2}{\sqrt{5}};$

$$k_{1,3} = \frac{EA}{\frac{\sqrt{5}L}{2}}\begin{bmatrix} \frac{1}{5} & \frac{2}{5} & -\frac{1}{5} & -\frac{2}{5} \\ \frac{2}{5} & \frac{4}{5} & -\frac{2}{5} & -\frac{4}{5} \\ -\frac{1}{5} & -\frac{2}{5} & \frac{1}{5} & \frac{2}{5} \\ -\frac{2}{5} & -\frac{4}{5} & \frac{2}{5} & \frac{4}{5} \end{bmatrix} = \frac{EA}{L}\begin{bmatrix} \frac{2}{5\sqrt{5}} & \frac{4}{5\sqrt{5}} & -\frac{2}{5\sqrt{5}} & -\frac{4}{5\sqrt{5}} \\ \frac{4}{5\sqrt{5}} & \frac{8}{5\sqrt{5}} & -\frac{4}{5\sqrt{5}} & -\frac{8}{5\sqrt{5}} \\ -\frac{2}{5\sqrt{5}} & -\frac{4}{5\sqrt{5}} & \frac{2}{5\sqrt{5}} & \frac{4}{5\sqrt{5}} \\ -\frac{4}{5\sqrt{5}} & -\frac{8}{5\sqrt{5}} & \frac{4}{5\sqrt{5}} & \frac{8}{5\sqrt{5}} \end{bmatrix}$$

$c = \frac{1}{\sqrt{5}};\ s = \frac{2}{\sqrt{5}};$

$$k_{3,2} = \frac{EA}{\frac{\sqrt{5}L}{2}}\begin{bmatrix} \frac{1}{5} & -\frac{2}{5} & -\frac{1}{5} & \frac{2}{5} \\ -\frac{2}{5} & \frac{4}{5} & \frac{2}{5} & -\frac{4}{5} \\ -\frac{1}{5} & \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \\ \frac{2}{5} & -\frac{4}{5} & -\frac{2}{5} & \frac{4}{5} \end{bmatrix} = \frac{EA}{L}\begin{bmatrix} \frac{2}{5\sqrt{5}} & -\frac{4}{5\sqrt{5}} & -\frac{2}{5\sqrt{5}} & \frac{4}{5\sqrt{5}} \\ -\frac{4}{5\sqrt{5}} & \frac{8}{5\sqrt{5}} & \frac{4}{5\sqrt{5}} & -\frac{8}{5\sqrt{5}} \\ -\frac{2}{5\sqrt{5}} & \frac{4}{5\sqrt{5}} & \frac{2}{5\sqrt{5}} & -\frac{4}{5\sqrt{5}} \\ \frac{4}{5\sqrt{5}} & -\frac{8}{5\sqrt{5}} & -\frac{4}{5\sqrt{5}} & \frac{8}{5\sqrt{5}} \end{bmatrix}$$

$c = 0;\ s = 1;$

$$k_{4,2} = \frac{EA}{L}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

$c = 1;\ s = 0;$

$$k_{3,4} = \frac{EA}{\frac{L}{2}}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \frac{EA}{L}\begin{bmatrix} 2 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 \\ -2 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Globalna matrica krutosti:

$$K_G = \frac{EA}{L}\begin{bmatrix} 1+\frac{2}{5\sqrt{5}} & \frac{4}{5\sqrt{5}} & -1 & 0 & -\frac{2}{5\sqrt{5}} & -\frac{4}{5\sqrt{5}} & 0 & 0 \\ \frac{4}{5\sqrt{5}} & \frac{8}{5\sqrt{5}} & 0 & 0 & -\frac{4}{5\sqrt{5}} & -\frac{8}{5\sqrt{5}} & 0 & 0 \\ -1 & 0 & 1+\frac{2}{5\sqrt{5}} & -\frac{4}{5\sqrt{5}} & -\frac{2}{5\sqrt{5}} & \frac{4}{5\sqrt{5}} & 0 & 0 \\ 0 & 0 & -\frac{4}{5\sqrt{5}} & 1+\frac{8}{5\sqrt{5}} & \frac{4}{5\sqrt{5}} & -\frac{8}{5\sqrt{5}} & 0 & -1 \\ -\frac{2}{5\sqrt{5}} & -\frac{4}{5\sqrt{5}} & -\frac{2}{5\sqrt{5}} & \frac{4}{5\sqrt{5}} & 2+\frac{4}{5\sqrt{5}} & 0 & -2 & 0 \\ -\frac{4}{5\sqrt{5}} & -\frac{8}{5\sqrt{5}} & \frac{4}{5\sqrt{5}} & -\frac{8}{5\sqrt{5}} & 0 & \frac{16}{5\sqrt{5}} & 0 & 0 \\ 0 & 0 & 0 & 0 & -2 & 0 & 2 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Umetanje rubnih uvjeta: iz gornje matrice izostavljamo one retke i stupce za koje su poznati pomaci (za čvor 1 obje translacije i za čvor 3 vertikalni pomak)

$$K = \frac{EA}{L}\begin{bmatrix} 1+\frac{2}{5\sqrt{5}} & -\frac{4}{5\sqrt{5}} & -\frac{2}{5\sqrt{5}} & 0 & 0 \\ -\frac{4}{5\sqrt{5}} & 1+\frac{8}{5\sqrt{5}} & \frac{4}{5\sqrt{5}} & 0 & -1 \\ -\frac{2}{5\sqrt{5}} & \frac{4}{5\sqrt{5}} & 2+\frac{4}{5\sqrt{5}} & -2 & 0 \\ 0 & 0 & -2 & 2 & 0 \\ 0 & -1 & 0 & 0 & 1 \end{bmatrix}$$

Formulacija dinamičkog problema slobodnih oscilacija:

$$[M]\{\ddot{x}\} + [K]\{x\} = \{0\}$$

Pošto nemaju svi čvorovi pridruženu masu potrebno je izvršiti dinamičku kondenzaciju tih stupnjeva slobode i tako svesti dinamički problem na sustav s dva dinamička stupnja slobode:

$$\begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{m_{t22}} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{x}}_{\mathbf{t1}} \\ \ddot{\mathbf{x}}_{\mathbf{t2}} \end{Bmatrix} + \begin{bmatrix} \mathbf{k_{t11}} & \mathbf{k_{t12}} \\ \mathbf{k_{t21}} & \mathbf{k_{t22}} \end{bmatrix} \begin{Bmatrix} \mathbf{x_{t1}} \\ \mathbf{x_{t2}} \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{0} \end{Bmatrix}$$

Gdje su $\mathbf{x_{t1}}$ pomaci čvorova koji nemaju pridruženu masu, a $\mathbf{x_{t2}}$ pomaci čvorova s konačnom masom.

Dinamička kondenzacija:

$$[\mathbf{m_{t22}}]\{\ddot{\mathbf{x}}_{\mathbf{t2}}\} + \underbrace{\left([\mathbf{k_{t22}}] - [\mathbf{k_{t21}}][\mathbf{k_{t11}}]^{-1}[\mathbf{k_{t12}}]\right)}_{[\bar{\mathbf{k}}]}\{\mathbf{x_{t2}}\} = \{\mathbf{0}\}$$

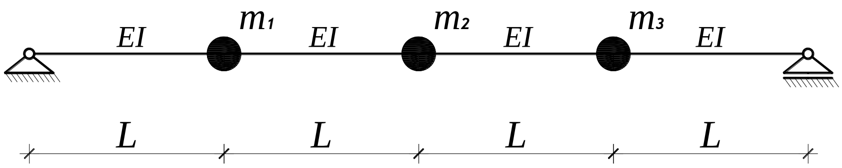
U našem primjeru:

$$\mathbf{k_{t22}} = \frac{EA}{L}\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}; \quad \mathbf{k_{t12}} = \frac{EA}{L}\begin{bmatrix} 0 & 0 \\ 0 & -1 \\ -2 & 0 \end{bmatrix}; \quad \mathbf{k_{t21}} = \frac{EA}{L}\begin{bmatrix} 0 & 0 & -2 \\ 0 & -1 & 0 \end{bmatrix}; \quad \mathbf{k_{t11}} = \frac{EA}{L}\begin{bmatrix} 1+\frac{2}{5\sqrt{5}} & -\frac{4}{5\sqrt{5}} & -\frac{2}{5\sqrt{5}} \\ -\frac{4}{5\sqrt{5}} & 1+\frac{8}{5\sqrt{5}} & \frac{4}{5\sqrt{5}} \\ -\frac{2}{5\sqrt{5}} & \frac{4}{5\sqrt{5}} & 2+\frac{4}{5\sqrt{5}} \end{bmatrix}$$

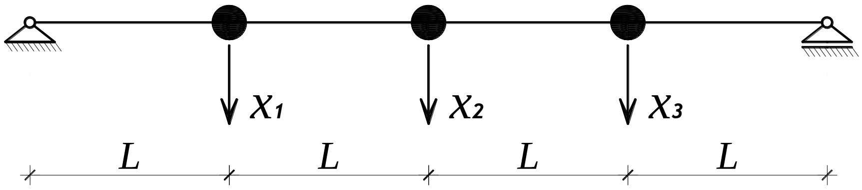
$$\bar{\mathbf{k}} = \begin{bmatrix} \frac{4\left(5\sqrt{5}-11\right)EA}{3L} & \frac{4\left(9\sqrt{5}-20\right)EA}{3L} \\ \frac{4\left(9\sqrt{5}-20\right)EA}{3L} & \frac{8\left(41\sqrt{5}-91\right)EA}{3L} \end{bmatrix}$$

Dobili smo istu matricu kao i inverijom matrice fleksibilnosti.

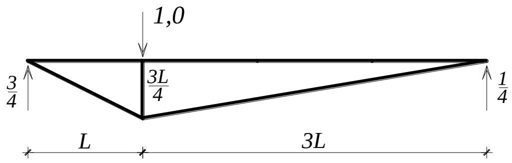
GREDA

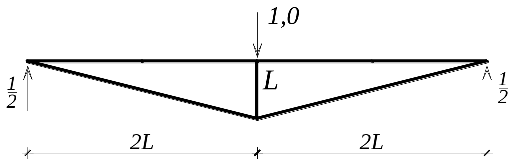


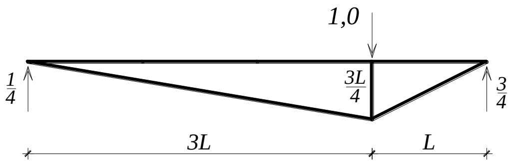
Dinamički stupnjevi slobode:



Matica fleksibilnosti:







$$\delta_{11} = \frac{1}{EI}\left[\frac{1}{2}\cdot L\cdot\frac{3L}{4}\cdot\frac{2}{3}\cdot\frac{3L}{4}+\frac{1}{2}\cdot 3L\cdot\frac{3L}{4}\cdot\frac{2}{3}\cdot\frac{3L}{4}\right]=\frac{3L^3}{4EI}$$

$$\delta_{12} = \frac{1}{EI}\left[\frac{1}{2}\cdot\frac{3L}{4}\cdot L\cdot\frac{2}{3}\cdot\frac{L}{2}+\frac{L}{2}\cdot L\cdot\frac{3L}{4}+\frac{1}{2}\cdot\frac{L}{4}\cdot L\cdot\left(\frac{L}{2}+\frac{1}{3}\cdot\frac{L}{2}\right)+\frac{1}{2}\cdot\frac{L}{2}\cdot 2L\cdot\frac{2}{3}\cdot L\right]=\frac{11L^3}{12EI}$$

$$\delta_{13} = \frac{1}{EI}\left[\frac{1}{2}\cdot\frac{3L}{4}\cdot L\cdot\frac{2}{3}\cdot\frac{1}{3}\cdot\frac{3L}{4}+2L\cdot\frac{L}{4}\cdot\frac{L}{2}+\frac{1}{2}\cdot\frac{L}{2}\cdot 2L\cdot\left(\frac{L}{4}+\frac{1}{3}\cdot\frac{L}{2}\right)+\frac{1}{2}\cdot\frac{L}{4}\cdot L\cdot\frac{2}{3}\cdot\frac{3L}{4}\right]=\frac{7L^3}{12EI}$$

$$\delta_{22} = \frac{1}{EI}\left[\frac{1}{2}\cdot 2L\cdot L\cdot\frac{2}{3}\cdot L\cdot 2\right]=\frac{4L^3}{3EI}$$

$$\delta_{23} = \frac{1}{EI}\left[\frac{1}{2}\cdot 2L\cdot L\cdot\frac{2}{3}\cdot\frac{L}{2}+\frac{L}{2}\cdot L\cdot\frac{5L}{8}+\frac{1}{2}\cdot\frac{L}{2}\cdot L\cdot\left(\frac{L}{2}+\frac{1}{3}\cdot\frac{L}{4}\right)+\frac{1}{2}\cdot\frac{L}{2}\cdot L\cdot\frac{2}{3}\cdot\frac{3L}{4}\right]=\frac{11L^3}{12EI}$$

$$\delta_{33} = \frac{1}{EI}\left[\frac{1}{2}\cdot\frac{3L}{4}\cdot 3L\cdot\frac{2}{3}\cdot\frac{3L}{4}+\frac{1}{2}\cdot\frac{3L}{4}\cdot L\cdot\frac{2}{3}\cdot\frac{3L}{4}\right]=\frac{3L^3}{4EI}$$

$$[F]=\begin{bmatrix} \frac{3L^3}{4EI} & \frac{11L^3}{12EI} & \frac{7L^3}{12EI} \\ \frac{11L^3}{12EI} & \frac{4L^3}{3EI} & \frac{11L^3}{12EI} \\ \frac{7L^3}{12EI} & \frac{11L^3}{12EI} & \frac{3L^3}{4EI} \end{bmatrix}=\frac{L^3}{12EI}\begin{bmatrix} 9 & 11 & 7 \\ 11 & 16 & 11 \\ 7 & 11 & 9 \end{bmatrix}$$

Matica krutosti:

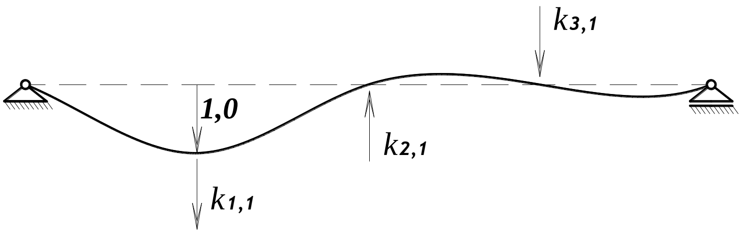
$$[K]=[F]^{-1}=\begin{bmatrix} \frac{69EI}{7L^3} & -\frac{66EI}{7L^3} & \frac{27EI}{7L^3} \\ -\frac{66EI}{7L^3} & \frac{96EI}{7L^3} & -\frac{66EI}{7L^3} \\ \frac{27EI}{7L^3} & -\frac{66EI}{7L^3} & \frac{69EI}{7L^3} \end{bmatrix}$$

<u>Direktno formiranje matice krutosti</u>

$k_{i,j}$ – sila (moment) na mjestu ″i″ od jediničnog pomaka (zaokreta) na mjestu ″j″ kad su svi ostali pomaci (zaokreti) u smjeru definiranih stupnjeva slobode spriječeni.

$$\{x\}=[F]\{P\}$$

x_1=1,0 , x_2=x_3=0

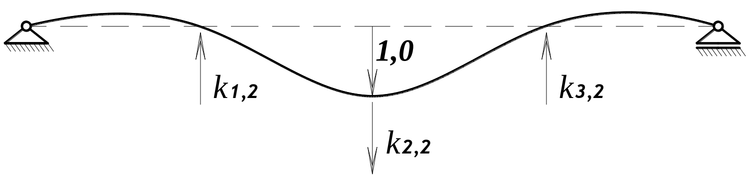


$$x_1=1,0=k_{1,1}\cdot\delta_{1,1}+k_{2,1}\cdot\delta_{1,2}+k_{3,1}\cdot\delta_{1,3}$$
$$x_2=0=k_{1,1}\cdot\delta_{2,1}+k_{2,1}\cdot\delta_{2,2}+k_{3,1}\cdot\delta_{2,3}$$
$$x_3=0=k_{1,1}\cdot\delta_{3,1}+k_{2,1}\cdot\delta_{3,2}+k_{3,1}\cdot\delta_{3,3}$$

rješenje sustava:

$$k_{1,1}=\frac{69EI}{7L^3},\ k_{2,1}=-\frac{66EI}{7L^3},\ k_{3,1}=\frac{27EI}{7L^3}$$

x_2=1,0 , x_1=x_3=0

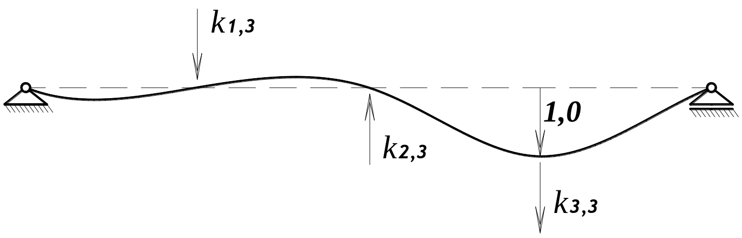


$$x_1=0=k_{1,2}\cdot\delta_{1,1}+k_{2,2}\cdot\delta_{1,2}+k_{3,2}\cdot\delta_{1,3}$$
$$x_2=1,0=k_{1,2}\cdot\delta_{2,1}+k_{2,2}\cdot\delta_{2,2}+k_{3,2}\cdot\delta_{2,3}$$
$$x_3=0=k_{1,2}\cdot\delta_{3,1}+k_{2,2}\cdot\delta_{3,2}+k_{3,2}\cdot\delta_{3,3}$$

rješenje sustava:

$$k_{1,2}=-\frac{66EI}{7L^3},\ k_{2,2}=\frac{96EI}{7L^3},\ k_{3,2}=-\frac{66EI}{7L^3}$$

x_3=1,0 , x_1=x_2=0



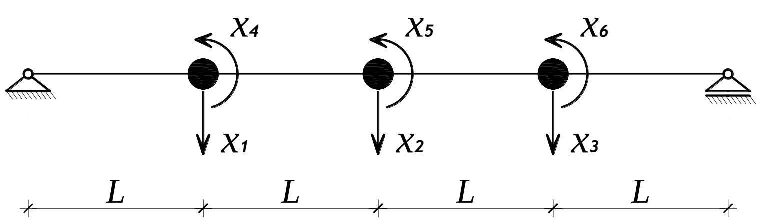
$$x_1=0=k_{1,3}\cdot\delta_{1,1}+k_{2,3}\cdot\delta_{1,2}+k_{3,3}\cdot\delta_{1,3}$$
$$x_2=0=k_{1,3}\cdot\delta_{2,1}+k_{2,3}\cdot\delta_{2,2}+k_{3,3}\cdot\delta_{2,3}$$
$$x_3=1,0=k_{1,3}\cdot\delta_{3,1}+k_{2,3}\cdot\delta_{3,2}+k_{3,3}\cdot\delta_{3,3}$$

rješenje sustava:

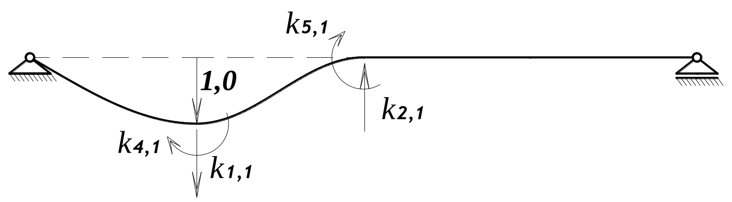
$$k_{1,3}=\frac{27EI}{7L^3},\ k_{2,3}=-\frac{66EI}{7L^3},\ k_{3,3}=\frac{69EI}{7L^3}$$

Definiranje matice krutosti metodom pomaka

Za svaku masu definiramo translacijski i rotacioni stupanj slobode, te tako oblikovanu matricu krutosti kondenziramo.

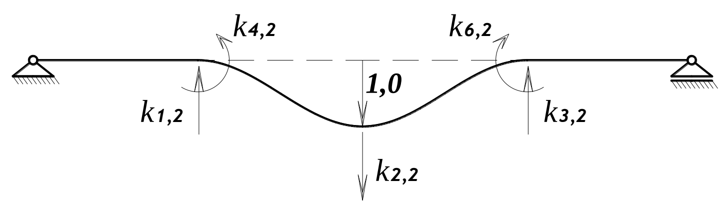


x_1=1,0 , x_2=…=x_6=0



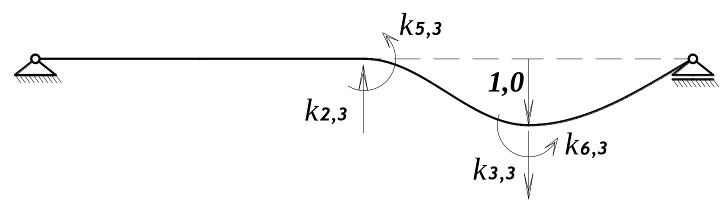
$$k_{1,1}=\frac{12EI}{L^3}+\frac{3EI}{L^3}=\frac{15EI}{L^3},\quad k_{2,1}=-\frac{12EI}{L^3},\quad k_{3,1}=0,$$
$$k_{4,1}=\frac{3EI}{L^2}-\frac{6EI}{L^2}=-\frac{3EI}{L^2},\quad k_{5,1}=-\frac{6EI}{L^3},\quad k_{6,1}=0.$$

x_2=1,0 , x_1=x_3=…=x_6=0



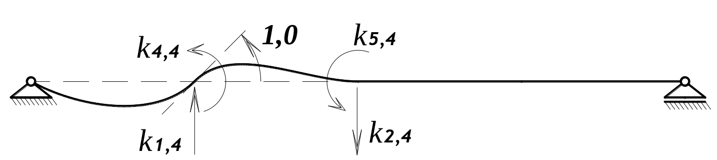
$$k_{1,2}=-\frac{12EI}{L^3},\quad k_{2,2}=\frac{24EI}{L^3},\quad k_{3,2}=-\frac{12EI}{L^3},$$
$$k_{4,2}=\frac{6EI}{L^2},\quad k_{5,2}=0,\quad k_{6,2}=-\frac{6EI}{L^2}.$$

x_3=1,0 , x_1=x_2=x_4=…=x_6=0



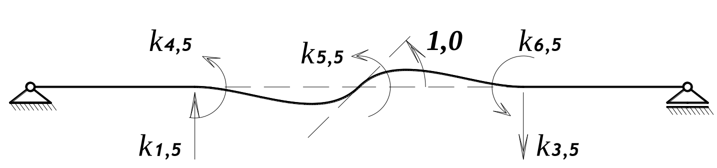
$$k_{1,3}=0,\quad k_{2,3}=-\frac{12EI}{L^3},\quad k_{3,3}=\frac{12EI}{L^3}+\frac{3EI}{L^3}=\frac{15EI}{L^3},$$
$$k_{4,3}=0,\quad k_{5,3}=\frac{6EI}{L^3},\quad k_{6,3}=\frac{6EI}{L^2}-\frac{3EI}{L^2}=\frac{3EI}{L^2}.$$

x_3=1,0 , x_1=x_2=x_4=…=x_6=0



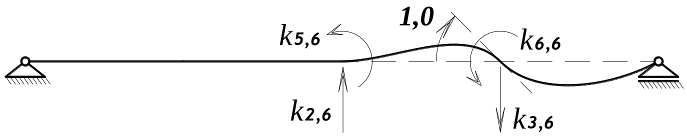
$$k_{1,4}=\frac{3EI}{L^2}-\frac{6EI}{L^2}=-\frac{3EI}{L^2},\quad k_{2,4}=\frac{6EI}{L^2},\quad k_{3,4}=0,$$
$$k_{4,4}=\frac{3EI}{L}+\frac{4EI}{L}=\frac{7EI}{L},\quad k_{5,4}=\frac{2EI}{L},\quad k_{6,4}=0.$$

x_4=1,0 , x_1=…=x_3=x_5=x_6=0



$$k_{1,5}=-\frac{6EI}{L^2},\quad k_{2,5}=0,\quad k_{3,5}=\frac{6EI}{L^2},$$
$$k_{4,5}=\frac{2EI}{L},\quad k_{5,5}=\frac{4EI}{L}\cdot 2=\frac{8EI}{L},\quad k_{6,5}=\frac{2EI}{L}.$$

$x_6=1,0$, $x_1=\ldots=x_5=0$



$$k_{1,6}=0,\quad k_{2,6}=-\frac{6EI}{L^2},\quad k_{3,6}=\frac{6EI}{L^2}-\frac{3EI}{L^2}=\frac{3EI}{L^2},$$
$$k_{4,6}=0,\quad k_{5,6}=\frac{2EI}{L},\quad k_{6,6}=\frac{3EI}{L}+\frac{4EI}{L}=\frac{7EI}{L}.$$

Dinamička kondenzacija

$$[\mathbf{M}]\{\ddot{\mathbf{x}}\}+[\mathbf{K}]\{\mathbf{x}\}=\mathbf{0}$$

$$\begin{bmatrix}\mathbf{m} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}\end{bmatrix}\begin{Bmatrix}\ddot{\mathbf{x}}_{\mathbf{t}}\\ \ddot{\mathbf{x}}_{\boldsymbol{\varphi}}\end{Bmatrix}+\begin{bmatrix}\mathbf{K}_{\mathbf{tt}} & \mathbf{K}_{\mathbf{t}\boldsymbol{\varphi}}\\ \mathbf{K}_{\boldsymbol{\varphi}\mathbf{t}} & \mathbf{K}_{\boldsymbol{\varphi\varphi}}\end{bmatrix}\begin{Bmatrix}\mathbf{x}_{\mathbf{t}}\\ \mathbf{x}_{\boldsymbol{\varphi}}\end{Bmatrix}=\begin{Bmatrix}\mathbf{0}\\ \mathbf{0}\end{Bmatrix}$$

Gdje je **m** matrica masa, a **I** je matica momenata inercije mase. Gornju jednadžbu rastavljamo na dvije matrične jednadžbe:

$$[\mathbf{m}]\{\ddot{\mathbf{x}}_{\mathbf{t}}\}+[\mathbf{K}_{\mathbf{tt}}]\{\mathbf{x}_{\mathbf{t}}\}+[\mathbf{K}_{\mathbf{t}\boldsymbol{\varphi}}]\{\mathbf{x}_{\boldsymbol{\varphi}}\}=\{\mathbf{0}\}$$
$$[\mathbf{I}]\{\ddot{\mathbf{x}}_{\boldsymbol{\varphi}}\}+[\mathbf{K}_{\boldsymbol{\varphi}\mathbf{t}}]\{\mathbf{x}_{\mathbf{t}}\}+[\mathbf{K}_{\boldsymbol{\varphi\varphi}}]\{\mathbf{x}_{\boldsymbol{\varphi}}\}=\{\mathbf{0}\}$$
$$\text{dinamička kondenzacija} -> [\mathbf{I}]\{\ddot{\mathbf{x}}_{\boldsymbol{\varphi}}\}=\{\mathbf{0}\}$$

Iz druge jednadžbe izražavamo rotacijske stupnjeve slobode:

$$\{\mathbf{x}_{\boldsymbol{\varphi}}\}=-[\mathbf{K}_{\boldsymbol{\varphi\varphi}}]^{-1}[\mathbf{K}_{\boldsymbol{\varphi}\mathbf{t}}]\{\mathbf{x}_{\mathbf{t}}\}$$

$$[\mathbf{m}]\{\ddot{\mathbf{x}}_{\mathbf{t}}\}+[\mathbf{K}_{\mathbf{tt}}]\{\mathbf{x}_{\mathbf{t}}\}-[\mathbf{K}_{\mathbf{t}\boldsymbol{\varphi}}][\mathbf{K}_{\boldsymbol{\varphi\varphi}}]^{-1}[\mathbf{K}_{\boldsymbol{\varphi}\mathbf{t}}]\{\mathbf{x}_{\mathbf{t}}\}=\{\mathbf{0}\}$$
$$[\mathbf{m}]\{\ddot{\mathbf{x}}_{\mathbf{t}}\}+\left([\mathbf{K}_{\mathbf{tt}}]-[\mathbf{K}_{\mathbf{t}\boldsymbol{\varphi}}][\mathbf{K}_{\boldsymbol{\varphi\varphi}}]^{-1}[\mathbf{K}_{\boldsymbol{\varphi}\mathbf{t}}]\right)\{\mathbf{x}_{\mathbf{t}}\}=\{\mathbf{0}\}$$
$$[\mathbf{m}]\{\ddot{\mathbf{x}}_{\mathbf{t}}\}+[\bar{\mathbf{K}}]\{\mathbf{x}_{\mathbf{t}}\}=\{\mathbf{0}\}$$

$$[K]=\left[\begin{array}{ccc:ccc}
\frac{15EI}{L^3} & -\frac{12EI}{L^3} & 0 & -\frac{3EI}{L^2} & -\frac{6EI}{L^2} & 0\\
-\frac{12EI}{L^3} & \frac{24EI}{L^3} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{6EI}{L^2}\\
0 & -\frac{12EI}{L^3} & \frac{15EI}{L^3} & 0 & \frac{6EI}{L^2} & -\frac{3EI}{L^2}\\
\hdashline
-\frac{3EI}{L^2} & \frac{6EI}{L^2} & 0 & \frac{7EI}{L} & \frac{2EI}{L} & 0\\
-\frac{6EI}{L^2} & 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & \frac{8EI}{L} & \frac{2EI}{L}\\
0 & -\frac{6EI}{L^2} & -\frac{3EI}{L^2} & 0 & \frac{2EI}{L} & \frac{7EI}{L}
\end{array}\right]$$

Nakon kondenzacije dobivamo:

$$\left[\bar{K}\right]=\begin{bmatrix} \frac{69EI}{7L^3} & -\frac{66EI}{7L^3} & \frac{27EI}{7L^3} \\ -\frac{66EI}{7L^3} & \frac{96EI}{7L^3} & -\frac{66EI}{7L^3} \\ \frac{27EI}{7L^3} & -\frac{66EI}{7L^3} & \frac{69EI}{7L^3} \end{bmatrix}$$