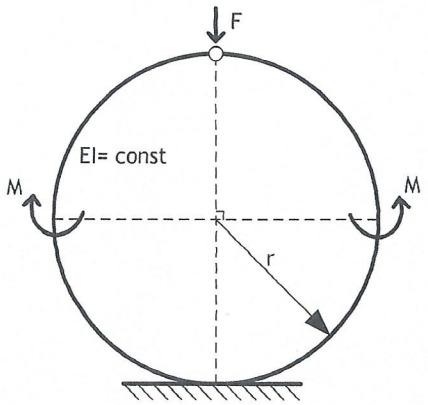
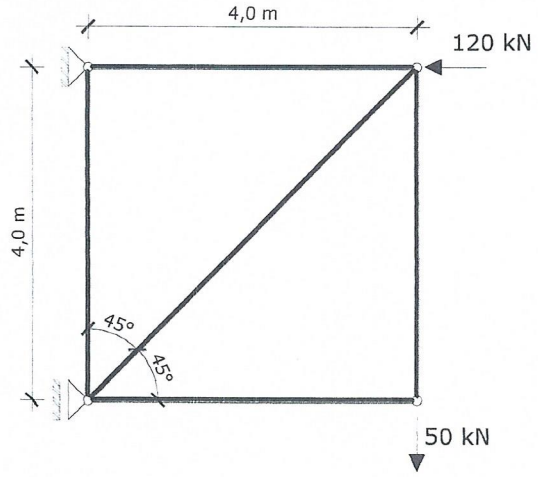
OTPORNOST MATERIJALA II, 2. kolokvij – 16.05.2016.
grupa B

Ime i prezime:

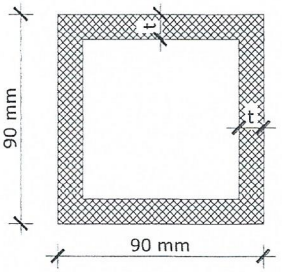
1.] Za zadani sustav prikazan na slici treba principom o minimumu potencijalne energije deformacija nacrtati dijagrame momenta savijanja, poprečnih i uzdužnih sila. Zadano je: $M = 80$ **kNm,** $F = 30$ **kN,** $r = 2$ **m.**



2.] Odredi minimalno potrebnu debljinu stjenke poprečnog presjeka – **t** kako ne bi došlo do gubitka stabilnosti (izvijanja) sustava prikazanog na slici. Zadano je: $E = 2{,}1 \cdot 10^5$ **MPa,** $\sigma_P = 200$ **MPa.** Za dobiveni poprečni presjek potrebno je odrediti da li se sustav izvija u elastičnom području.

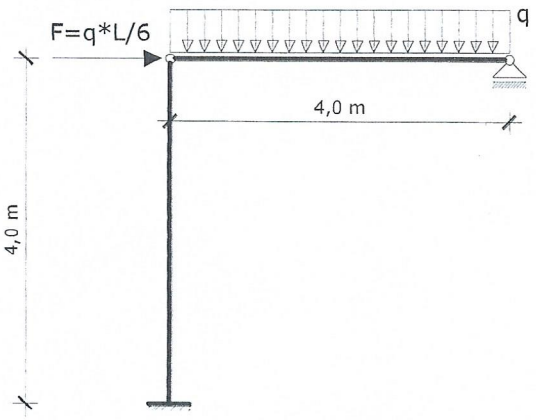


POPREČNI PRESJEK

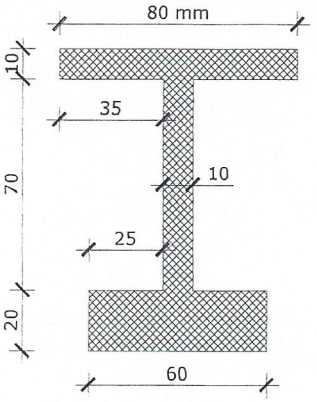


3.] Prema teoriji plastičnosti potrebno je odrediti dopuštene vrijednosti opterećenja (F i q) zadanog sustava, ako je granica tečenja materijala $\sigma_T = 300$ MPa. Koliki je faktor oblika zadanog poprečnog presjeka?

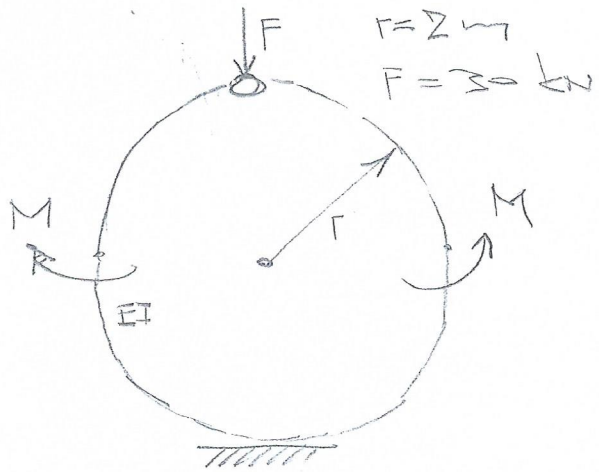
$k = 1{,}5$

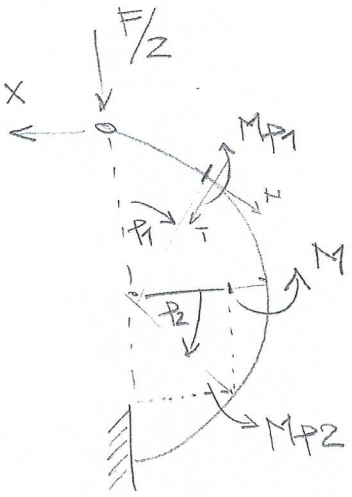


POPREČNI PRESJEK



1. ZADATAK: $M = 60\ kNm$





$$\frac{\partial U}{\partial X} = 0 \qquad U = 2\int_0^{\pi/2} \frac{M_{p1}^2 r\,d\varphi_1}{\Sigma EI} + 2\int_0^{\pi/2} \frac{M_{p2}^2 r\,d\varphi_2}{\Sigma EI}$$

$$\frac{\partial U}{\partial X} = \frac{2}{EI}\int_0^{\pi/2} M_{p1}\frac{\partial M_{p1}}{\partial X} r\,d\varphi_1 + \frac{2}{EI}\int_0^{\pi/2} M_{p2}\frac{\partial M_{p2}}{\partial X} r\,d\varphi_2 = 0$$

$$M_{p1} = -\frac{F}{2}\cdot r\sin\varphi_1 - X\cdot r(1-\cos\varphi_1) \qquad \frac{\partial M_{p1}}{\partial X} = -r(1-\cos\varphi_1)$$

$$M_{p2} = -\frac{F}{2} r\cos\varphi_2 - X\cdot r(1+\sin\varphi_2) - M \qquad \frac{\partial M_{p2}}{\partial X} = -r(1+\sin\varphi_2)$$

$$\frac{2}{EI}\int_0^{\pi/2}\left(-\frac{F}{2} r\sin\varphi_1 - Xr(1-\cos\varphi_1)\right)\left(-r(1-\cos\varphi_1)\right) r\,d\varphi_1 + \frac{2}{EI}\int_0^{\pi/2}\left(-\frac{F}{2} r\cos\varphi_2 - Xr(1+\sin\varphi_2) - M\right)$$

$$\cdot\left(-r(1+\sin\varphi_2)\right) r\,d\varphi_2 = 0$$

$$3\int_0^{\pi/2}\left(-\frac{F}{2}\sin\varphi_1\cos\varphi_1 - X\cos\varphi_1 + X\cos^2\varphi_1 + \frac{F}{2}\sin\varphi_1 + X - X\cos\varphi_1\right) d\varphi_1 +$$

$$+ r^2\int_0^{\pi/2}\left(\frac{F}{2} r\cos\varphi_2 + Xr + Xr\sin\varphi_2 + M + \frac{F}{2} r\cos\varphi_2\sin\varphi_2 + Xr\sin\varphi_2 + Xr\sin^2\varphi_2 + \right.$$

$$\left. + M\sin\varphi_2\right) d\varphi_2 = 0$$

$$3\left[-\frac{F}{2}\cdot\frac{1}{2} - X + X\frac{\pi}{4} + \frac{F}{2} + X\frac{\pi}{2} - X\right] + r^2\left[\frac{F}{2} r + Xr\frac{\pi}{2} + Xr + M\frac{\pi}{2} + \frac{F}{2} r\cdot\frac{1}{2} + Xr + Xr\frac{\pi}{4} + M\right] = 0$$

$$-3\frac{F}{4} - 2X + \frac{\pi}{4} X + \frac{\pi}{2} X + \frac{3}{2}\frac{F}{2} + 2X + X\frac{\pi}{2} + X\frac{\pi}{4} + M + M\frac{\pi}{2} = 0$$

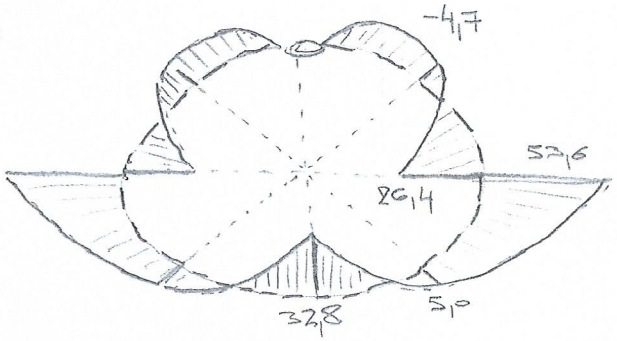
$$r + 3\frac{\pi}{2} X + M + M\frac{\pi}{2} = 0$$

$$3\frac{\pi}{2} X = -102{,}5$$

$$X = -28{,}19\ kN \quad (\text{TLAK})$$

DIJAGRAMI:

M



Dio 1:

$M_0 = \emptyset$

$M_{\frac{\pi}{4}} = -21,21 + 15,51 = -4,70$ kNm

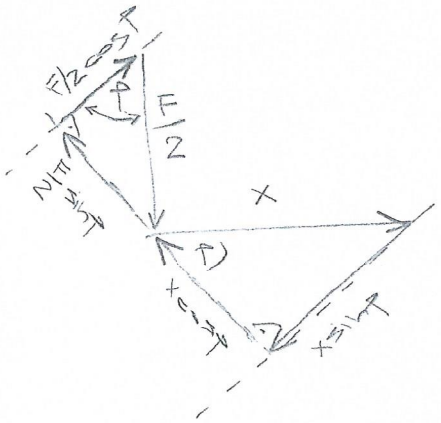
$M_{\frac{\pi}{2}} = -30 + 56,38 = 26,38$ kNm

Dio 2:

$M_0 = -30 + 56,38 - 80 = -53,62$ kNm

$M_{\frac{\pi}{4}} = -21,21 + 96,25 - 80 = -4,96$ kN.

$M_{\frac{\pi}{2}} = 32,76$ kNm



$T = X \sin\varphi - \frac{F}{2}\cos\varphi$

$T_0 = -15$ kN $\quad T_{45} = 9,33 \quad T_{90} = 28,19$ [kN]

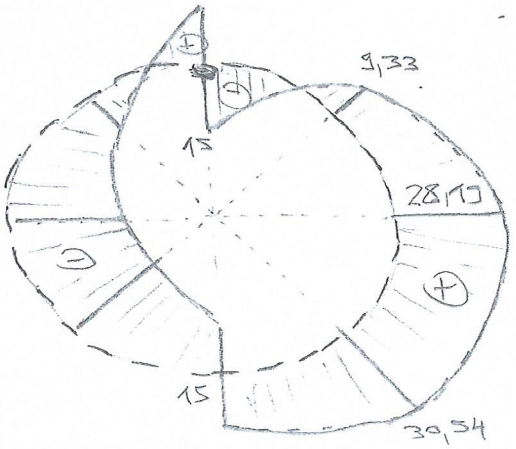
$T_{135} = 30,54 \quad T_{180} = +15$

$N = -X\cos\varphi - \frac{F}{2}\sin\varphi$

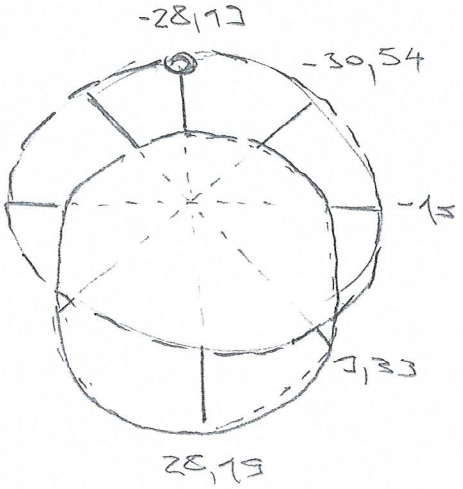
$N_0 = -28,19 \quad N_{45} = -30,54 \quad N_{90} = -15$ [kN]

$N_{135} = 9,33 \quad N_{180} = 28,19$

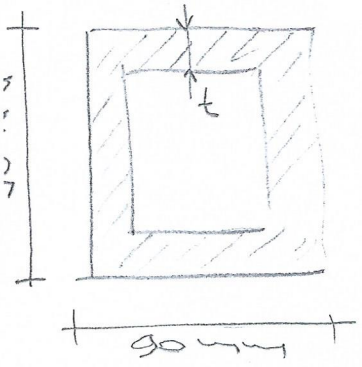
T

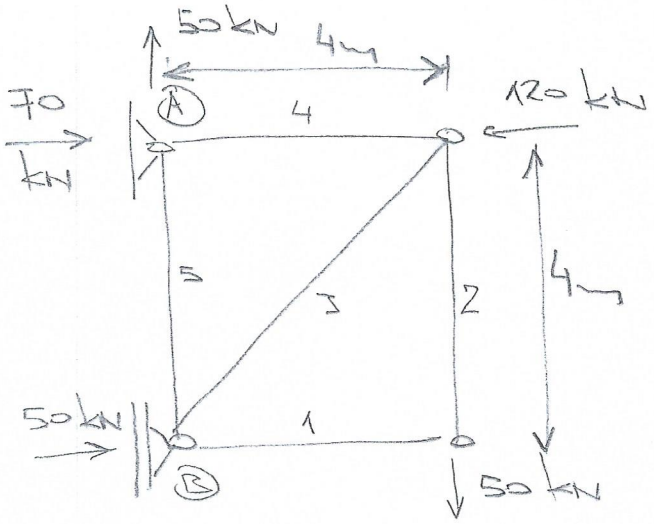


N



2. ZADATAK:





$\Sigma M_A = 0$

$F_B = 50\ kN$

$\Sigma F_H = 0$

$F_{HA} = 70\ kN$

$\Sigma F_V = 0$

$F_{VA} = 50\ kN$

$E = 2{,}1 \cdot 10^5\ MPa$

$\sigma_p = 200\ MPa$

$L_i = 4\sqrt{2}$

$L_i = 5656{,}9\ mm$

$S_1 = 0$

$S_2 = 50\ kN$ (VLAK)

$S_3 \cdot \cos 45^\circ = 50\ kN \rightarrow S_3 = 70{,}71\ kN$ (TLAK)

$S_4 = 70\ kN$ (TLAK)

$S_5 = 50\ kN$ (VLAK)

$$S_{kr} < \frac{\pi^2 \cdot E \cdot I_{min}}{L_i^2} = 70710$$

$$I_{min} = \frac{70710 \cdot (5656{,}9)^2}{\pi^2 \cdot 2{,}1 \cdot 10^5} = 1{,}09174 \cdot 10^6\ mm^4$$

$$I_{min} = \frac{90^4}{12} - \frac{(90-2t)^4}{12}$$

$$\frac{90^4}{12} - \frac{(90-2t)^4}{12} = 1{,}09174 \cdot 10^6$$

$$(90-2t)^4 = 52509120 \rightarrow 90 - 2t = 85{,}125$$

$$t = 2{,}44\ mm$$

$$t_{od} = 2{,}5\ mm$$

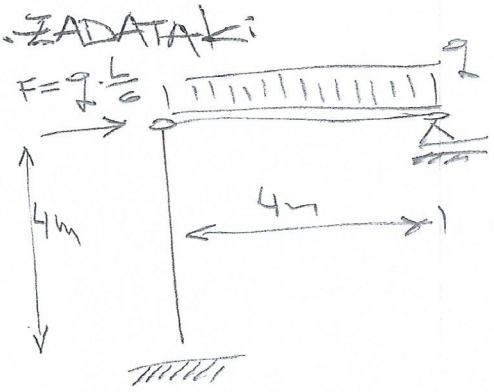
$$\lambda_p = \sqrt{\frac{\pi^2 \cdot E}{\sigma_p}} = 101{,}8$$

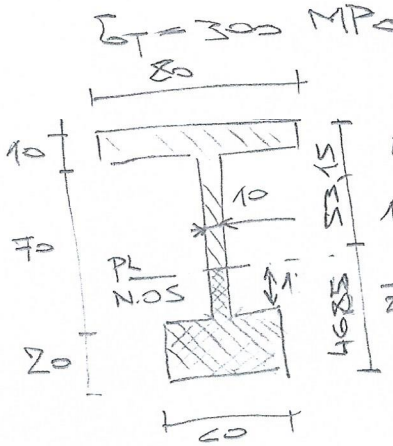
$$\lambda = \frac{L_i}{I_{min}} = 158{,}3$$

} izvijanje je u elast. području.

$$I_{min} = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{1{,}11745 \cdot 10^6}{875}} = 35{,}74\ mm$$

3. ZADATAK





$\sigma_T = 300$ MPa

$A = 2700$ mm^2

$1350 = 60 \cdot 20 + z_{PL} \cdot 10$

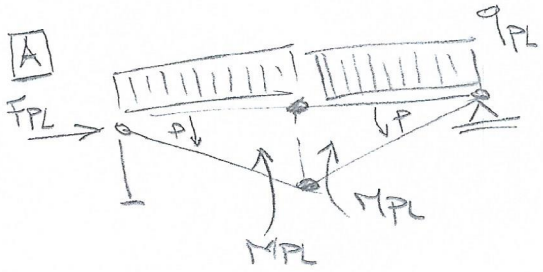
$z_{PL} = 15$ mm

$W_{PL} = 80 \cdot 10 \cdot 60 + 55 \cdot 10 \cdot 27{,}5 + 15 \cdot 10 \cdot 7{,}5 + 60 \cdot 20 \cdot 25$

$W_{PL} = 94250$ mm^3

$M_{PL} = \sigma_T \cdot W_{PL} = 28{,}275$ kNm

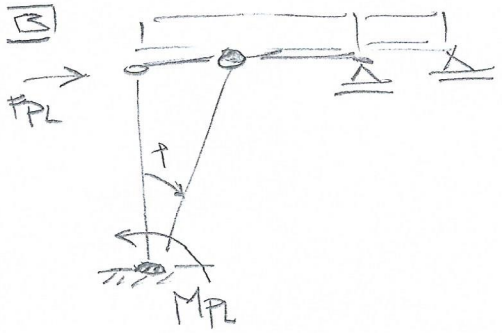
A



$q_{PL} \cdot 2 \cdot \frac{2}{2} P \cdot 2 = 2 M_{PL} \cdot P$

$q_{PL} = \frac{2}{4} M_{PL} = 14{,}14$ kN/m

B



$F_{PL} \cdot 4P - M_{PL} \cdot P = 0$

$F_{PL} = 7{,}069$ kN ODABRANO

$q \cdot \frac{4}{6} = 7{,}069$

$q = 10{,}603$ kN/m ODABRANO

$z_T = \frac{20 \cdot 60 \cdot 10 + 70 \cdot 10 \cdot 55 + 10 \cdot 80 \cdot 95}{2700} = 49{,}85$ mm

$y = \frac{60 \cdot 20^3}{12} + 60 \cdot 20 \cdot 36{,}85^2 + \frac{10 \cdot 70^3}{12} + 10 \cdot 70 \cdot 8{,}15^2 + \frac{80 \cdot 10^3}{12} + 80 \cdot 10 \cdot 48{,}15^2$

$I_y = 3{,}8632 \cdot 10^6$ mm^4

$W_y = \frac{I_y}{z_{max}} = 72{,}6856$ mm^3

$f_{OB} = \frac{M_{PL}}{M_y} = 1{,}297$

$M_y = W_y \cdot \sigma_T = 21{,}81$ kNm