Otpornost materijala 2
2. kolokvij
18. svibnja 2016.

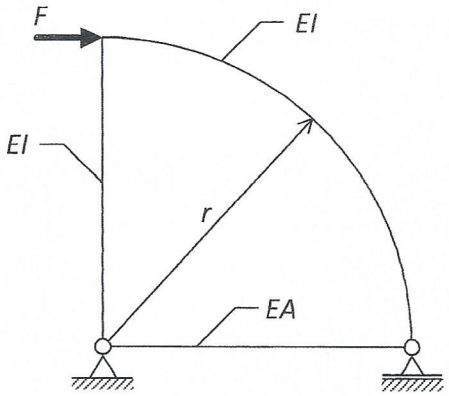
C

Ime i prezime: _____

1. Za zadani sustav odredite i nacrtajte M, T i N dijagrame primjenom principa o minimumu potencijalne energije deformacije.

35

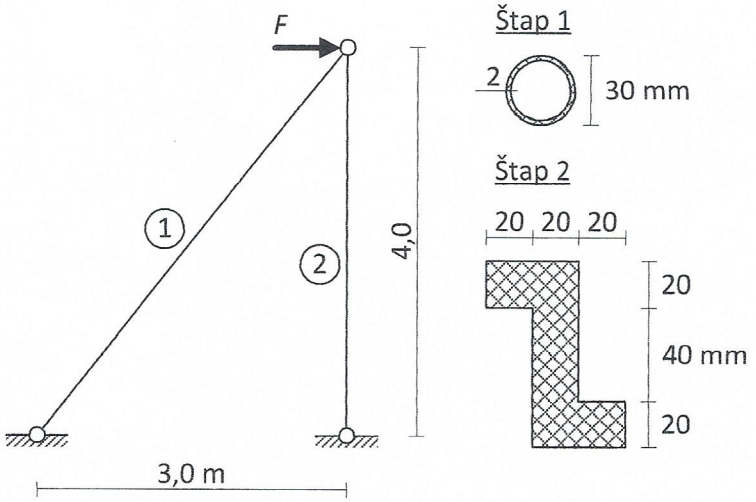
Zadano:
$F = 20$ kN
$I = 5{,}2 \cdot 10^5$ mm^4
$A = 78{,}5$ mm^2
$r = 2$ m
$E = 210$ GPa



2. Za zadani sustav odredite dopušteno opterećenje iz uvjeta čvrstoće i uvjeta stabilnosti štapova.

35

Zadano:
$\sigma_{vl,dop} = 160$ MPa
$\sigma_p = 210$ MPa
$E = 200$ GPa
$k_{izvijanje} = 2{,}1$

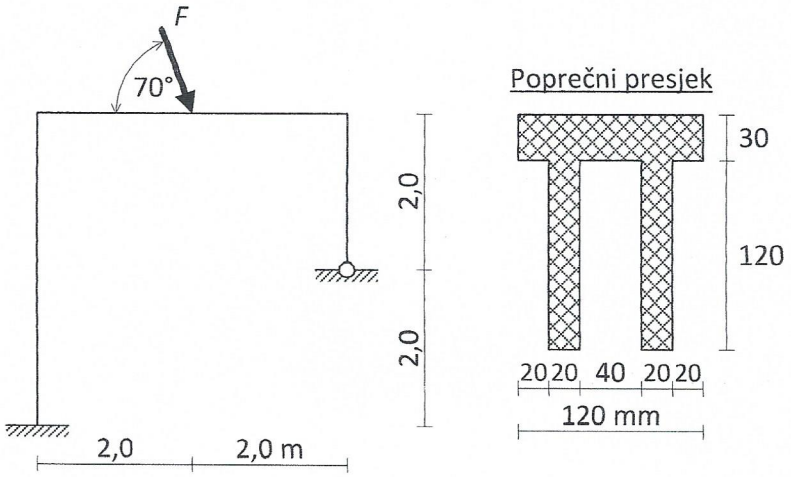


3. Za zadani sustav odredite dopušteno opterećenje prema teoriji plastičnosti.

30

Zadano:

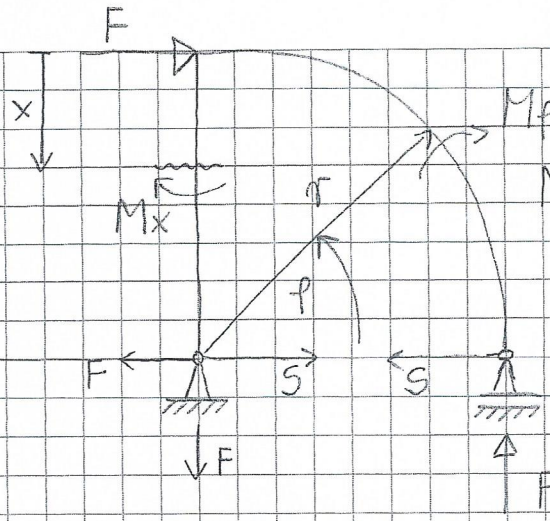
$\sigma_T = 210$ MPa
k = 1,2



ZADATAK 1

Zadano:

$F = 20\ kN$
$A = 78,5\ mm^2$
$J = 5,2 \cdot 10^5\ mm^4$
$r = 2\ m$
$E = 210\ GPa$



$$M_\varphi = -S \cdot r \sin\varphi + Fr(1-\cos\varphi)$$

$$M_x = F \cdot r - S \cdot (r - x) - F \cdot x$$

$$\frac{\partial M_\varphi}{\partial S} = -r\sin\varphi$$

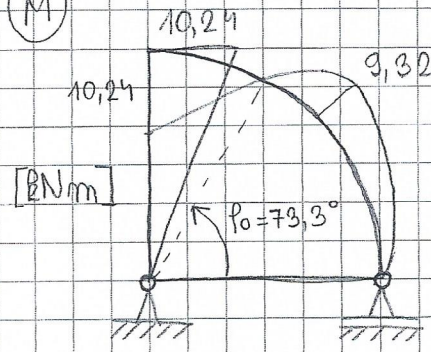
$$\frac{\partial M_x}{\partial S} = x - r$$

$$\frac{\partial \Delta l}{\partial x} = -\frac{S \cdot r}{E \cdot A}$$

$$\frac{1}{EJ}\int_0^{\pi/2}\left[-Sr\sin\varphi + Fr(1-\cos\varphi)\right]\cdot(-r\sin\varphi)\,r\,d\varphi + \frac{1}{EJ}\int_0^{r}\left[Fr - S(r-x) - F\cdot x\right]\cdot(x-r)\,dx = -\frac{S\cdot r}{E\cdot A}$$

$$S = \frac{\frac{5}{6}F}{\frac{1}{3} + \frac{\pi}{4} + \frac{J}{Ar^2}} = 0,744F \Rightarrow S = 14,88\ kN \Rightarrow \sigma = 189,6\ MPa$$

(M)



$$M_\varphi = -Sr\sin\varphi + Fr(1-\cos\varphi)$$

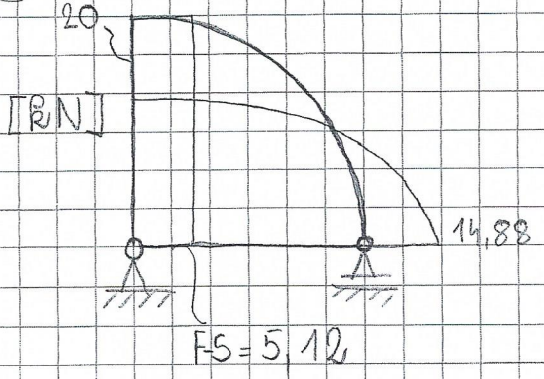
$$M(\varphi = 90°) = 10,24\ kNm$$

$$M(\varphi = 45°) = -9,32\ kNm$$

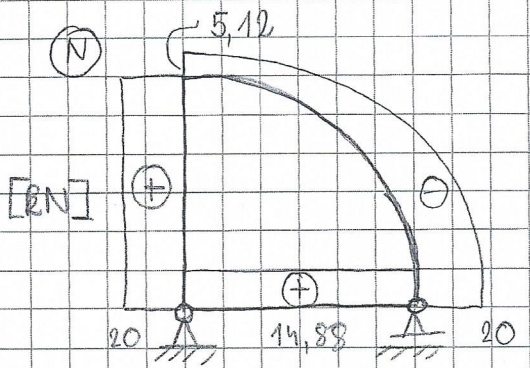
$$M_x = Fr - S(r-x) - Fx$$

$$M(x = 0) = 10,24\ kNm$$

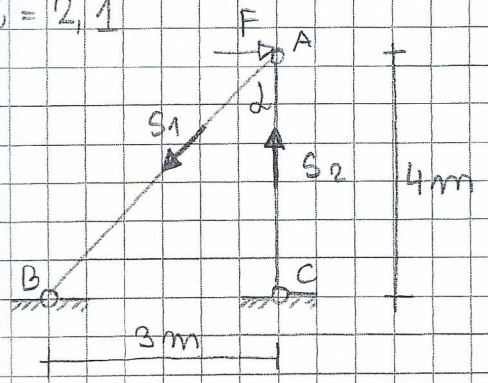
(T)



(N)



$\sigma_{vl,dop} = 160\ MPa$
$\sigma_P = 210\ MPa$
$E = 2 \cdot 10^5\ MPa$
$k_i = 2,1$



$\alpha = 36,87°$

$\sin\alpha = 0,6$

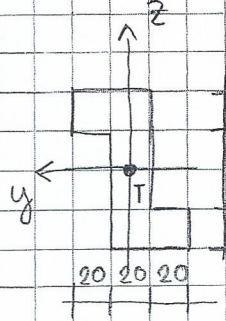
$\cos\alpha = 0,8$

$\operatorname{tg}\alpha = \frac{3}{4}$

$\Sigma F_H = 0: \quad S_1 = \frac{F}{\sin\alpha}$

$\Sigma F_V = 0: \quad S_2 = S_1 \cos\alpha = \frac{F}{\sin\alpha} \cdot \cos\alpha = \frac{F}{\operatorname{tg}\alpha}$

2-2



$A = 2400\ mm^2$

$J_y = 1,6 \cdot 10^6\ mm^4$

$J_z = 0,4 \cdot 10^6\ mm^4$

$J_{yz} = 0,48 \cdot 10^6\ mm^4$

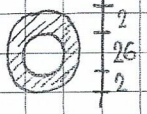
$J_u = 1,768 \cdot 10^6\ mm^4$

$J_v = 0,232 \cdot 10^6\ mm^4 = J_{min}$

$i_{min} = \sqrt{\frac{J_{min}}{A}}$

$i_{min} = 9,83\ mm$

1-1



$A = \frac{(30^2 - 26^2) \cdot \pi}{4} = 175,98\ mm^2$

ŠTAP AB → uvjet čvrstoće

$\sigma = \frac{S_1}{A} \leq \sigma_{vl,dop}; \quad \frac{F}{\sin\alpha A} \leq \sigma_{vl,dop} \rightarrow \underline{F \leq 16,89\ kN}$

ŠTAP AC → uvjet stabilnosti $\quad l_i = l = 4\ m$

$S_{KR}^{AC} = \frac{\pi^2 E J_{min}}{l_i^2} = 28,62\ kN$

$\lambda_P = \sqrt{\frac{\pi^2 E}{\sigma_P}} \approx 97$

$\lambda = \frac{l_i}{i_{min}} = 407 > \lambda_P \checkmark$

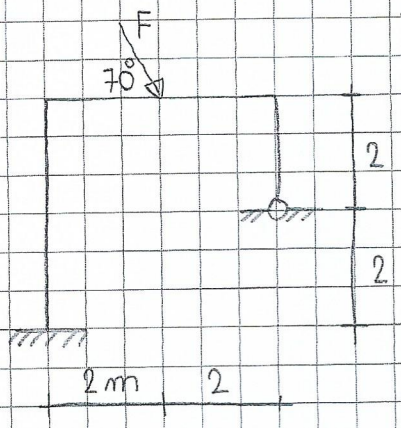
$S_{dop}^{AC} = \frac{S_{KR}^{AC}}{k_i} = 13,63\ kN$

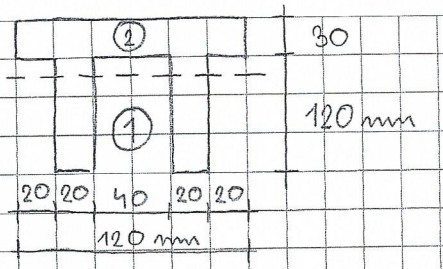
$S_2 = \frac{F}{\operatorname{tg}\alpha} \leq S_{dop}^{AC} \Rightarrow \underline{F \leq 10,22\ kN}$

Mjerodavno!

ZADATAK 3

$\sigma_T = 210$ MPa
$R = 1,2$





$A = 8400\ mm^2$

$\frac{A}{2} = z_{pl} \cdot 20 \cdot 2$

$z_{pl} = 105$ mm

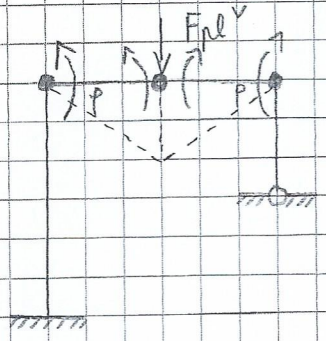
$S_1 = 220500\ mm^3$

$S_2 = 112500\ mm^3$

$W_{pl} = S_1 + S_2 = 333000\ mm^3$

$M_{pl} = W_{pl} \cdot \sigma_T = 69,93$ kN m

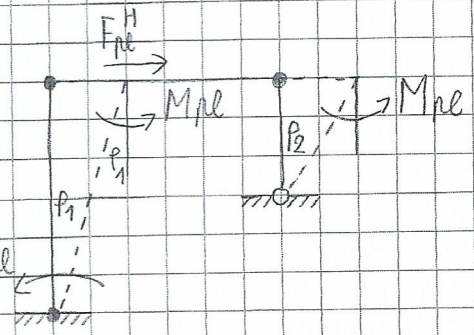
a)



$F_{pl}^V \cdot f \cdot 2 = 4 M_{pl} \cdot f$; $F_{pl}^V = F_{pl} \sin 70° = 0,94 F_{pl}$

$F_{pl} = 2,13 M_{pl} = 148,95$ kN

b)



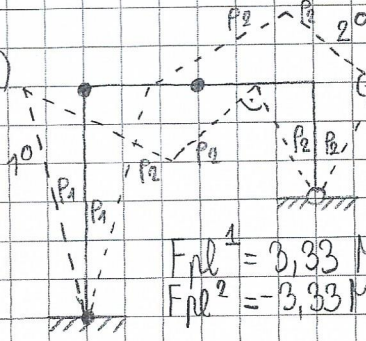
$P_1 \cdot 4 = P_2 \cdot 2$; $P_1 = 0,5 P_2$; $P_2 = 2 P_1$

$F_{pl}^H \cdot P_1 \cdot 4 = 2 M_{pl} P_1 + M_{pl} P_2$; $F_{pl}^H = F_{pl} \cos 70° = 0,34 F_{pl}$

$1,36 F_{pl} \cdot f = 2 M_{pl} f + 2 M_{pl} f = 4 M_{pl}$

$F_{pl} = 2,94 M_{pl} = 205,59$ kN

c)



Nije mjerodavno!

$F_{pl}^1 = 3,33 M_{pl} = 232,87$ kN

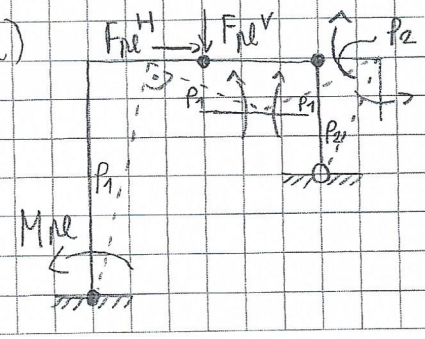
$F_{pl}^2 = -3,33 M_{pl} = -232,87$ kN

① $-F_{pl}^H \cdot P_1 \cdot 4 + F_{pl}^V \cdot P_2 \cdot 2 = 2 M_{pl} P_1 + 3 M_{pl} P_2$

$-F_{pl}^H \cdot 2 P_2 + F_{pl}^V \cdot 2 P_2 = M_{pl} P_2 + 3 M_{pl} P_2$

$F_{pl}^{1°} = 3,33 M_{pl}$

d)



$F_{pl}^H \cdot P_1 \cdot 4 + F_{pl}^V \cdot P_1 \cdot 2 = 4 M_{pl} \cdot P_1 + M_{pl} P_2$

$0,34 F_{pl} \cdot 4 f + 0,94 F_{pl} \cdot 2 f = 4 M_{pl} f + 2 M_{pl} f$

$F_{pl} = 1,85 M_{pl} = 129,37$ kN $= F_{pl,min}$

$F_{dop} = F_{pl,min} / R = 107,81$ kN