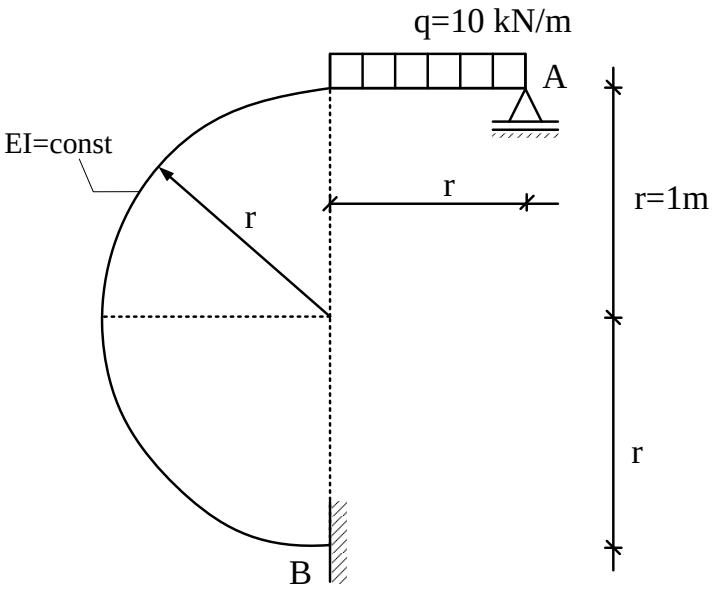
2. KOLOKVIJ iz "OTPORNOSTI MATERIJALA 2" GRUPA A

SVEUČILIŠTE U ZAGREBU
GRAĐEVINSKI FAKULTET
ZAVOD ZA TEHNIČKU MEHANIKU

27. 05. 2017.

Ime i prezime:_____

1. Za zadani sistem prikazan na slici treba odrediti dijagrame unutarnjih sila M, T i N.



2. Potrebno je odrediti koeficijent sigurnosti sustava ako je zadano:

$\sigma_P = 210\,MPa$

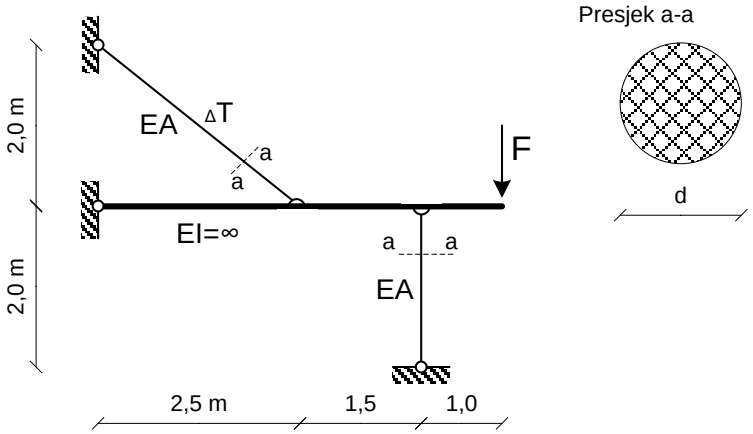
$\Delta T = 10\,K$

$\alpha_T = 1,2 \cdot 10^{-5}\,K^{-1}$

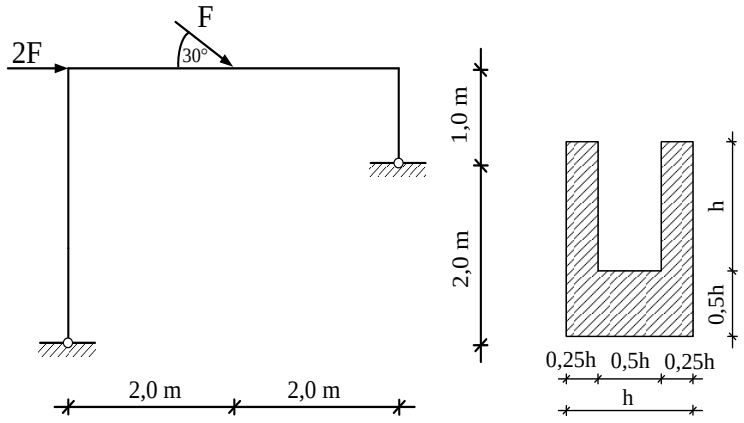
$E = 2,1 \cdot 10^5\,MPa$

$d = 50\,mm$

$F = 10\,kN$

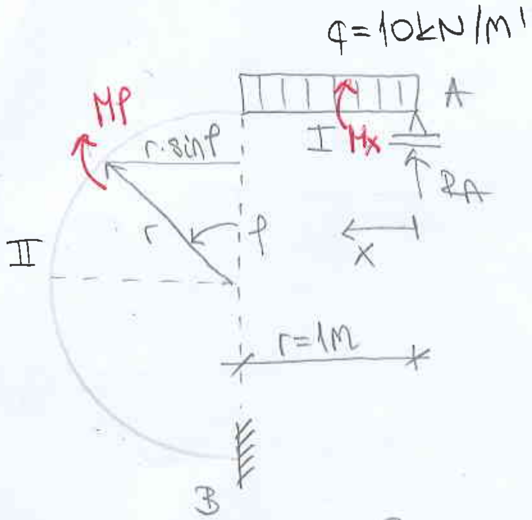


3. Za sistem opterećen prema slici (*F*=20 kN) potrebno je po teoriji plastičnosti odrediti dimenziju ''h'' prikazanog poprečnog presjeka, ako je granica tečenja $\sigma_T = 240\,Mpa$ i koeficijent sigurnosti $k = 2,0$.
Koliki je faktor oblika zadanog poprečnog presjeka?



30bod

$EI = const$



2

$$\frac{\partial U}{\partial R_A} = \emptyset \quad 1$$

$$M_x = R_A \cdot x - \frac{q \cdot x^2}{2} \quad 2$$

$$\frac{\partial M_x}{\partial R_A} = x$$

$$M_\varphi = R_A \cdot r(1+\sin\varphi) - q \cdot r^2\left(\frac{1}{2}+\sin\varphi\right)$$

$$\frac{\partial M_\varphi}{\partial R_A} = r(1+\sin\varphi) \quad 3$$

3

$$\frac{\partial U}{\partial R_A} = \frac{1}{EI}\int_0^r \left(R_A \cdot x - \frac{q \cdot x^2}{2}\right) \cdot x \cdot dx + \frac{1}{EI}\int_0^\pi r\left[R_A(1+\sin\varphi) - q \cdot r\left(\frac{1}{2}+\sin\varphi\right)\right] \cdot r(1+\sin\varphi)\, r\, d\varphi = \emptyset$$

$$\int_0^r \left(R_A \cdot x^2 - \frac{q \cdot x^3}{2}\right)dx + r^3\int_0^\pi R_A \cdot (1+2\sin\varphi+\sin^2\varphi) - q \cdot r\left(\frac{1}{2}+\sin\varphi+\frac{1}{2}\sin\varphi+\sin^2\varphi\right)d\varphi = \emptyset$$

$$R_A \cdot \frac{r^3}{3} - \frac{q \cdot r^4}{8} + r^3 R_A \cdot \left(\pi + 2 \cdot 2 + \frac{\pi}{2}\right) - r^4 \cdot q\left(\frac{1}{2}\pi + 2 + \frac{1}{2} \cdot 2 + \frac{\pi}{2}\right) = 0 \,/ : r^3$$

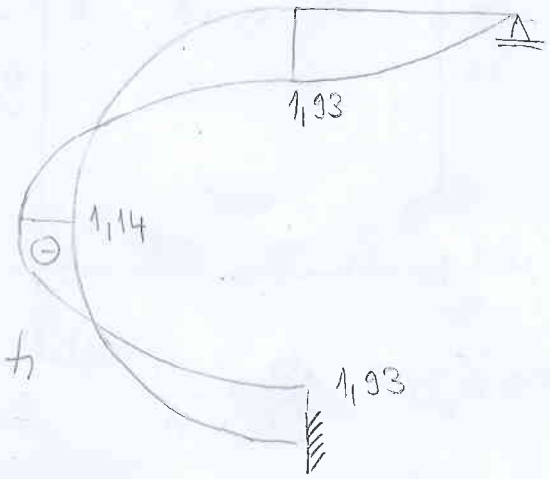
2

$$\frac{R_A}{3} - \frac{q \cdot r}{8} + R_A\left(\frac{3}{2}\pi + 4\right) - q \cdot r(\pi + 3) = \emptyset$$

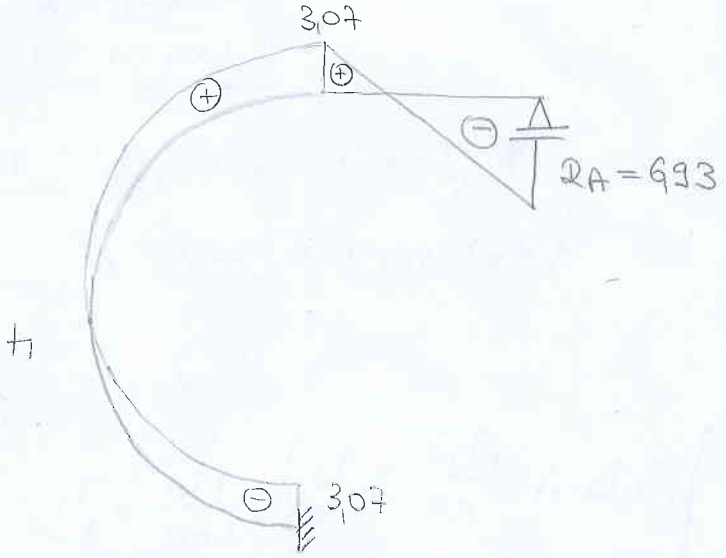
$$R_A\left(\frac{1}{3} + \frac{3}{2}\pi + 4\right) = q \cdot r\left(\pi + 3 + \frac{1}{8}\right) \qquad r = 1m$$

$$R_A \cdot 9{,}046 = q \cdot 6{,}27 \longrightarrow \boxed{R_A = 6{,}93\,kN} \quad 5$$

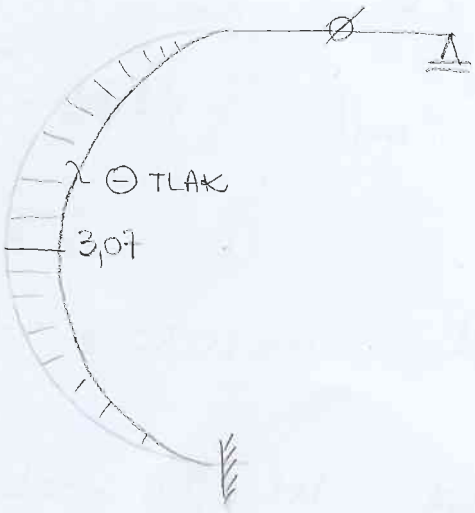
(M)

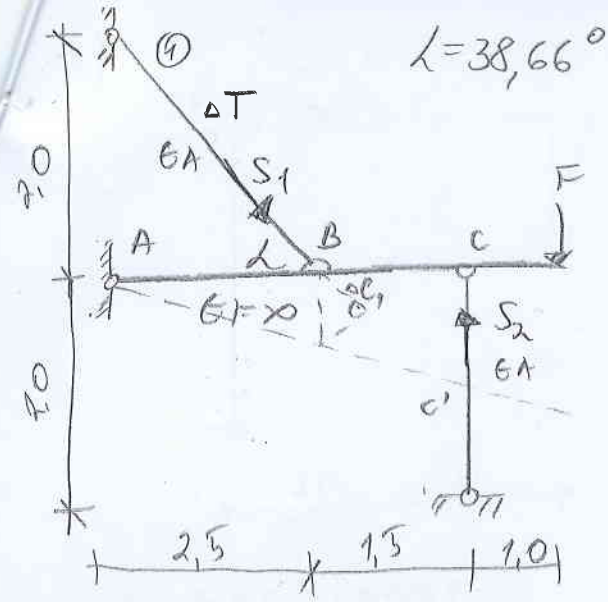


(T)



(N)





$\lambda = 38,66°$

$\sigma_p = 210\ MPa$

$\Delta T = 10\ K$

$\alpha_T = 1,2 \cdot 10^{-5}\ K^{-1}$

$E = 2,1 \cdot 10^5\ MPa$

$d = 50\ mm$

$F = 10\ kN$

U.R. $S_1 2,5 \sin\lambda + F \cdot 5 = S_2 \cdot 4$

$S_2 = 0,625 \cdot S_1 \sin\lambda + 1,25 F$

U.D. $\frac{\delta_C}{4} = \frac{\delta_B}{2,5}$ $\quad \delta_B = 0,625 \cdot \delta_C$ $\quad \delta_C = \frac{S_2 \cdot l_2}{EA}$

$\delta_B = \frac{\Delta l_1}{\sin\lambda} = \left| \Delta l_1 = -\frac{S_1 l_1}{EA} + \alpha_T \Delta T l_1 \right|$

$0,625 \cdot \sin\lambda\ S_2 l_2 = -S_1 l_1 + \alpha_T \Delta T l_1 EA$

$0,625^2 \cdot \sin^2\lambda \cdot l_2 S_1 + 0,625 \cdot \sin\lambda \cdot l_2 \cdot 1,2 F = -S_1 l_1 + \alpha_T \Delta T l_1 EA$

$$S_1 = \frac{\alpha_T \Delta T l_1 EA - 0,625 \cdot l_2 \cdot 1,2 F \sin\lambda}{l_1 + 0,625^2 \cdot \sin^2\lambda \cdot l_2}$$

$S_1 = 42,51\ kN$

$S_2 = 29,09\ kN$

$i_{min} = \frac{d}{4} = 12,5$; $\quad \lambda_1 = \frac{l_i^{(1)}}{i_{min}} = \frac{3202}{12,5} = 236,16$; $\quad \lambda_2 = \frac{l_{i1}^{(2)}}{i_{min}} = 160$

$\lambda_p = \sqrt{\frac{\pi^2 \cdot E}{\sigma_p}} = 99,34$ $\quad I_{min} = \frac{d^4 \pi}{64} = 306,79 \cdot 10^3\ mm^4$

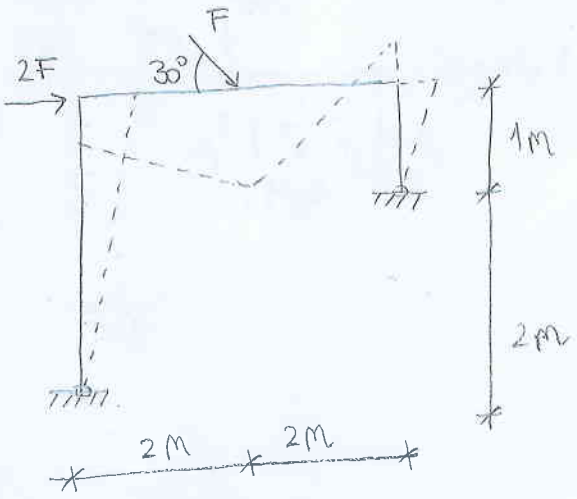
KRITIČNA SILA

$F_{kr}^{(1)} = \frac{\pi^2 \cdot E \cdot I_{min}}{l_{i(1)}^2} = 62,02\ kN$ $\quad k_1 = \frac{F_{kr}^{(1)}}{S_1} = 1,46$

$F_{kr}^{(2)} = \frac{\pi^2 \cdot E \cdot I_{min}}{l_{i(2)}^2} = 158,97\ kN$ $\quad k_2 = \frac{F_{kr}^{(2)}}{S_2} = 5,46$

KOEFICIJENT SUSTAVA $k_1 = 1,46$

3.)



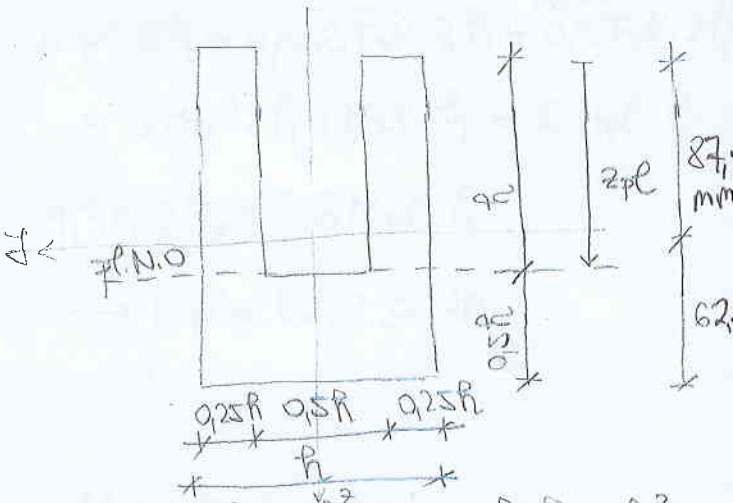
$F_V = F \cdot \sin 30° = 0{,}5F$

$F_H = F \cdot \cos 30° = 0{,}866F$

$k = 2{,}0$

$F = 20kN$

$\sigma_f = 240MPa$



$A = R \cdot 0{,}5R + 2 \cdot 0{,}25R \cdot R = R^2$

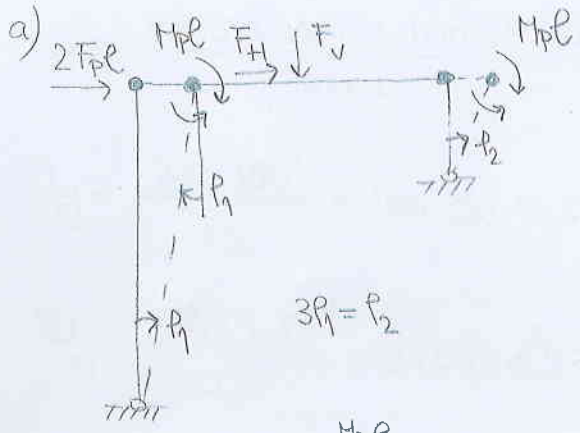
$\frac{A}{2} = 2 \cdot 0{,}25R \cdot 2pl \rightarrow \underline{2pl = R}$

$W_{pl} = 0{,}5 \cdot R \cdot R \cdot \frac{0{,}5 \cdot R}{2} + 2 \cdot R \cdot 0{,}25R \cdot \frac{R}{2}$

$W_{pl} = 0{,}375R^3$

$F_{pl} = F \cdot k = 20 \cdot 2{,}0 = 40kN$

a)



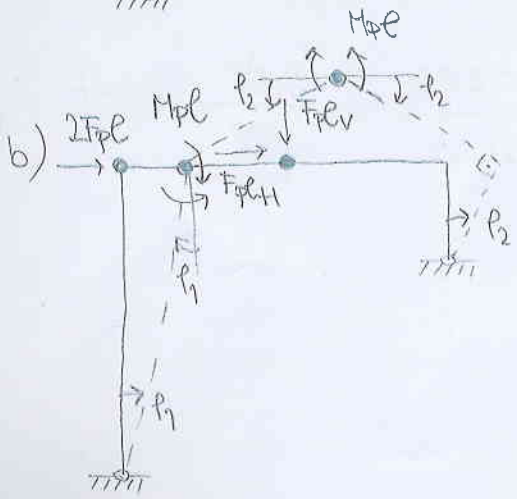
F_{pl}^H

$2F_{pl} \cdot 3f_1 + 0{,}866F_{pl} \cdot 3f_1 = M_{pl} \cdot f_1 + M_{pl} \cdot f_2 = 4M_{pl} \cdot f_1$

$F_{pl} = 0{,}465M_{pl} \rightarrow \underline{M_{pl} = 86{,}02kNm}$

Mjerodavno

b)



F_{pl}^H F_{pl}^V

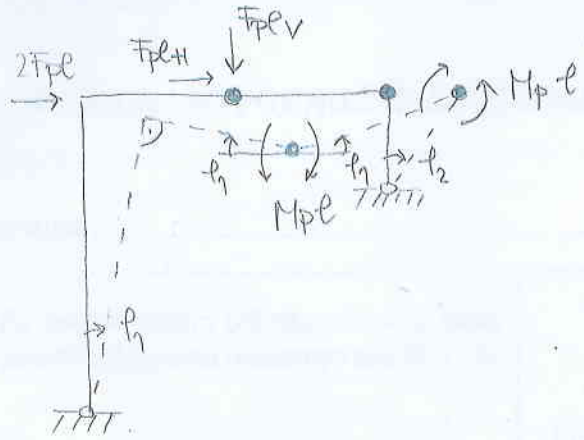
$2 \cdot F_{pl} \cdot 3f_1 + 0{,}866 \cdot F_{pl} \cdot 3f_1 - 0{,}5F_{pl} \cdot f_2 \cdot 2 =$

$= 3M_{pl}f_2 + M_{pl}f_1$

$5{,}598F_{pl} = 10M_{pl}$

$F_{pl} = 1{,}786M_{pl} \rightarrow M_{pl} = 22{,}39kNm$

c)



$$2F_{pl}\cdot 3f_1 + \underbrace{0{,}866F_{pl}}_{F_{plH}}\cdot 3f_1 + \underbrace{0{,}5F_{pl}}_{F_{plV}}\cdot 2f_1 = 3M_{pl}\cdot f_1 + M_{pl}\cdot f_2 = 6M_{pl}\cdot f_1$$

$$9{,}598F_{pl} = 6M_{pl}f_1$$

$$\longrightarrow M_{pl} = 63{,}99\,kNm$$

Mjerodavno: $M_{pl} = 86{,}02\,kNm$ $\qquad M_{pl} = W_{pl}\cdot\sigma_T$

$$M_{pl} = 86{,}02\,kNm = \sigma_T\cdot 0{,}375\,R^3$$

$$R = \sqrt[3]{\frac{86{,}02\cdot 10^6}{240\cdot 0{,}375}} = 98{,}50\,mm$$

odaberano: $R = 100\,mm$ $\qquad W_{pl} = 0{,}375\cdot 100^3 = 375000\,mm^3$

FAKTOR OBLIKA

$$z_T = \frac{100\cdot 50\cdot 25 + 2\cdot 100\cdot 25\cdot 100}{10000} = 62{,}5\,mm$$

$$I_y = \frac{100\,150^3}{12} + 100\cdot 150\cdot 12{,}5^2 - \left(\frac{50\cdot 100^3}{12} + 50\cdot 100\cdot 37{,}5^2\right) = 19{,}27\cdot 10^6\,mm^4$$

$$W_y = \frac{19{,}27\cdot 10^6}{87{,}5} = 220228{,}57\,mm^3$$

$$f_{OB} = \frac{W_{pl}}{W_y} = \frac{375000}{220228{,}57} = 1{,}70$$