2. KOLOKVIJ iz "OTPORNOSTI MATERIJALA 2" GRUPA B

SVEUČILIŠTE U ZAGREBU
GRAĐEVINSKI FAKULTET
ZAVOD ZA TEHNIČKU MEHANIKU

27. 05. 2017.

Ime i prezime:_____

1. Čelični konzolni štap ABCD okruglog poprečnog presjeka opterećen je prema slici. Treba dimenzionirati štap po I. i III. teoriji čvrstoće.

$a = 2{,}0\,m$

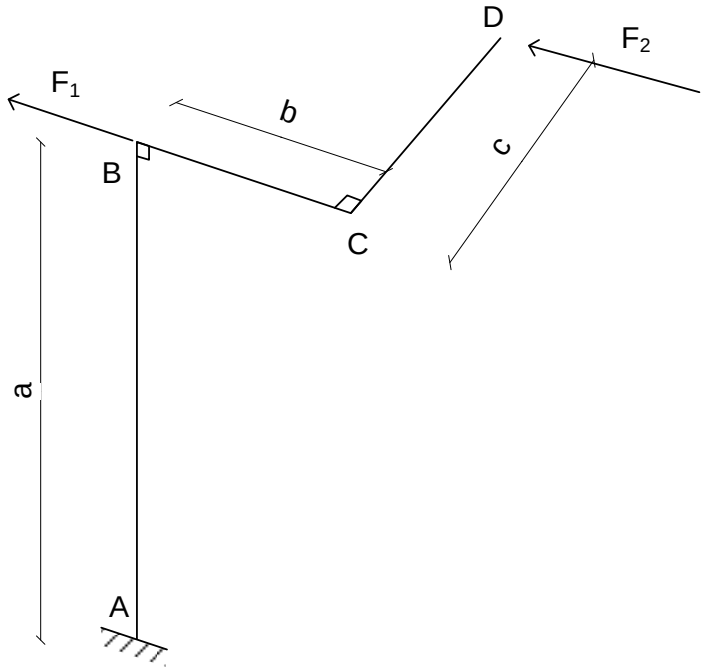
$b = 1{,}0\,m$

$c = 1{,}5\,m$

$F_1 = 0{,}5\,kN$

$F_2 = 1{,}5\,kN$

$\sigma_{dop} = 240\,MPa$



2. Potrebno je odrediti koeficijent sigurnosti sustava ako je zadano:

$\sigma_P = 210\,MPa$

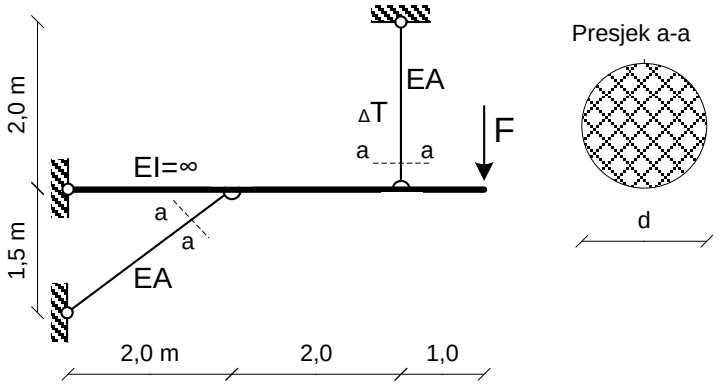
$\Delta T = 50\,K$

$\alpha_T = 1{,}3 \cdot 10^{-5}\,K^{-1}$

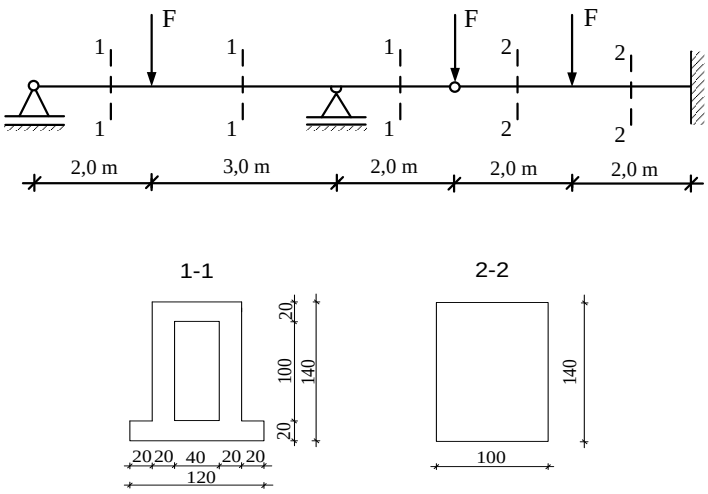
$E = 2{,}1 \cdot 10^5\,MPa$

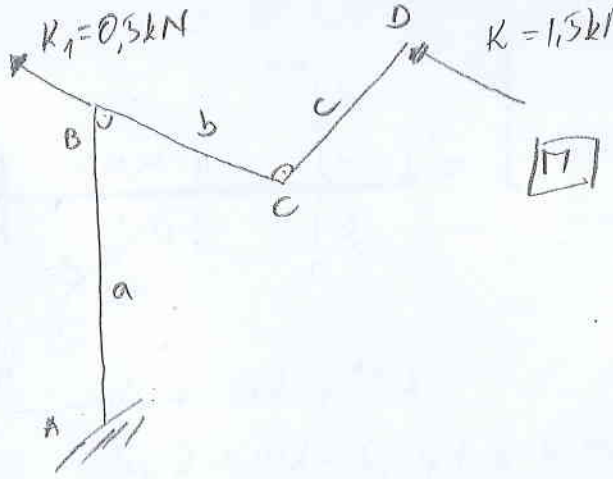
$d = 45\,mm$

$F = 5\,kN$



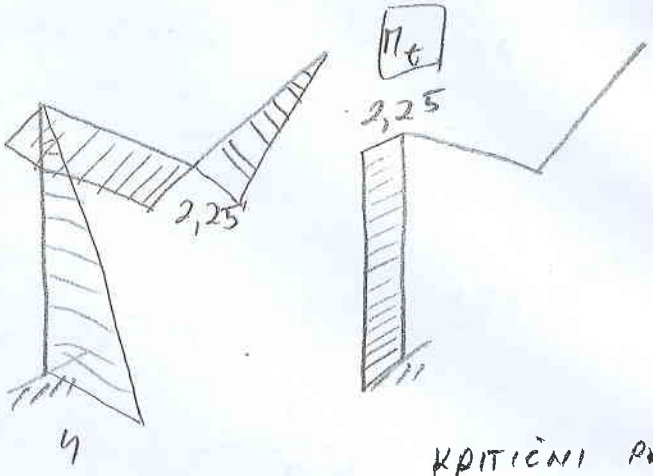
3. Prema teoriji plastičnosti treba odrediti dopušteno opterećenje sile *F*, ako je granica tečenja $\sigma_T = 240\,Mpa$ i koeficijent sigurnosti $k = 1{,}6$.
Koliki je faktor oblika zadanih poprečnih presjeka?





$a = 2m;\ b = 1m,\ c = 1{,}5\ m$

$\sigma_{dop} = 240\ MPa$



KRITIČNI PREREZ A

GLAVNA NAPREZANJA:

$\sigma_s = M_s / W_y$; $\tau = \frac{M_t}{W_p} = \frac{M_t}{2W_y}$

$W_y = \frac{I_y}{d/2} = \frac{\pi d^3}{32}$; $W_p = \frac{I_p}{d/2} = \frac{\pi \cdot d^3}{16}$

$$\sigma_{1,2} = \frac{\sigma_s}{2} \pm \frac{1}{2}\sqrt{\sigma_s^2 + 4\tau^2} = \frac{M_s}{2W_y} \pm \frac{1}{2}\sqrt{\frac{M_s^2}{W_y^2} + 4\frac{M_t^2}{4W_y^2}} = \frac{1}{2W_y}\left(M_s \pm \sqrt{M_s^2 + M_t^2}\right)$$

1. TEORIJA ČVRSTOĆE:

$$\sigma_{ek} = \sigma_{max} = \sigma_1 = \frac{1}{2W_y}\left(M_s + \sqrt{M_s^2 + M_t^2}\right) \le \sigma_{dop}$$

$$W_y \ge \frac{1}{2\sigma_{dop}}\left(M_s + \sqrt{M_s^2 + M_t^2}\right) = 17894{,}36\ mm^3$$

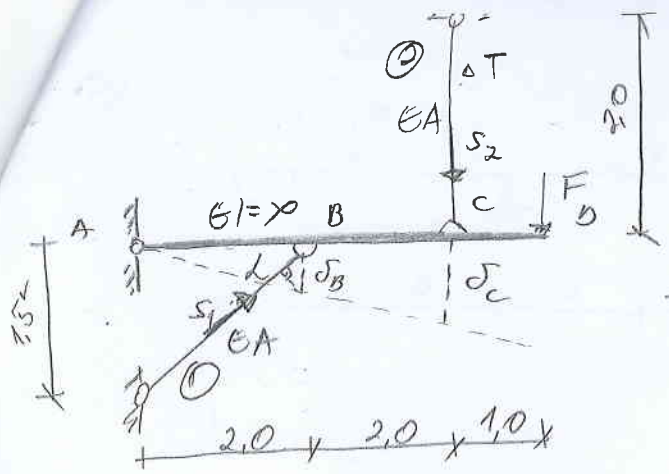
$$\boxed{d \ge 56{,}70\ mm}$$

3. TEORIJA ČVRSTOĆE

$$\sigma_{ek} = \sigma_1 - \sigma_2 = \frac{1}{2W_y}\left(M_s + \sqrt{M_s^2 + M_t^2}\right) - \frac{1}{2W_y}\left(M_s - \sqrt{M_s^2 + M_t^2}\right) \le \sigma_{dop}$$

$$\sigma_{ek} = \left(\sqrt{M_s^2 + M_t^2}\right) / W_y \le \sigma_{dop}$$

$$W_y \ge \frac{\sqrt{M_s^2 + M_t^2}}{\sigma_{dop}} = 19122{,}45\ mm^3 \rightarrow \boxed{d \ge 57{,}94\ mm}$$



$d = 45$ mm
$\Delta T = 50$ K
$\alpha_T = 1{,}3 \cdot 10^{-5}\ K^{-1}$
$E = 2{,}1 \cdot 10^5$ MPa
$\sigma_p = 210$ MPa

$\sin\alpha = 0{,}6$
$\cos\alpha = 0{,}8$
$I_{min} = \frac{d^4\pi}{64} = 20{,}128 \cdot 10^{-8}$

U.R. $\quad S_1 \cdot 2 \cdot \sin\alpha = S_2 \cdot 4 + K \cdot 5 \Rightarrow S_2 = S_1 \cdot 0{,}5 \cdot \sin\alpha - 1{,}25 \cdot K$

U.D. $\quad \frac{\delta_C}{4{,}0} = \frac{\delta_B}{2{,}0} \qquad \delta_B = \frac{S_1 \cdot l_1}{EA \sin\alpha}\ ; \quad \delta_C = -\frac{S_2 \cdot l_2}{E \cdot A_2} + \alpha_T \cdot \Delta T \cdot l_2$

$$-S_2 \cdot l_2 \sin\alpha + \alpha_T \cdot \Delta T \cdot l_2 EA \sin\alpha = 2 S_1 \cdot l_1$$

$$-S_1 \cdot 0{,}5 \sin^2\alpha \cdot l_2 + K \cdot 1{,}25 \cdot l_2 \sin\alpha + \alpha_T \cdot \Delta T \cdot l_2 EA \sin\alpha = 2 S_1 \cdot l_1$$

$$S_1 = \frac{1{,}25 \cdot K \cdot l_2 \sin\alpha + \alpha_T \cdot \Delta T \cdot l_2 EA \sin\alpha}{2 l_1 + 0{,}5 \cdot l_2 \sin^2\alpha}$$

$S_1 = 50$ kN ; $S_2 = 8{,}75$ kN

$$i_{min} = \sqrt{\frac{d^4\pi/64}{d^2\pi/4}} = \frac{d}{4} = 11{,}25\ ; \quad \lambda_p = \sqrt{\frac{\pi^2 E}{\sigma_p}} = 99{,}35$$

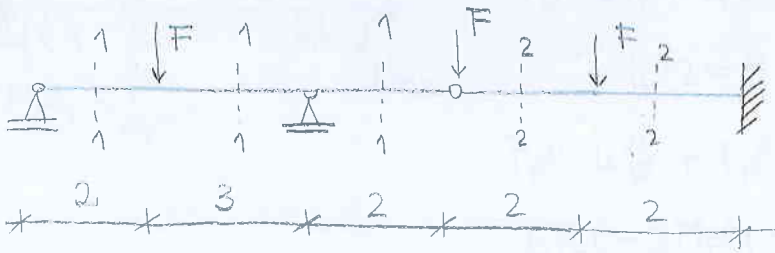
$$\lambda_1 = \frac{l_i^{(1)}}{i_{min}} = \frac{2500}{11{,}25} = 222{,}2\ ; \quad \lambda_2 = \frac{l_i^{(2)}}{i_{min}} = \frac{2000}{11{,}25} = 177{,}78$$

$$F_{ka}^{(1)} = \frac{\pi^2 \cdot E \cdot I_{min}}{l_i^2} = 66{,}75\ kN \qquad k_1 = \frac{F_{ka}^{(1)}}{S_1} = \frac{66{,}75}{50} = 1{,}34$$

$$F_{ka}^{(2)} = \frac{\pi^2 \cdot E \cdot I_{min}}{l_i^2} = 104{,}30\ kN \qquad k_2 = \frac{F_{ka}^{(2)}}{S_2} = 11{,}92$$

KOEFICIJENT SUSTAVA JE $k_1 = 1{,}34$

3.)

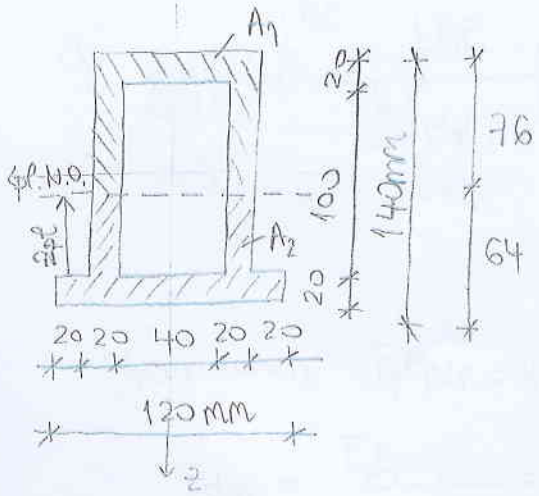


$\sigma_T = 240\,MPa$

$k = 1{,}6$

$F_{dop} = ?$

1–1



$A = 8000\,mm^2$

$A_1 = A_2 = \frac{A}{2} = 4000 = 120 \cdot 20 + 2pl \cdot 2 \cdot 20$

$\Rightarrow 2pl = 40\,mm$

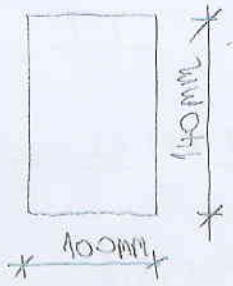
$W_{pl} = S_1 + S_2 = (20 \cdot 80 \cdot 70 + 2 \cdot 20 \cdot 60 \cdot 30) +$

$+ (2 \cdot 40 \cdot 20 \cdot 20 + 20 \cdot 120 \cdot 50) = 184000 + 152000$

$W_{pl_1} = 336000\,mm^3$

$M_{pl_1} = W_{pl_1} \cdot \sigma_T = 80{,}64\,kNm$

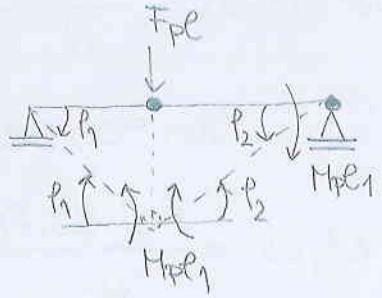
2–2



$W_{pl_2} = 2 \cdot 100 \cdot \frac{140}{2} \cdot \frac{140}{4} = 490000\,mm^3$

$M_{pl_2} = W_{pl_2} \cdot \sigma_T = 117{,}6\,kNm$

a)



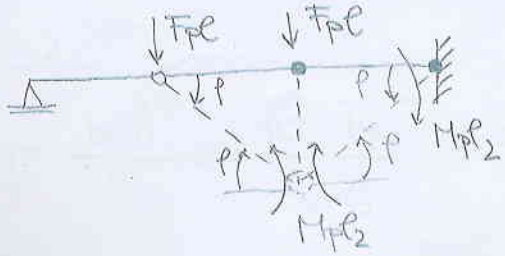
$F_{pl} \cdot 2p_1 - 2M_{pl_1} \cdot p_2 - M_{pl_1} \cdot p_1 = 0$

$2p_1 = 3p_2 \rightarrow p_1 = \frac{3}{2} p_2$

$3F_{pl} \cdot \cancel{p_2} = \frac{7}{2} M_{pl_1} \cdot \cancel{p_2}$

$F_{pl} = \frac{7}{6} M_{pl_1} = 94{,}08\,kN$

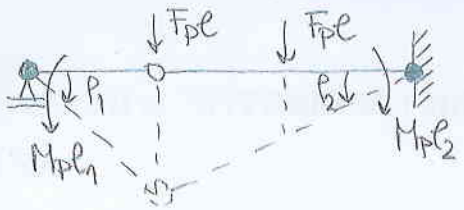
b)



$F_{pl} \cdot \cancel{2p} - 3M_{pl_2} \cdot \cancel{p} = 0$

$F_{pl} = \frac{3}{2} M_{pl_2} = 176{,}4\,kN$

c)



$F_{pl} \cdot 2f_1 + F_{pl} \cdot 2f_2 - M_{pl1} \cdot f_1 - M_{pl2} \cdot f_2 = 0$

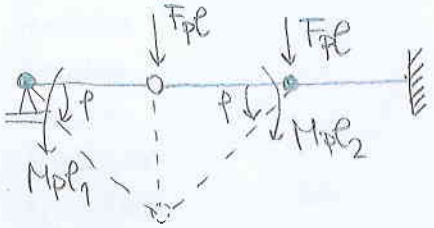
$f_1 \cdot 2 = f_2 \cdot 4 \rightarrow f_1 = 2f_2$

$F_{pl} \cdot 4f_2 + F_{pl} \cdot 2f_2 - M_{pl1} \cdot 2f_2 - M_{pl2} \cdot f_2 = 0$

$6F_{pl} - 2M_{pl1} - M_{pl2} = 0$

$F_{pl} = \frac{2M_{pl1} + M_{pl2}}{6} = 46,48\,kN$ → mjerodavno

d)



$F_{pl} \cdot 2f - M_{pl1} \cdot f - M_{pl2} \cdot f = 0$

$F_{pl} = \frac{M_{pl1} + M_{pl2}}{2} = 99,12\,kN$

Mjerodavno: $F_{plmin} = 46,48\,kN$

$F_{dop} = \frac{F_{plmin}}{k} = \frac{46,48}{1,6} = 29,05\,kN$

FAKTOR OBLIKA

1-1

$z_T = \frac{20 \cdot 120 \cdot 10 + 2 \cdot 20 \cdot 100 \cdot 70 + 2 \cdot 80 \cdot 130}{8000} = 64\,mm$

$I_y = \frac{120 \cdot 20^3}{12} + 20 \cdot 120 \cdot 54^2 + 2 \cdot \left(\frac{20 \cdot 100^3}{12} + 20 \cdot 100 \cdot 6^2 \right) + \frac{80 \cdot 20^3}{12} + 80 \cdot 20 \cdot 66^2$

$= 17,58 \cdot 10^6\,mm^4$

$W_y = \frac{17,58 \cdot 10^6}{76} = 23,13 \cdot 10^4\,mm^3$

$f_{oB} = \frac{W_{pl}}{W_y} = \frac{336000}{23,13 \cdot 10^4} = 1,45$

2-2

$W_y = \frac{b \cdot h^2}{6} = \frac{100 \cdot 140^2}{6} = 326666,67\,mm^3$

$f_{oB} = \frac{W_{pl}}{W_y} = \frac{490000}{326666,67} = 1,49 \approx 1,50$