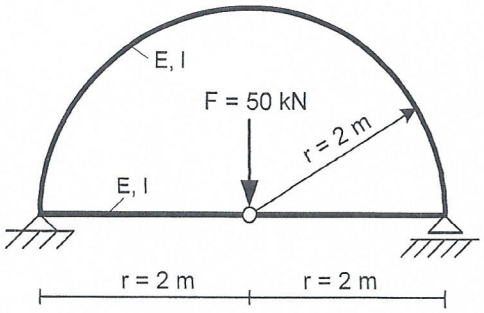
OTPORNOST MATERIJALA 2

2. kolokvij, 16. svibnja 2016.

A

Ime i prezime:

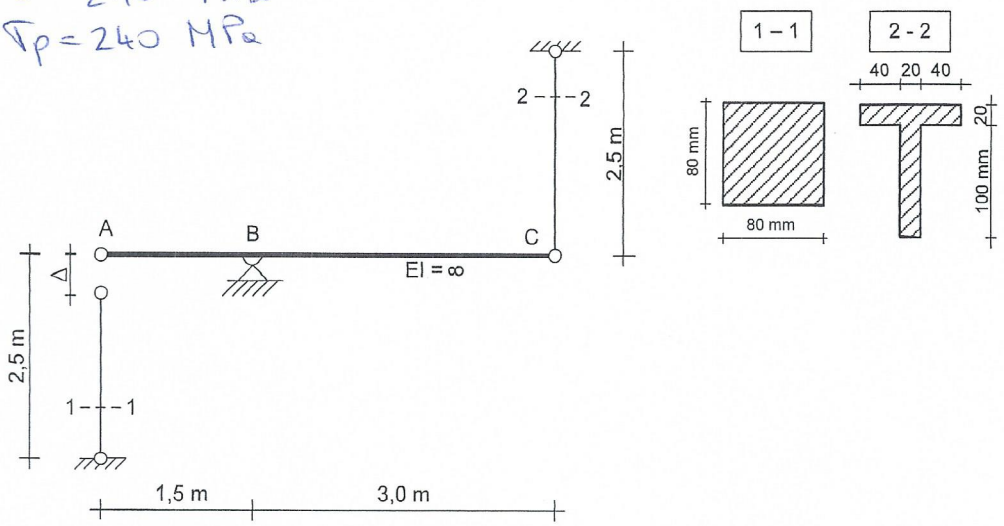
1. Za zadani sustav treba odrediti i nacrtati dijagrame unutarnjih sila ***M, T*** i ***N*** primjenom principa o minimumu potencijalne energije.



2. Koliki je faktor sigurnosti štapa *2* na izvijanje ako je štap *1* izveden kraći za $\Delta = 1\ mm$?

$E = 2{\cdot}10^5\ MPa$

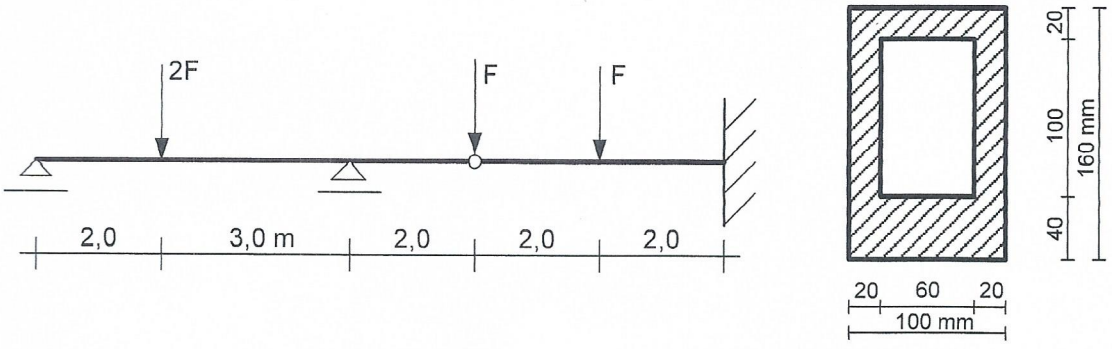
$\sigma_p = 240\ MPa$



3. Za zadani statički sustav treba po teoriji plastičnosti odrediti dopušteno opterećenje *F* ako je zadano:

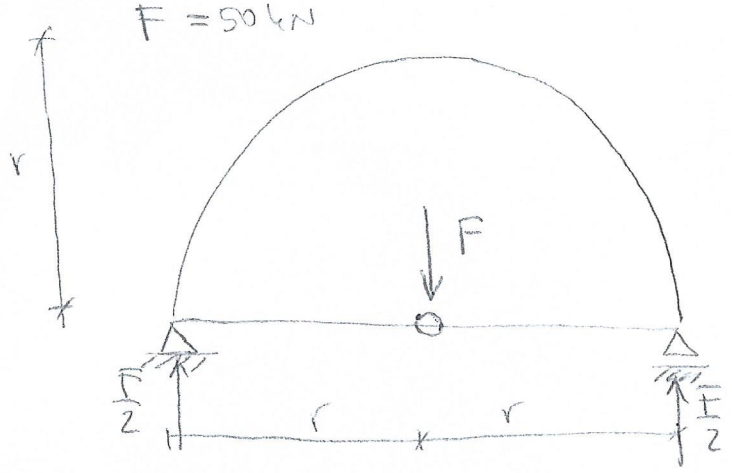
$\sigma_T = 250\ MPa$

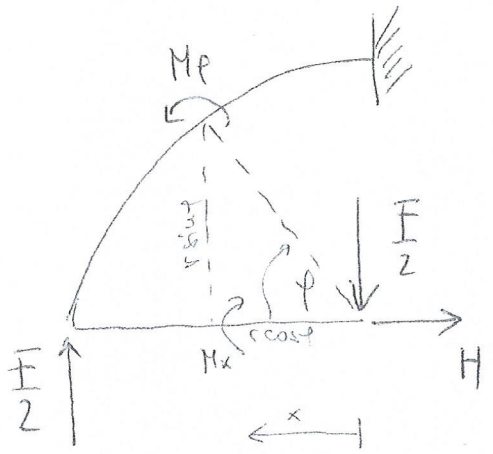
$k = 1{,}5$



(1) $r = 2\,m$

$F = 50\,kN$





$$\delta_H = \frac{\partial U}{\partial H} = 0$$

$$\frac{1}{EI}\int_0^r M_x \underbrace{\frac{\partial M_x}{\partial H}}_{0} dx - \frac{1}{EI}\int_0^{\pi/2} M_\varphi \frac{\partial M_\varphi}{\partial H} r\,d\varphi = 0 \;/\cdot EI$$

$$M_x = -\frac{F}{2}\cdot x \qquad \frac{\partial M_x}{\partial H} = 0$$

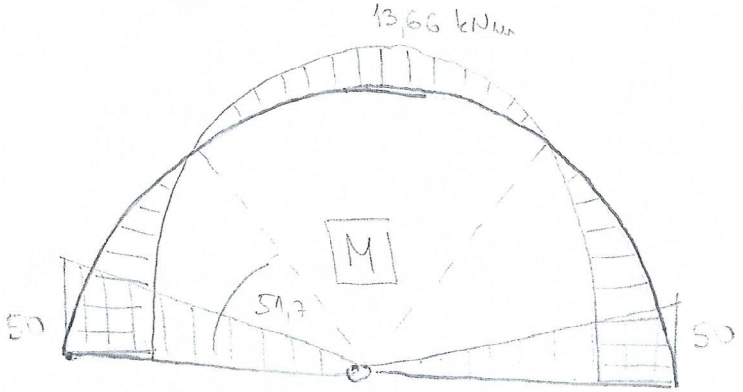
$$M_\varphi = -H\cdot r\cdot \sin\varphi + \frac{F}{2} r\cos\varphi + \frac{F}{2} r(1-\cos\varphi)$$
$$= -H\cdot r\sin\varphi + \frac{F}{2}\cdot r$$

$$\frac{\partial M_\varphi}{\partial H} = -r\sin\varphi$$

$$\int_0^{\pi/2}\left(-H\cdot r\sin\varphi + \frac{F}{2}\cdot r\right)(-r\sin\varphi)\,r\,d\varphi = 0$$

$$\int_0^{\pi/2}\left(Hr^2\sin^2\varphi - \frac{F}{2}r^2\sin\varphi\right)d\varphi = 0$$

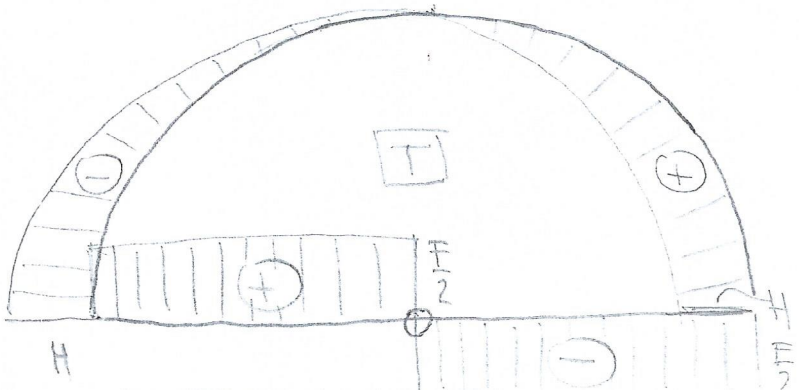
$$H\cdot r^2\frac{\pi}{4} - \frac{F}{2}r^2\cdot 1 = 0 \qquad H = 0{,}637\,F = \underline{31{,}83\ kN}$$

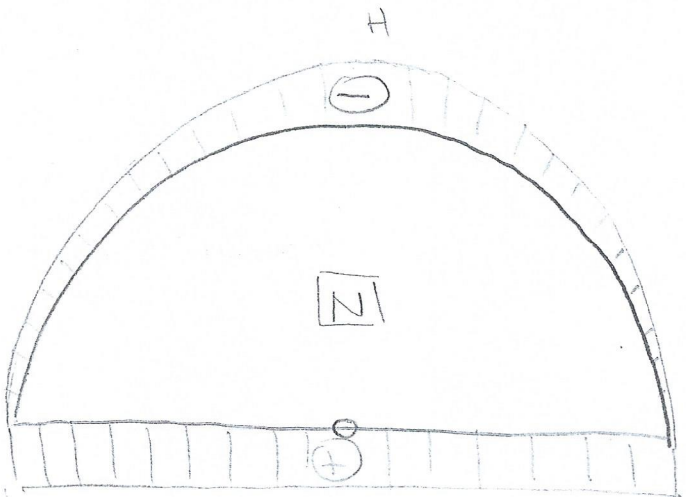


$$M(\varphi=\pi/2) = -31{,}83\cdot 2\cdot 1 + 25\cdot 2 = -13{,}66$$

$$0 = -31{,}83\cdot 2\cdot \sin\varphi_0 + 25\cdot 2$$

$$\varphi_0 = 51{,}7°$$

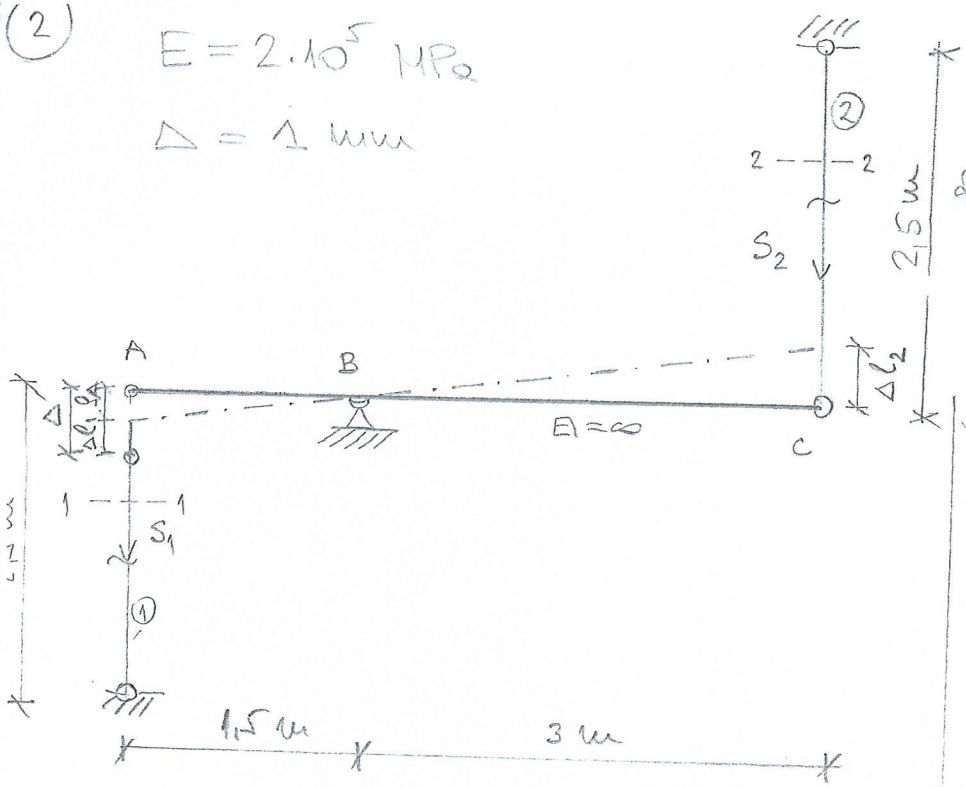


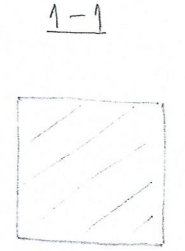


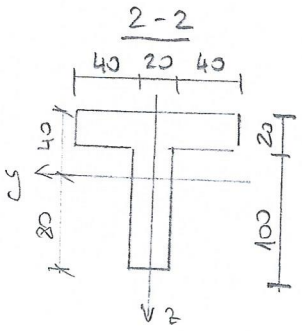
(2)

$E = 2 \cdot 10^5$ MPa

$\Delta = 1$ mm







$A_2 = 100 \cdot 20 + 100 \cdot 20 = 4000$ mm^2

$y_T = \frac{100 \cdot 20 \cdot 110 + 100 \cdot 20 \cdot 50}{4000} = 80$ mm

$I_y = \frac{20 \cdot 100^3}{12} + 20 \cdot 100 \cdot 30^2 + \frac{100 \cdot 20^3}{12} + 100 \cdot 20 \cdot$

$I_y = 5{,}133 \cdot 10^6$ mm^4

$I_z = \frac{100 \cdot 20^3}{12} + \frac{20 \cdot 100^3}{12} = 1{,}733 \cdot 10^6$ mm^4 $= I_{min}$

$A_1 = 6400$ mm^2

U.R. $\quad \Sigma M_B = 0 \Rightarrow S_1 = 2\,S_2$

U.D. $\quad \Delta - \Delta l_1 = \delta_A$

$\frac{\delta_A}{1{,}5} = \frac{\Delta l_2}{3{,}0} \Rightarrow \frac{\Delta - \Delta l_1}{1{,}5} = \frac{\Delta l_2}{3{,}0} \Rightarrow \Delta - \Delta l_1 = 0{,}5\,\Delta l_2$

$\Delta - \frac{S_1 \cdot l}{E \cdot A_1} = 0{,}5 \frac{S_2 \cdot l}{E \cdot A_2} \quad / \cdot \frac{E \cdot A_2}{l}$

$\frac{\Delta E A_2}{l} - 2 \frac{A_2}{A_1} S_2 = 0{,}5\,S_2$

$1{,}75\,S_2 = \frac{\Delta \cdot E \cdot A_2}{l} \qquad S_2 = \frac{1 \cdot 2 \cdot 10^5 \cdot 4000}{1{,}75 \cdot 2500} = 182{,}9$ kN

$i_{min} = 20{,}81$

$\lambda = \frac{l_i}{i_{min}} = 120{,}2$

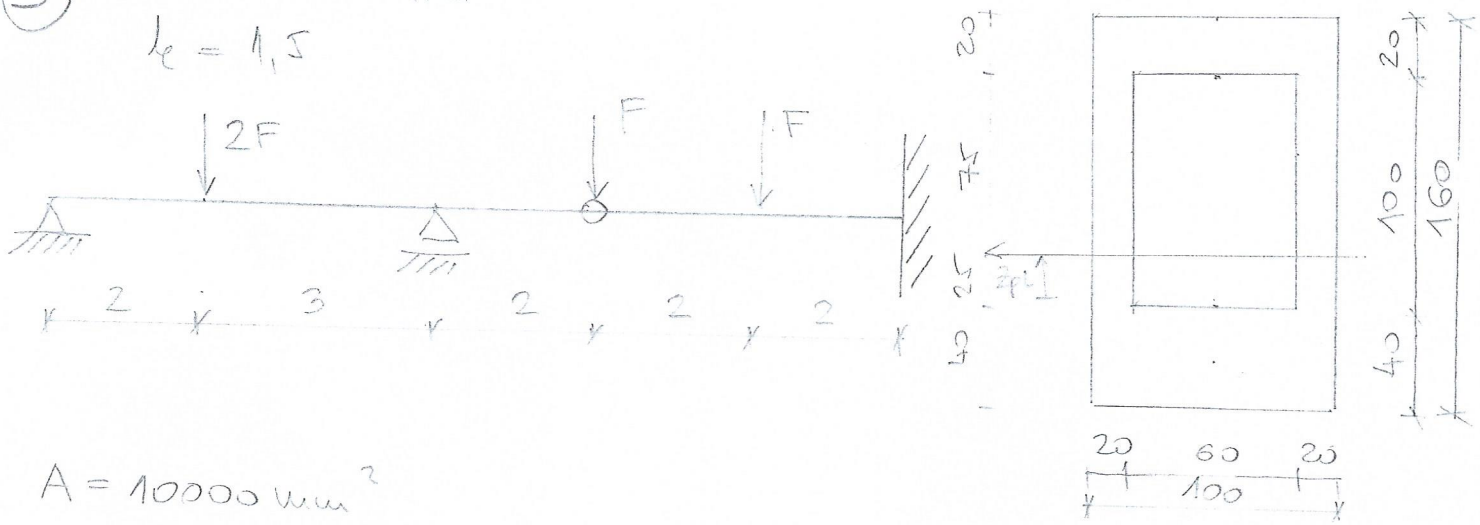
$\lambda_p = \sqrt{\frac{E \pi}{\sigma_p}} = 97$

$S_{KE}^{(2)} = \frac{\pi^2 \cdot I_{min} \cdot E}{l_i^2} = \frac{\pi^2 \cdot 1{,}733 \cdot 10^6 \cdot 2{,}0 \cdot 10^5}{2500^2} = 547{,}3$ kN

$S_{dop}^{(2)} = \frac{S_{KE}^{(2)}}{k_i} \qquad k_i = \frac{S_{dop}}{S_{KE}} = 2{,}99$

(3) $\sigma_T = 250$ MPa

$k_c = 1{,}5$



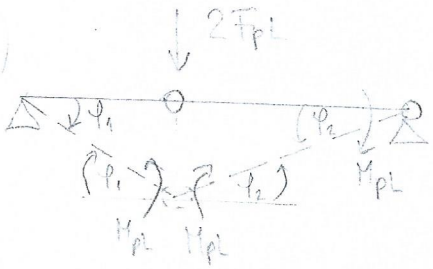
$A = 10000\ mm^2$

$\frac{A}{2} = 100 \cdot 40 + 2 \cdot z_{pL} \cdot 20 \qquad z_{pL} = 25$

$W_{pL} = 100 \cdot 40 \cdot 45 + 2 \cdot 20 \cdot 25 \cdot 12{,}5 + 100 \cdot 20 \cdot 85 + 2 \cdot 75 \cdot 20 \cdot 37{,}5 = 0{,}475 \cdot 10^6\ mm^3$

$M_{pL} = \sigma_T \cdot W_{pL} = 118{,}75\ kNm$

a)



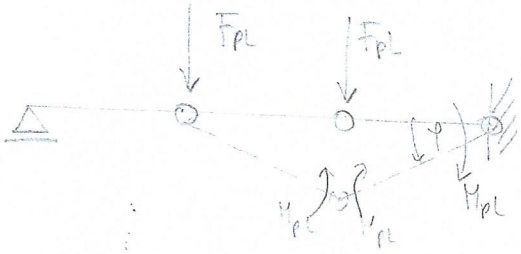
$2F_{pL} \cdot 2\varphi_1 - 2M_{pL}\varphi_2 - M_{pL}\varphi_1 = 0$

$\varphi_1 = \frac{3}{2}\varphi_2$

$2F_{pL} \cdot 2 \cdot \frac{3}{2}\varphi_2 = 2M_{pL}\varphi_2 + \frac{3}{2}M_{pL}\varphi_2$

$F_{pL} = \frac{7}{12}M_{pL}$

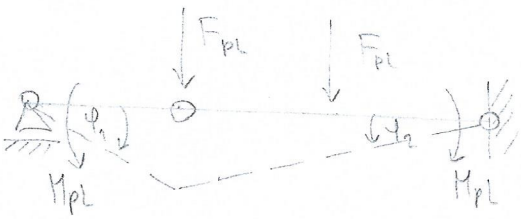
b)



$F_{pL} \cdot 2 \cdot \varphi - 3M_{pL} \cdot \varphi = 0$

$F_{pL} = \frac{3}{2}M_{pL}$

c)



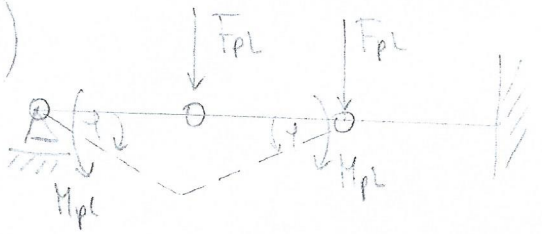
$\varphi_1 = 2\varphi_2$

$F_{pL} \cdot 4\varphi_2 + F_{pL} \cdot 2\varphi_2 - M_{pL}\varphi_1 - M_{pL}\varphi_2 = 0$

$6F_{pL} \cdot \varphi_2 = 3M_{pL}\varphi_2$

$F_{pL} = \frac{1}{2}M_{pL}$ Mijenjamo!

d)



$F_{pL} \cdot 2\varphi - 2M_{pL} \cdot \varphi = 0$

$F_{pL} = M_{pL}$

$F_{pL,min} = \frac{1}{2}M_{pL} = 59{,}4\ kN$

$F_{dop} = \frac{F_{pL,min}}{} = 39{,}6\ kN$